

Recitation 3
February 16, 2006

1. A particular class has had a history of low attendance. The annoyed professor decides that she will not lecture unless at least k of the n students enrolled in the class are present. Each student will independently show up with probability p_g if the weather is good, and with probability p_b if the weather is bad. Given the probability of bad weather on a given day, calculate the probability that the professor will teach her class on that day.
2. Consider two coins, a blue and a red one. We choose one of the two coins at random, each being chosen with probability $1/2$. Let H1 be the event that the first toss results in heads, and H2 be the event that the second toss results in heads. The coins are biased: with the blue coin, the probability of heads in any given toss is 0.99, whereas for the red coin it is 0.01.
 - (a) Are the events H1 and H2 (unconditionally) independent?
 - (b) Given that the blue coin was selected, are the events H1 and H2 (conditionally) independent?
3. For each one of the following statements, indicate whether it is true or false, and provide a brief explanation.
 - (a) If $P(A | B) = P(A)$, then $P(B | A^c) = P(B)$.
 - (b) If 5 out of 10 independent fair coin tosses resulted in tails, the events “first toss was tails” and “10th toss was tails” are independent.
 - (c) If 10 out of 10 independent fair coin tosses resulted in tails, the events “first toss was tails” and “10th toss was tails” are independent.
 - (d) If the events A_1, \dots, A_n form a partition of the sample space, and if B, C are some other events, then

$$P(B | C) = \sum_{i=1}^n P(A_i | C)P(B | A_i).$$