

## LECTURE 2

- **Readings:** Sections 1.3-1.4

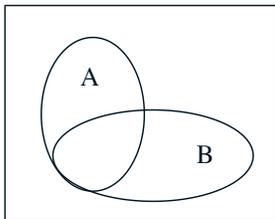
### Lecture outline

- Review
- Conditional probability
- Three **important** tools:
  - Multiplication rule
  - Total probability theorem
  - Bayes' rule

## Review of probability models

- **Sample space  $\Omega$** 
  - Mutually exclusive
  - Collectively exhaustive
  - Right granularity
- **Event:** Subset of the sample space
- Allocation of probabilities to events
  1.  $P(A) \geq 0$
  2.  $P(\Omega) = 1$
  3. If  $A \cap B = \emptyset$ , then  $P(A \cup B) = P(A) + P(B)$
  - 3'. If  $A_1, A_2, \dots$  are disjoint events, then:  $P(A_1 \cup A_2 \cup \dots) = P(A_1) + P(A_2) + \dots$
- Problem solving:
  - Specify sample space
  - Define probability law
  - Identify event of interest
  - Calculate...

## Conditional probability



- $P(A|B)$  = probability of  $A$ , given that  $B$  occurred
  - $B$  is our new universe

- **Definition:** Assuming  $P(B) \neq 0$ ,

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$P(A|B)$  undefined if  $P(B) = 0$

## Die roll example

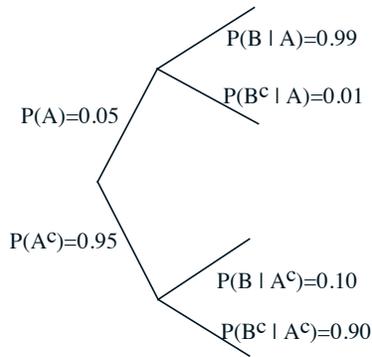
	1	2	3	4
4				
3				
2				
1				

$X$  = First roll

- Let  $B$  be the event:  $\min(X, Y) = 2$
- Let  $M = \max(X, Y)$
- $P(M = 1 | B) =$
- $P(M = 2 | B) =$

### Models based on conditional probabilities

- Event  $A$ : Airplane is flying above
- Event  $B$ : Something registers on radar screen



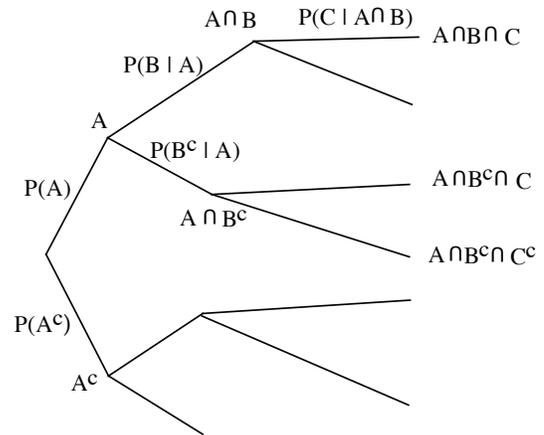
$P(A \cap B) =$

$P(B) =$

$P(A | B) =$

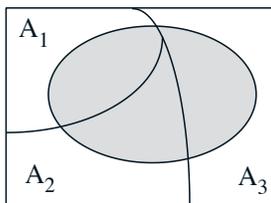
### Multiplication rule

$$P(A \cap B \cap C) = P(A) \cdot P(B | A) \cdot P(C | A \cap B)$$



### Total probability theorem

- Divide and conquer
- Partition of sample space into  $A_1, A_2, A_3$
- Have  $P(B | A_i)$ , for every  $i$

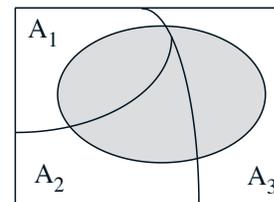


- One way of computing  $P(B)$ :

$$P(B) = P(A_1)P(B | A_1) + P(A_2)P(B | A_2) + P(A_3)P(B | A_3)$$

### Bayes' rule

- "Prior" probabilities  $P(A_i)$ 
  - initial "beliefs"
- We know  $P(B | A_i)$  for each  $i$
- Wish to compute  $P(A_i | B)$ 
  - revise "beliefs", given that  $B$  occurred



$$P(A_i | B) = \frac{P(A_i \cap B)}{P(B)} = \frac{P(A_i)P(B | A_i)}{P(B)} = \frac{P(A_i)P(B | A_i)}{\sum_j P(A_j)P(B | A_j)}$$

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6.041 / 6.431 Probabilistic Systems Analysis and Applied Probability  
Fall 2010

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