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6.02 Practice Problems: MAC protocols

Please read Chapter 15 before trying to solve these problems. Please also solve the problems at the end of Chapter 15.

Problem 1.

Which of these statements are true for correctly implemented versions of stabilized unslotted Aloha, stabilized slotted Aloha, and Time Division Multiple Access (TDMA)? Assume that the slotted and unslotted versions of Aloha use the same stabilization method and parameters.

A. When the number of nodes is large, unslotted Aloha has a lower maximum throughput than slotted Aloha.

Hide Answer

True. By a factor of 2: 1/(2e) instead of 1/e.

B. When the number of nodes is large and nodes transmit data according to a Poisson process, there exists *some* offered load for which the throughput of unslotted Aloha is higher than the throughput of slotted Aloha.

Hide Answer

False.

C. TDMA has no packet collisions.

Hide Answer

True. TDMA eliminates collisions by explicitly allotting time slots.

D. There exists some offered load pattern for which TDMA has lower throughput than slotted Aloha.

Hide Answer

True. For example, a skewed workload in which some nodes have much more traffic to send than others.

Problem 2.

Binary exponential backoff is a mechanism used in some MAC protocols. Which of the following statements is correct?

A. It ensures that two nodes that experience a collision in a time slot will *never* collide with each other when they each retry that packet.

B. It ensures that two or more nodes that experience a collision in a time slot will experience a lower probability of colliding with each ther when they each retry that packet.

C. It can be used with slotted Aloha but not with carrier sense multiple access.

D. Over short time scales, it improves the fairness of the throughput achieved by different nodes compared to not using the mechanism.

Hide Answer

B is true. The others are false.

Problem 3.

In the Aloha stabilization protocols we studied, when a node experiences a collision, it decreases its transmission probability, but sets a lower bound, p_min. When it transmits successfully, it increases its transmission probability, but sets an upper bound, p_max.

A. Why would we set a lower bound on p_min that is not too close to 0?

   Hide Answer

   To avoid starvation where some nodes are denied access to the medium for long periods of time.

B. Why would we set p_max to be significantly smaller than 1?

   Hide Answer

   To avoid the capture effect, in which a successful node hogs the medium for multiple time slots even when other nodes are backlogged.

C. Let N be the average number of backlogged nodes. What happens if we set p_min >> 1/N?

   Hide Answer

   The rate of collisions will be high and the utilization close to 0.

Problem 4.

Consider a shared medium with N backlogged nodes running the slotted Aloha MAC protocol without any backoffs. An idle slot is one in which no node sends data. We will refer to the fraction of time during which no node uses the medium as the "idletime" of the protocol.

A. If each node has a sending probability of p, what is the idletime? What are the smallest and largest possible values of the idletime?

   Hide Answer

   $(1-p)^N$. 0 and 1.

B. Assume N is large. If the Aloha sending probability, p, for each node is picked so as to maximize the utilization, what is the corresponding idletime?

   Hide Answer

1/e --> same as the utilization!

Problem 5.

True or false?

Assume that the shared medium has N nodes and they are always backlogged.

A. In a slotted Aloha MAC protocol using binary exponential backoff, the probability of transmission will always eventually converge to some value p, and all nodes will eventually transmit with probability p.

Hide Answer

False - In a binary exponential backoff, the probability of transmission constantly changes. If the a transmission succeeds, then the probability goes up. If the transmission fails, the probability goes down.

B. Using carrier sense multiple access (CSMA), suppose that a node "hears" that the channel is busy at time slot t. To maximize utilization, the node should not transmit in slot t and instead transmit the packet in the next time slot with probability 1.

Hide Answer

False - The node should not transmit at time t+1 with 100% probability. Other nodes may have also "heard" that the channel is busy and would want to send a packet at time t+1. So the node should send with a probability less than 100% to reduce the possibility of a collision.

C. There is some workload for which an unslotted Aloha with perfect CSMA will not achieve 100% utilization.

Hide Answer

True - Multiple nodes may think that the channel is idle and send a packet at the same time, resulting in a collision. Thus, the utilization can never reach 100%.

Problem 6.

