MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering & Computer Science
6.041/6.431: Probabilistic Systems Analysis
(Spring 2006)

Recitation 08 Answers
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1. (a) The marginal distributions are obtained by integrating the joint distribution along the X and Y axes and is shown in the following figure.

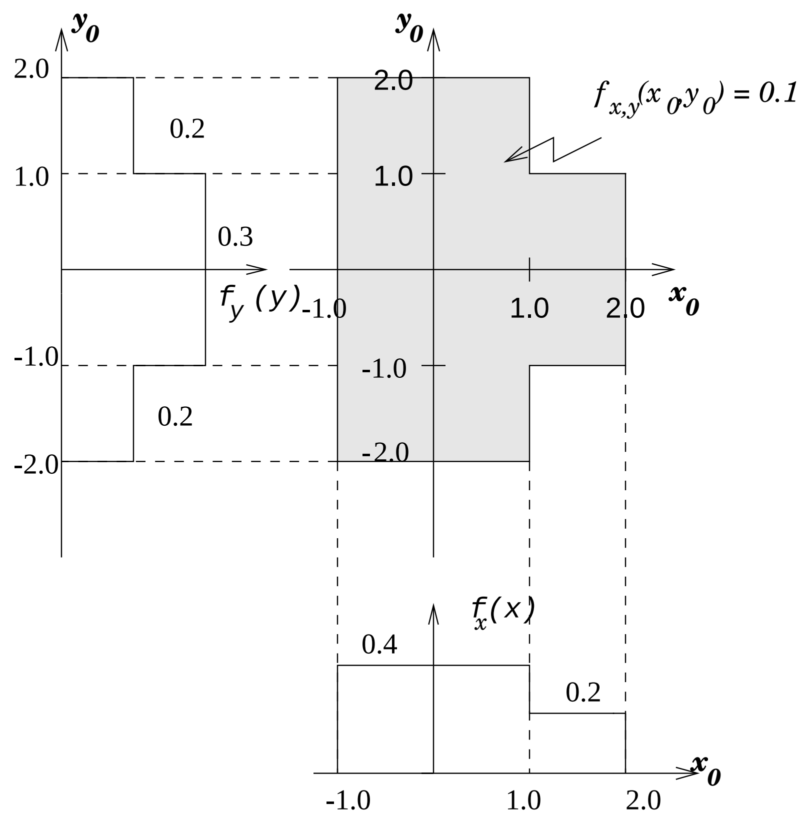

Figure 1: Marginal probabilities $f_X(x)$ and $f_Y(y)$ obtained by integration along the y and x axes respectively

The conditional PDFs are as shown in the figure below.

(b) X and Y are **NOT** indepenent since $f_{XY}(x, y) \neq f_X(x)f_Y(y)$. Also, from the figures we have $f_{X|Y}(x|y) \neq f_X(x)$.

(c)

$$f_{X,Y|A}(x, y) = \begin{cases} \frac{f_{X,Y}((x,y)}{\mathbf{P}(A)} & (x, y) \in A \\ 0 & \text{otherwise} \end{cases}$$
$$= \begin{cases} \frac{0.1}{\pi 0.1} & (x, y) \in A \\ 0 & \text{otherwise} \end{cases}$$

(d)

$$\mathbf{E}[X|Y = y] = \left\{ \begin{array}{ll} 0 & -2.0 \leq y \leq -1.0 \\ \frac{1}{2} & -1.0 \leq y \leq 1.0 \\ 0 & 1.0 \leq y \leq 2.0 \end{array} \right\}$$

