Tutorial 7
October 28/29, 2010

1. Alice and Bob alternate playing at the casino table. (Alice starts and plays at odd times $i = 1, 3, \ldots$; Bob plays at even times $i = 2, 4, \ldots$.) At each time i, the net gain of whoever is playing is a random variable G_i with the following PMF:

$$p_G(g) = \begin{cases} \frac{1}{3} & g = -2, \\ \frac{1}{2} & g = 1, \\ \frac{1}{6} & g = 3, \\ 0 & \text{otherwise} \end{cases}$$

   Assume that the net gains at different times are independent. We refer to an outcome of -2 as a "loss."

   (a) They keep gambling until the first time where a loss by Bob immediately follows a loss by Alice. Write down the PMF of the total number of rounds played. (A round consists of two plays, one by Alice and then one by Bob.)

   (b) Write down the PMF for Z, defined as the time at which Bob has his third loss.

   (c) Let N be the number of rounds until each one of them has won at least once. Find $\mathbf{E}[N]$.

2. Problem 6.6, page 328 in text.

   Sum of a geometric number of independent geometric random variables

   Let $Y = X_1 + \cdots + X_N$, where the random variable X_i are geometric with parameter p, and N is geometric with parameter q. Assume that the random variables $N, X_1, X_2, \cdots$ are independent. Show that Y is geometric with parameter pq. *Hint:* Interpret the various random variables in terms of a split Bernoulli process.

3. A train bridge is constructed across a wide river. Trains arrive at the bridge according to a Poisson process of rate $\lambda = 3$ per day.

   (a) If a train arrives on day 0, find the probability that there will be no trains on days 1, 2, and 3.

   (b) Find the probability that the next train to arrive after the first train on day 0, takes more than 3 days to arrive.

   (c) Find the probability that no trains arrive in the first 2 days, but 4 trains arrive on the 4^{th} day.

   (d) Find the probability that it takes more than 2 days for the 5^{th} train to arrive at the bridge.

6.041 / 6.431 Probabilistic Systems Analysis and Applied Probability
Fall 2010