

6.02 Practice Problems: Frequency Response of LTI Systems & Filters

Note: In these problems, we sometimes refer to $H(\Omega)$ as $H(e^{j\Omega})$. The reason is that in some previous terms we used the latter convention, and it's too painful to redraw all the figures, which embed the latter notation. Anyway, there should be no confusion because the convention should be apparent from the context.

Problem 1.

Give an expression for the magnitude of a complex exponential with frequency φ , i.e., $|e^{j\varphi}|$.

Show Answer

Problem 2.

A. Prove the validity of the following formula, often referred to as the *finite sum formula*:

$$\sum_{n=0}^{N-1} \alpha^n = \begin{cases} N & \alpha = 1 \\ \frac{1-\alpha^N}{1-\alpha} & \text{for any complex number } \alpha \neq 1 \end{cases}$$

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B. Use the finite sum formula to show:

$$\sum_{n=\langle N \rangle} e^{jk\frac{2\pi}{N}n} = \begin{cases} N & k = 0, \pm N, \pm 2N, \dots \\ 0 & \text{otherwise} \end{cases}$$

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Problem 3.

Consider an LTI system characterized by the unit-sample response $h[n]$.

A. Give an expression for the frequency response of the system $H(e^{j\Omega})$ in terms of $h[n]$.

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B. If $h[0]=1$, $h[1]=0$, $h[2]=1$, and $h[n]=0$ for all other n , what is $H(e^{j\Omega})$?

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C. Let $h[n]$ be defined as in part B and $x[n] = \cos(\varphi n)$. Is there a value of φ such that $y[n]=0$ for all n and $0 \leq \varphi \leq \pi$?

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D. Let $h[n]$ be defined as in part B. Find the maximum magnitude of $y[n]$ if $x[n] = \cos(\pi n/4)$.

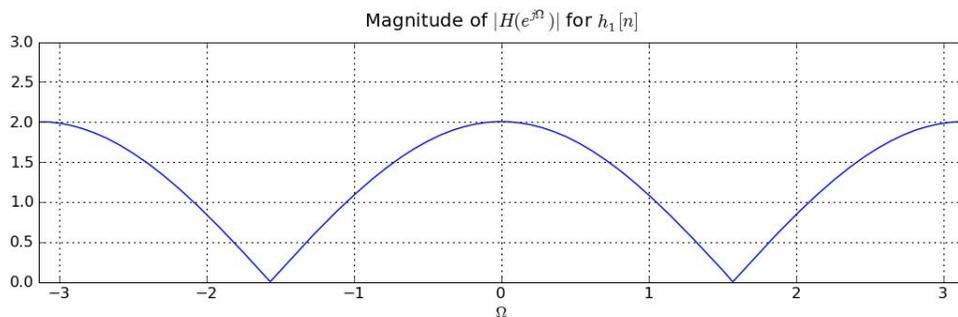
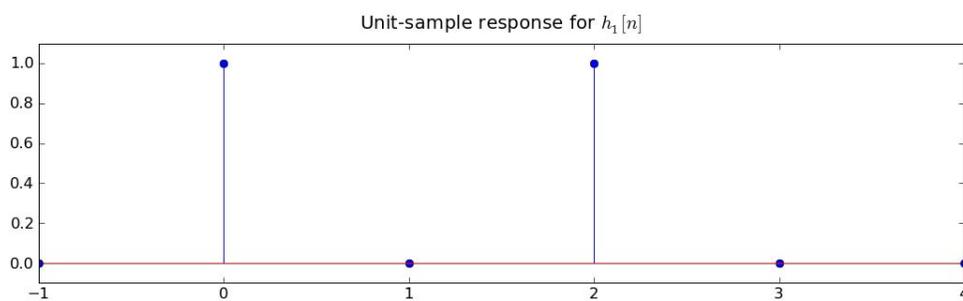
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E. Let $h[n]$ be defined as in part B. Find the maximum magnitude of $y[n]$ if $x[n] = \cos(-(\pi/2)n)$.

