LECTURE 4

- Readings: Sections 1.6

Lecture outline

- Principles of counting
  - Many examples
- Binomial probabilities

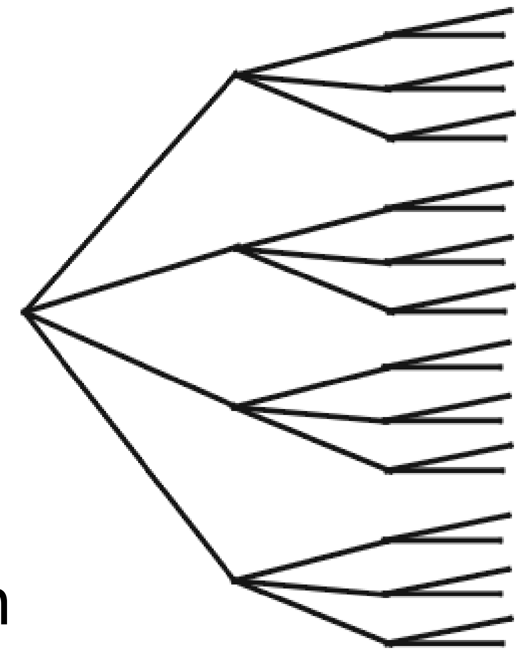
Discrete Uniform Law

- Let all sample points be equally likely.
- Then,

$$\mathbf{P}(A) = \frac{\text{number of elements of } A}{\text{total number of sample points}}$$

- Just count…

Basic Counting Principle



- r steps
- n_i choices at step i
- Number of choices is:

$$n_1 n_2 \cdots n_r$$

- Number of license plates with 3 letters a 4 digits =
- … if repetition is prohibited =
- **Permutations**: Number of ways of ordering n elements is=
- Number of subsets of $\{1,\ldots, n\} =$

Example

- Probability that six rolls of a six-sided die all give different numbers?
  - Number of outcomes that make the event happen=
  - Number of elements in the sample space=
  - Answer=

Combinations

- $\binom{n}{k}$: number of k-element subsets of a given n element set.
- Two ways of constructing an ordered sequence of k **distinct** items:
  - Choose the k items one at a time: $n(n-1)\cdots(n-k+1) = \frac{n!}{(n-k)!}$ choices
  - Choose k items, then order them ($k!$ possible orders)
- Hence: $\binom{n}{k} \cdot k! = \frac{n!}{(n-k)!}$ $\binom{n}{k} = \frac{n!}{k!(n-k)!}$
- Identity: $\sum_{k=0}^{n} \binom{n}{k} =$

Summary: Different Ways of Sampling

Draw k balls from an urn with n numbered balls.

- Sampling with replacement and ordering:
- Sampling without replacement and ordering:
- Sampling w/o replacement and w/o ordering:
- Sampling w/ replacement and w/o ordering:

Binomial Probabilities

- n independent coin tosses
  - $\mathbf{P}(H) = p$
- $\mathbf{P}(HTTHHH) =$
- $\mathbf{P}(\text{sequence}) = p^{\#\ \text{heads}}(1-p)^{\#\ \text{tails}}$

$$\mathbf{P}(k\ \text{heads}) = \sum_{k\ \text{head seq.}} \mathbf{P}(\text{seq.})$$

$$= (\#\ \text{of}\ k\text{-head seqs.}) \cdot p^k(1-p)^{n-k}$$

$$= \binom{n}{k} p^k(1-p)^{n-k}$$

Coin Tossing Problem

- Event B: 3 out of 10 tosses were "heads".
  - What is the (conditional) probability that the first 2 tosses were heads, given that B occurred?
- All outcomes in conditioning set B are equally likely:
  - Probability: $p^3(1 - p)^7$
  - Conditional probability law is uniform.
- Number of outcomes in B:
- Out of the outcomes in B,
  how many start with HH?

Partitions

- 52-card deck, dealt to 4 players.
- Find **P**(each gets an ace)
- Count size of the sample space (possible combination of "hands")
- Count number of ways of distributing the four aces: $4 \cdot 3 \cdot 2$
- Count number of ways of dealing the remaining 48 cards
- Answer: