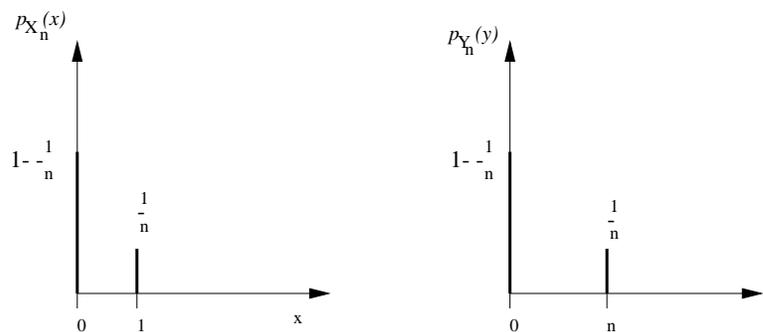


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1. In your summer internship, you are working for the world's largest producer of lightbulbs. Your manager asks you to estimate the quality of production, that is, to estimate the probability p that a bulb produced by the factory is defectless. You are told to assume that all lightbulbs have the same probability of having a defect, and that defects in different lightbulbs are independent.
 - (a) Suppose that you test n randomly picked bulbs, what is a good estimate Z_n for p , such that Z_n converges to p in probability?
 - (b) If you test 50 light bulbs, what is the probability that your estimate is in the range $p \pm 0.1$?
 - (c) The management asks that your estimate falls in the range $p \pm 0.1$ with probability 0.95. How many light bulbs do you need to test to meet this specification?

- 2.



Let X_n and Y_n have the distributions shown above.

- (a) Find the expected value and variance of X_n and Y_n .
- (b) What does the Chebyshev inequality tell us about the convergence of X_n ? Y_n ?
- (c) Is Y_n convergent in probability? If so, to what value?
- (d) If a sequence of random variables converges in probability to a , does the corresponding sequence of expected values converge to a ? Prove or give a counter example.

A sequence of random variables is said to converge to a number c in the **mean square**, if

$$\lim_{n \rightarrow \infty} \mathbf{E} [(X_n - c)^2] = 0.$$

- (e) Use Markov's inequality to show that convergence in the mean square implies convergence in probability.
- (f) Give an example that shows that convergence in probability does not imply convergence in the mean square.

3. Random variable X is uniformly distributed between -1.0 and 1.0 . Let X_1, X_2, \dots , be independent identically distributed random variables with the same distribution as X . Determine which, if any, of the following sequences (all with $i = 1, 2, \dots$) are convergent in probability. Give reasons for your answers. Include the limits if they exist.

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering & Computer Science
6.041/6.431: Probabilistic Systems Analysis
(Fall 2010)

(a) X_i

(b) $Y_i = \frac{X_i}{i}$

(c) $Z_i = (X_i)^i$

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