Eight Cell Processor cores are connected together with a shared bus. To simplify bus arbitration, Ben Bittdidle, young IBM engineer, suggested time-domain multiplexing (TDM) as an arbitration mechanism. Using TDM each of the processors is allocated a equal-sized time slots on the common bus in a round-robin fashion. He's been asked to evaluate the proposed scheme on two types of applications: 1) core-to-core streaming, 2) random loads.

1) Core-to-core streaming setup: Assume each core has the same stream bandwidth requirement.

2) Random loads setup: core 1 load = 20%, core 2 load = 30%, core 3 load = 10%, core 4 = 5%, core 5 = 1%, core 6 = 3%, core 7 = 1%, core 8 load = 30%

Help Ben out by evaluating the effectiveness (bus utilization) of TDM under these two traffic scenarios.

Hide Answer

1) All cores have same bandwidth requirements during streaming, so, bus utilization is 100%

2) Each core is given 12.5% of bus bandwidth, but not all cores can use it, and some need more than that. So bus utilization is: 12.5% + 12.5% + 10% + 5% + 1% + 3% + 1% + 12.5% = 57.5%

Problem 7.

Randomized exponential backoff is a mechanism used to stabilize contention MAC protocols. Which of the following statements is correct?

A. It ensures that two nodes that experience a collision in a time-slot will *never* collide with each other when they each retry that packet.

Hide Answer

False. They might get unlucky and back-off the same amount.

B. It ensures that two or more nodes that experience a collision n a time-slot will experience a lower probability of colliding with ach other when they each retry that packet.

Hide Answer

True. The probability of a repeat collision is a smaller in each subsequent backoff.

C. It can be used with slotted Aloha but not with CSMA.

Hide Answer

False. It can be used in any contention protocol.

Problem 8.

Three users X, Y and Z use a shared link to connect to the Internet. Only one of X, Y or Z can use the link at a given time. The link has a capacity of 1 Megabit/s. There are two possible strategies for accessing the shared link:

- TDMA: equal slots of 0.1 seconds.
- "Taking turns": adds a latency of 0.05 seconds before taking the turn. The user can then use the link for as long as it has data to send. A user requests the link only when it has data to send.

In each of the following two cases, which strategy would you pick and why?

A. X, Y and Z send a 40 Kbytes file every 1sec.

Hide Answer

TDMA. Why: Each of the users generate a load of 40KB/s = 0.32 Megabits/s, which can be fully transmitted given the share of 0.33 Megabits/s available per user when partitioning the channel with TDMA. Taking turns on the other hand does not offer enough capacity for all the files to be transmitted: 3*0.32 + 3*0.05 = 1.11s > 1s, and would incur extra overhead.

B. X sends 80 Kbytes files every 1sec, while Y and Z send 10 Kbytes files every 1sec.

Hide Answer

Taking Turns Why: First, by using TDMA, X does not have enough capacity to transmit, 80 Kbytes/s = 0.640 Megabits/s > 0.33 Megabigs/s. Second, with TDMA, Y and Z waste 3 out of 4 slots. On the other hand, when taking turns, there is enough capacity to transmit all the data:

0.64+0.05+0.08+0.05+0.08+0.05=0.95s.

Problem 9.

Alyssa P. Hacker is setting up an 8-node broadcast network in her apartment building in which all nodes can hear each other. Nodes send packets of the same size. If packet collisions occur, both packets are corrupted and lost; no other packet losses occur. All nodes generate equal load on average.

Alyssa observes a utilization of 0.5. Which of the following are consistent with the observed utilization?

A. True/False: Four nodes are backlogged on average, and the network is using Slotted Aloha with stabilization, and the fairness is close to 1.

Hide Answer

False. With four backlogged nodes and fairness close to 1, the probability of of transmission is 1/4. That would put the utilization at $4*(1/4)*(1-1/4)^3 = 0.42 < 0.5$.

B. True/False: Four nodes are backlogged on average, and the network is using TDMA, and the fairness is close to 1.

Hide Answer

True. TDMA gives each node an equal share of the network, but 4 nodes have nothing to send, giving a utilization of 0.5.

Now suppose Alyssa's 8-node network runs the Carrier Sense Multiple Access (CSMA) MAC protocol. The maximum data rate of the network is 10 Megabits/s. Including retries, each node sends traffic according to some unknown random process at an average rate of 1 Megabit/s per node. Alyssa measures the network's utilization and finds that it is 0.75. No packets get dropped in the network except due to collisions.

