

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
 Department of Electrical Engineering & Computer Science
6.041/6.431: Probabilistic Systems Analysis
 (Spring 2006)

Recitation 07
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1. The random variable X is exponentially distributed with parameter λ .

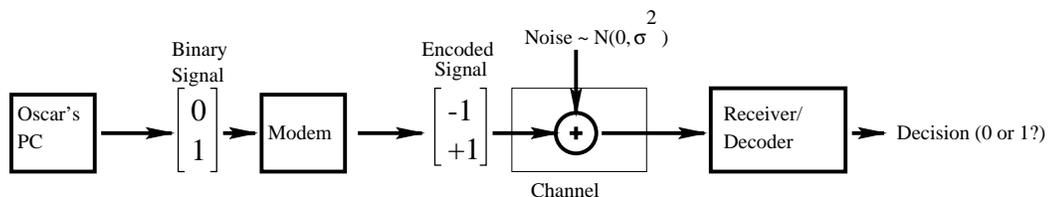
$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

- (a) Calculate $\mathbf{E}[X]$, $\text{var}(X)$ and find $\mathbf{P}(X \geq E[X])$.

Hint:

$$\mathbf{P}(x \geq k) = \int_k^{\infty} f_X(x) dx$$

- (b) Find $\mathbf{P}(X \geq t + k | X > t)$.
2. You are allowed to take a certain test three times, and your final score will be the maximum of the test scores. Your score in test i , where $i = 1, 2, 3$ takes one of the values from i to 10 with equal probability $1/(11 - i)$, independently of the scores in other tests. What is the PMF of the final score?
3. Wanting to browse the net, Oscar uses his high-speed 300-baud modem to connect through his Internet Service Provider. The modem transmits bits in such a fashion that -1 is sent if a given bit is zero and +1 is sent if a given bit is one. The telephone line has additive zero-mean Gaussian (normal) noise with variance σ^2 (so, the receiver on the other end gets a signal which is the sum of the transmitted signal and the channel noise). The value of the noise is assumed to be independent of the encoded signal value.



We assume that the probability of the modem sending -1 is p and the probability of sending 1 is $1 - p$.

- (a) Suppose we conclude that an encoded signal of -1 was sent when the value received on the other end of the line is less than a (where $-1 < a < +1$), and conclude $+1$ was sent when the value is more than a . What is the probability of making an error?
- (b) Answer part (a) assuming that $p = 2/5$, $a = 1/2$ and $\sigma^2 = 1/4$.