6.041: Probabilistic Systems Analysis
6.431: Applied Probability

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Course Outline

- Introductions
- Recitation Assignment
- Tutorial Assignment
- Text Book
  - Introduction to Probability: Bertsekas and Tsitsiklis
- Grading Policy:
  - Q1: 25%,
  - Q2: 25%,
  - Final: 35%,
  - Homework: 10%,
  - Participation: 5%.
- Homework Policy
- **Read the General Information Handout**

LECTURE 1

- Readings: Sections 1.1, 1.2

Lecture outline

- Motivation
- Sample space of an experiment
  - Examples
- Axioms of probability
  - More examples

Motivation

- Why do we study probability theory?
  - An effective model of uncertainty
  - Decision Making under uncertainty

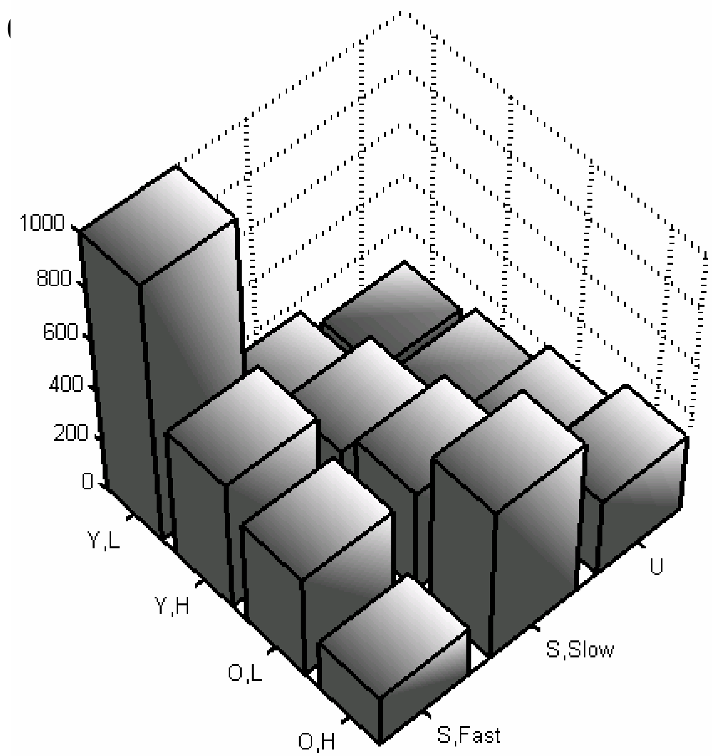
- Examples:
  - Measurement sensors
  - Waiting time at a Bank's teller.
  - Value of a stock at a given day.
  - Outcome of a medical procedure.
  - A customer buying behavior.

- One Decision Making Process: Collect Data, Model the Phenomenon, Extrapolate and make decisions.

From Frequency to Probability (1)

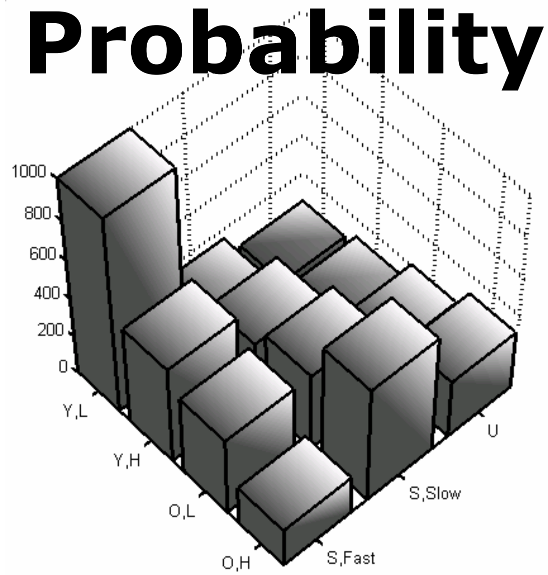
- The time of recovery (Fast, Slow, Unsuccessful) from an ACL knee surgery was seen to be a function of the patient's age (Young, Old) and weight (Heavy, Light). The medical department at MIT
data:

	S,Fast	S,Slow	U
Y,L	1000	150	50
Y,H	500	300	100
O,L	400	400	200
O,H	200	600	300



From Frequency to Probability (2)

	S, Fast	S, Slow	U
Y,L	1000	150	50
Y,H	500	300	100
O,L	400	400	200
O,H	200	600	300



- What is the “likelihood” that a 40 years old man (Old!) will have a successful surgery with a speedy recovery?
- If a patient undergoes an operation, what is the “likelihood” that the result is unsuccessful?
- Need a measure of “likelihood”.
- Ingredients: Sample space, Events, Probability.