C. What fraction of packets sent by the nodes (including retries) experience a collision?

Hide Answer

The offered load presented to the network is 8 Megabits/s in aggregate. The throughput of the protocol is 0.75*10 = 7.5 Megabits/s. The packet collision rate is therefore equal to

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1-7.5/8 = 1/16 = 6.25%.
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Problem 10.

Note: this problem is useful to review how to set up and solve problems related to Aloha-like access protocols, but the calculations shown in the answer are more complex than we would ask on a quiz.

Consider a network with four nodes, where each node has a dedicated channel to each other node. The probability that any node transmits is p. Each node can only send OR receive one packet at a time. What is the utilization of the network? Assume each packet takes 1 time slot. Assume each node has 3 queues -- one for each other node -- and that each is backlogged.

Hide Answer

Utilization is the expected number of packets that get through in each slot divided by the maximum (2, i.e. when A always transmits to B and C always transmits to D).

P[4 nodes transmit] = p^4
P[3 nodes transmit] = $4p^3(1-p)$
P[2 nodes transmit] = $6p^2(1-p)^2$
P[1 node transmits] = $4p(1-p)^3$
p[0 nodes transmit] = $(1-p)^4$

For each case, we compute the expected number of packets that get through, then remove conditioning using the total probability theorem.

Expected packets through if 4 nodes transmit = 0
No one is free to receive a packet.

Expected packets through if 3 nodes transmit = 1*(3/27) = 1/9
Without loss of generality, assume that A,B and C transmit. There are 27 combinations of destinations that they can transmit to (each can choose one of 3). Of these, only 3 result in the successful transmission of one packet (e.g. A->D,B->C,C->B) or (B->D,A<->C) or (C->D,A<->B)

Expected packets through if 2 nodes transmit = 2*(2/9) = 4/9
Similar to above, assume that A and B transmit. There are 9 combinations of destinations. Of these, only 2 result in the successful transmission of 2 packets (A->D,B->C) or (A->C,B->D).

Expected packets through if 1 node transmits = 1
No matter who the node transmits to, it will get through.

Expected packets through if 0 nodes transmit = 0
No packets sent.

Thus, the expected number of packets through is

$(1/9)*4p^3(1-p) + (4/9)*6p^2(1-p)^2 + (1)*4p(1-p)^3 = 4/9*p^3(1-p) + (4/3)p^2(1-p)^2 + 4p(1-p)^3$

The utilization is half of that.

Problem 11.

Suppose that there are three nodes seeking access to a shared medium using slotted Aloha, where each packet takes one slot to transmit. Assume that the nodes are always backlogged, and that each has probability p_i of sending a packet in each slot, where i = 1, 2 and 3 indexes the node. Suppose that we assign more the sending probabilities so that

$p_1 = 2(p_2)$ and $p_2 = p_3$

A. What is the utilization of the shared medium?

Hide Answer

$U = (p_1)(1 - p_2)(1 - p_3) + (1 - p_1)(p_2)(1 - p_3) + (1 - p_1)(1 - p_2)(p3)$

If p = p3

$U = 2p(1-p)^2 + 2(1 - 2p)p(1 - p) = 4p - 10p^2 + 6p^3$

B. What are the probabilities that maximize the utilization and the corresponding utilization?

Hide Answer

Differentiating U and sett the result equal to zero we obtain

$dU/dp = 4 - 20p + 18p^2 = 0$

has two roots at p=0.2616 and 0.8495. However, only the root at 0.2616 is feasible since the other leads to a value of p_1 that is greater than 1. Thus,

$p_1 = 0.5232$, $p_2 = 0.2616$ and $p_3 = 0.2616$.

The corresponding utilization is 0.4695.

Problem 12.

Suppose that two nodes are seeking access to a shared medium using slotted Aloha with binary exponential backoff subject to maximum and minimum limits of the probability pmax = 0.8 and pmin = 0.1. Suppose that both nodes are backlogged, and at slot n, the probabilities the two nodes transmit packets are p_1 = 0.5 and p_2 = 0.3.