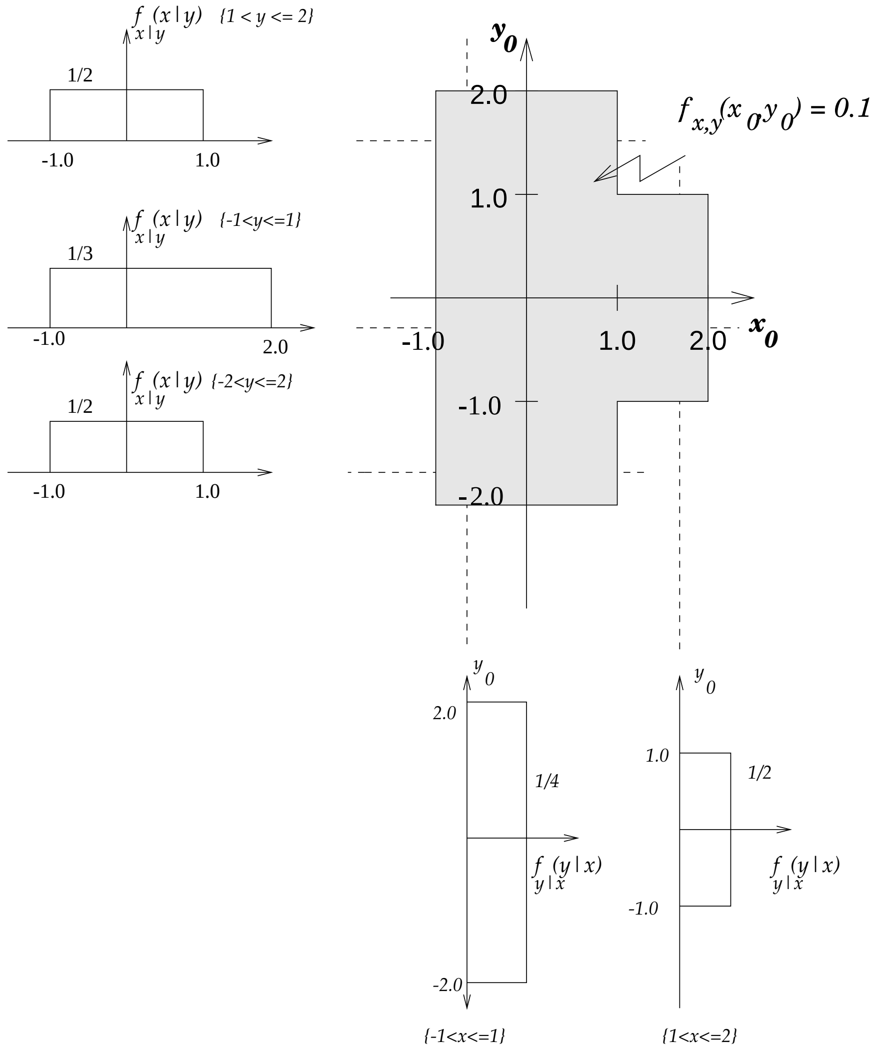

Figure 2: Conditional Probabilities

The conditional variance $var(X|Y = y)$ is given by

$$\text{var}(X|Y = y) = \left\{ \begin{array}{ll} \frac{4}{12} & -2.0 \le y \le -1.0 \\ \frac{9}{12} & -1.0 \le y \le 1.0 \\ \frac{4}{12} & 1.0 \le y \le 2.0 \end{array} \right\}$$

2. (a) We have $a = 1/800$, so that

$$f_{XY}(x, y) = \left\{ \begin{array}{ll} 1/1600 & \text{if } 0 \le x \le 40 \text{ and } 0 \le y \le 2x \\ 0, & \text{otherwise.} \end{array} \right\}$$

(b) $\mathbf{P}(Y > X) = 1/2$

(c) Let $Z = Y - X$. We have

$$f_Z(z) = \left\{ \begin{array}{ll} \frac{1}{1600}z + \frac{1}{40}, & \text{if } -40 \le z \le 0, \\ -\frac{1}{1600} + \frac{1}{40}, & \text{if } 0 \le z \le 40, \\ 0, & \text{otherwise.} \end{array} \right\}$$

$\mathbf{E}[Z] = 0$.