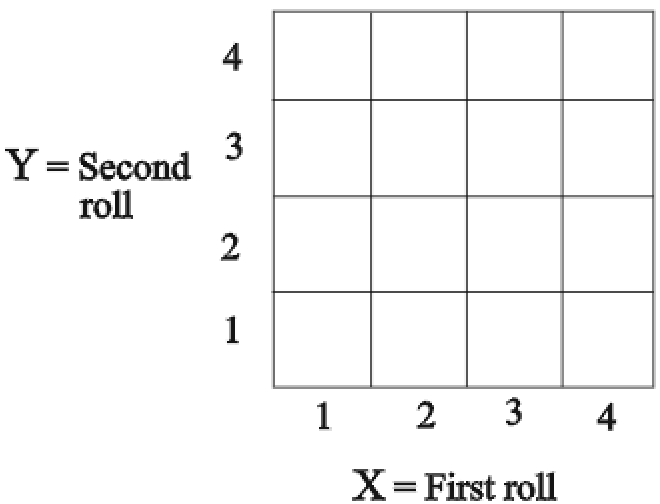
Think of Probability as Frequency....

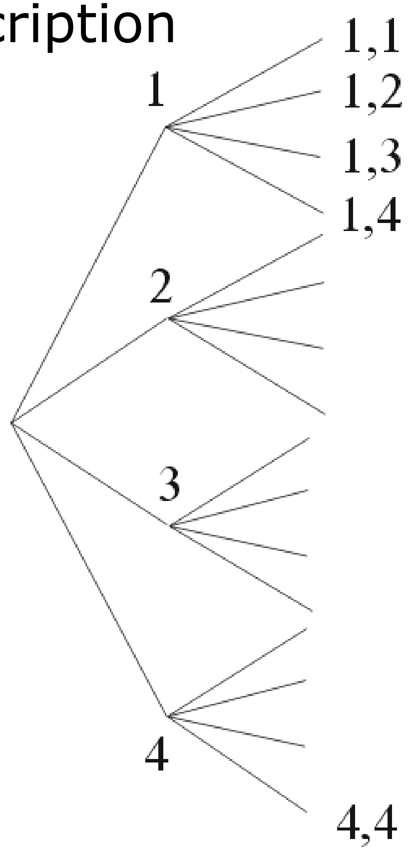
Sample Space

- List of possible outcomes
- List must be:
  - Mutually exclusive
  - Collectively exhaustive
  - At the “right” granularity

Sample Space Example (1)

- Two rolls of a tetrahedral die
  - Sample space vs. sequential description

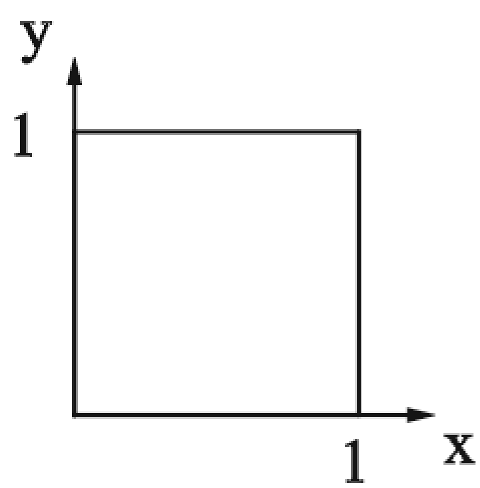




Sample Space Example (2)

- A continuous sample space:

  (x, y) such that $0 \leq x, y \leq 1$



Axioms of probability

- Event: a subset of the sample space
- Probability is assigned to events
- Axioms:

1. $P(A) \geq 0$
2. $P(\text{universe}) = 1$
3. If $A \cap B = \emptyset$, then $P(A \cup B) = P(A) + P(B)$

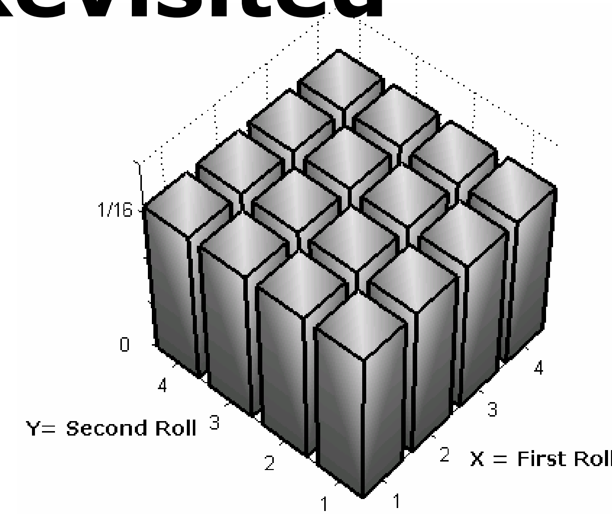
$$P(\{s_1, s_2, \ldots, s_k\}) = P(s_1) + \cdots + P(s_k)$$

- Axiom 3 needs strengthening
- Do weird sets have probabilities?