A. What are the possible values of p_1 at slot n+1? What are the probabilities assocated with each possible value?

Hide Answer

p_1 increases to 0.8 if node 1 transmits a packet and node 2 does not transmit a packet. Probability of this event:

$(p_1)(1 - p_2) = 0.5*0.7 = 0.35$

p_1 decreases to 0.25 if node 1 transmits a packeet and node 2 also transmits a packet. Probability of this event:

$(p_1)(p_2) = 0.15$

Otherwise p_1 stays the same with probability 1 - 0.35 - 0.15 = 0.5.

B. What are the possible values of p_2 at slot n+1? What are the probabilities associated with each possible value?

Hide Answer

p_2 increases to 0.6 if node 2 transmits a packet and node 1 does not transmit a packet. Probability of this event:

(p_2)(1 - p_1) = 0.3*0.5 = 0.15

p_2 decreases to 0.15 if node 2 transmits a packeet and node 1 also transmits a packet. Probability of this event:

(p_1)(p_2) = 0.15

Otherwise p_2 stays the same with probability 1 - 0.15 - 0.15 = 0.7.

Problem 13. Bluetooth is a wireless technology found on many mobile devices, including laptops, mobile phones, GPS navigation devices, headsets, and so on. It uses a MAC protocol called Time Division Duplex (TDD). In TDD, the shared medium network has 1 master and N slaves. You may assume that the network has already been configured with one device as the master and the others as slaves. Each slave has a unique identifier (ID) that serves as its address, an integer between 1 and N . Assume that no devices ever turn off during the operation of the protocol. Unless otherwise mentioned, assume that no packets are lost.

The MAC protocol works as follows. Time is slotted and each packet is one time slot long.

In every *odd* time slot (1, 3, 5, …, 2t–1, …), the master sends a packet addressed to some slave for which it has packets backlogged, in round-robin order (i.e., cycling through the slaves in numeric order).

In every *even* time slot (2, 4, 6, …, 2t, …), the slave that received a packet from the master in the immediately preceding time slot gets to send a packet to the master, if it has a packet to send. If it has no packet to send, then that time slot is left unused, and the slot is wasted.

A. Alyssa P. Hacker finds a problem with the TDD protocol described above, and implements the following rule in addition:

> From time to time, in an odd time slot, the master sends a "dummy" packet addressed to a slave even if it has no other data packets to send to the slave (and even if it has packets for other slaves).

Why does Alyssa's rule improve the TDD protocol?

Hide Answer

Because it will prevent a slave from being starved; without it, a slave that has packets to send will never send data packets if the master never has packets to send to it.

Henceforth, the term "TDD" will refer to the protocol described above, augmented with Alyssa's rule. Moreover, whenever a "dummy" packet is sent, that time slot will be considered a wasted slot.

B. Alyssa's goal is to emulate a round-robin TDMA scheme amongst the N slaves. Propose a way to achieve this goal by specifying the ID of the slave that the master should send a data or dummy packet to, in time slot 2t–1 (note that $1 \le t \le \infty$).

Hide Answer

Send to the slave whose ID is (t mod N)+1. This is almost exactly the same problem as in PSet #6, Task

1 (TDMA). The only difference is that the nodes begin with ID 1 here, not 0.

Henceforth, assume that the TDD scheme implements round-robin TDMA amongst the slaves. Suppose the master always has data packets to send only to an arbitrary (but fixed) subset of the N slaves. In addition, a (possibly different) subset of the slaves always has packets to send to the master. Each subset is of size r, a fixed value. Answer the questions below (you may find it helpful to think about different subsets of slaves).

C. What is the *maximum possible* utilization of such a configuration?

Hide Answer

The protocol is TDMA, so the master sends useful data packets in r of the 2N slots in an epoch, and receives useful packets in the r of the 2N slots. So the utilization is r/N. If one N seeks to maximize that over r, it's clear that the maximum happens when r = N, giving us a maximum of 1.

D. What is the *minimum possible* utilization (for a given value of r) of such a configuration? Assume that r > N/2. Note that if the master does not have a data packet to send to a slave in a round, it sends a "dummy" packet to that slave instead. A dummy packet does not count toward the utilization of the medium.

Hide Answer

r/N. When minimizing over all r, the smallest value becomes 1/2 + 1/N when N is even and 1/2 + 1/2N when N is odd.

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