Massachusetts Institute of Technology
Department of Electrical Engineering & Computer Science
6.041/6.431: Probabilistic Systems Analysis
(Spring 2006)

Recitation 4 Solutions
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1. Problem 1.45, page 66. See online solutions.

2. (a) The N_is are the numbers of times each ball is selected, so the sum of the N_is must be the total number of draws from the urn.

   (b) There is a nice visualization for this. Make a dot for each drawn ball, grouped according the ball's identity:

   $$\underbrace{\cdots}_{N_1}\ \underbrace{\cdots}_{N_2}\ \underbrace{\quad}_{\cdots}\ \underbrace{\cdots}_{N_n}$$

   There is a total of k dots put in n groups. Think of there being a separator mark between groups, so there are $n-1$ separator marks:

   $$\underbrace{\cdots}_{N_1}\ |\ \underbrace{\cdots}_{N_2}\ |\ \underbrace{\quad}_{\cdots}\ |\ \underbrace{\cdots}_{N_n}$$

   This gives a grand total of $k+n-1$ dots and marks. The number of solutions is the number of ways to place k dots in $k+n-1$ locations: $\binom{k+n-1}{k}$.

   (c) If we know that $X_1 = \ell$, then applying the result of the previous part to the remaining balls and remaining draws from the urn gives $\binom{(k-\ell)\ +(n-1)-1}{k-\ell}$ as the desired number. Since this is just a way of breaking down the problem of the previous part, we have

   $$\sum_{\ell=0}^{k} \binom{k+n-\ell-2}{k-\ell} = \binom{k+n-1}{k}.$$

3. (a) Students might say they are equal (both being the average number of students per bus) or have the correct intuition.

   (b) Make sure to define the PMFs of X and Y. Then

   $$\begin{aligned} E[X] &= \frac{40}{148}\cdot 40 + \frac{33}{148}\cdot 33 + \frac{25}{148}\cdot 25 + \frac{50}{148}\cdot 50 \ \approx\ 39.3 \\ E[Y] &= \frac{1}{4}\cdot 40 + \frac{1}{4}\cdot 33 + \frac{1}{4}\cdot 25 + \frac{1}{4}\cdot 50 \ =\ 37 \end{aligned}$$