Example (1) Revisited

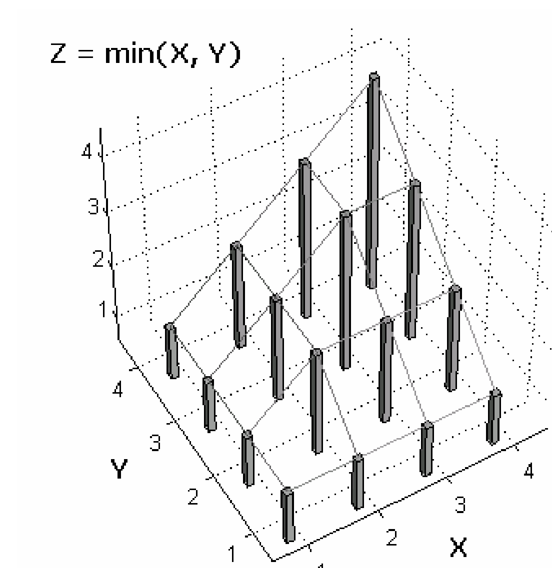
- Let every possible outcome have probability $\frac{1}{16}$

$$P(X=1) = 1/4$$



- Define $Z = \min(X, Y)$

$$P(Z=1) = 7/16$$
$$P(Z=2) = 5/16$$
$$P(Z=3) = 3/16$$
$$P(Z=4) = 1/16$$



Discrete Uniform Law

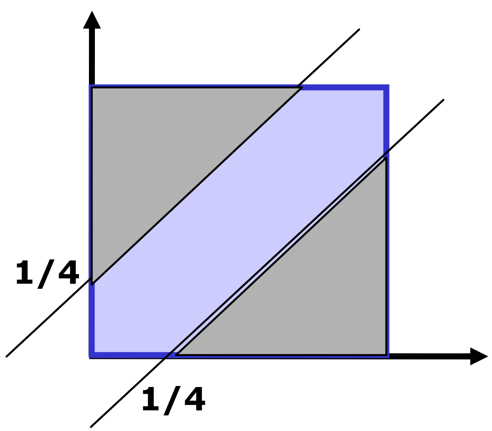
- Let all sample points be equally likely
- Then,

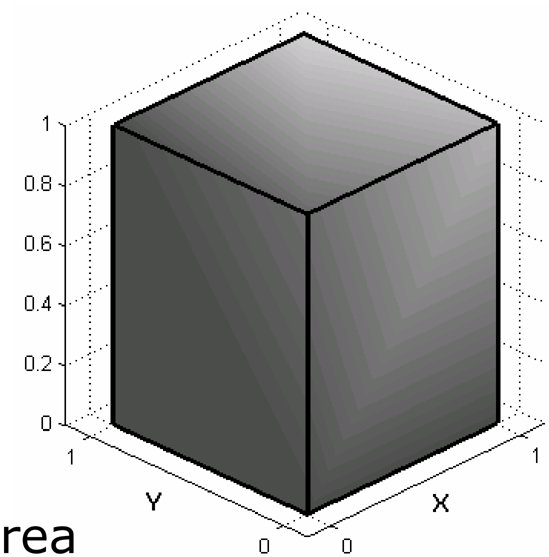
$$P(A) = \frac{\text{number of elements of } A}{\text{total number of sample points}}$$

- Just count …

Example (2) Revisited

- Each of two people choose a number between zero and one. What is the probability that they are at most 1/4 apart?
- Draw sample space and event of interest:





- Need to choose a probability law:
  - Choose **uniform** law: probability = area

The probability is: $1-(3/4).(3/4)=7/16$

A Word About Infinite Sample Spaces

- Sample space: $\{1, 2, \ldots\}$
  - We are given $P(n) = 2^{-n}$
  - Find $P(\text{outcome is even})$
- Solution:

$$\begin{aligned} P(\{2,4,6,\ldots\}) &= P(2) + P(4) + \cdots \\ &= \frac{1}{2^2} + \frac{1}{2^4} + \frac{1}{2^6} + \cdots = \frac{1}{3} \end{aligned}$$

- Axiom needed:

If $A_1, A_2, \ldots$ are disjoint events, then:

$$P(A_1 \cup A_2 \cup \cdots) = P(A_1) + P(A_2) + \cdots$$

Probability and the "Real World"

- Probability is a branch of math:
  - Axioms $\Rightarrow$ Theorems
  - One theorem: Frequency of event A is $P(A)$

- But are probabilities frequencies?
  - $P(\text{coin toss yields heads}) = 1/2$
  - $P(\text{The Iliad was written by Homer}) = 0.95$
  - $P(\text{a piece of equipment aboard the space shuttle fails}) = 10^{-8}$

- Probability models as a way of describing uncertainty:
  - Use for consistent reasoning
  - Use for predictions, decisions