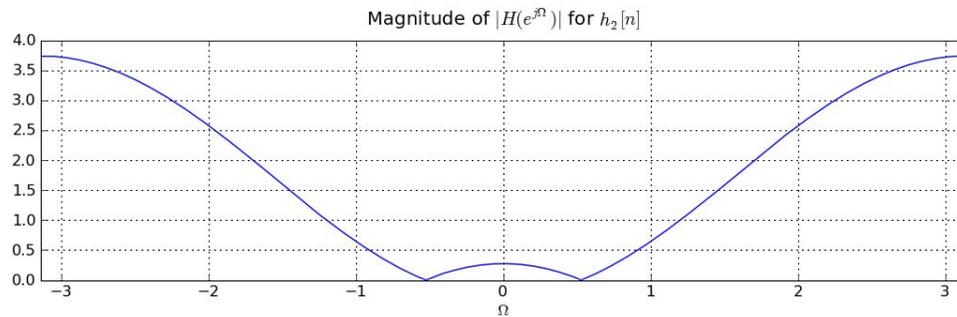
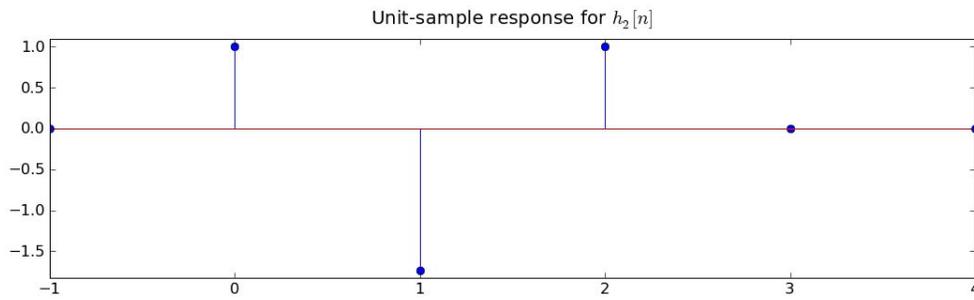
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Problem 4.

In answering the questions below, please consider the unit sample response and frequency response of two filters, H_1 and H_2 , plotted below.

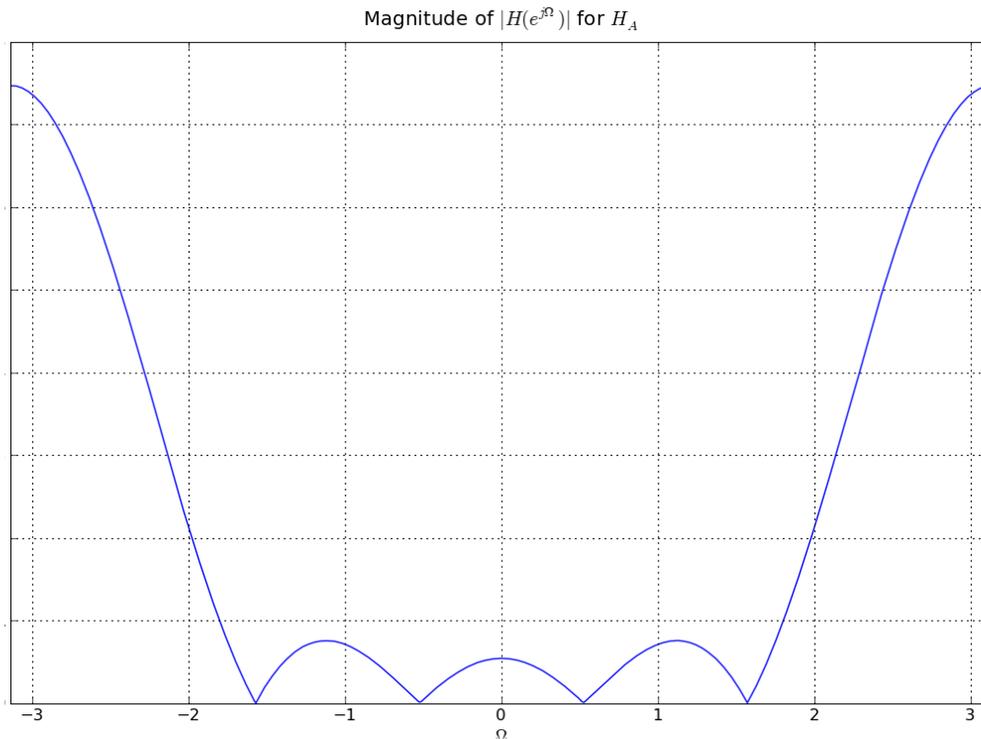


Note: the only nonzero values of unit sample response for H_1 are : $h_1[0] = 1$, $h_1[1]=0$, $h_1[2]=1$.

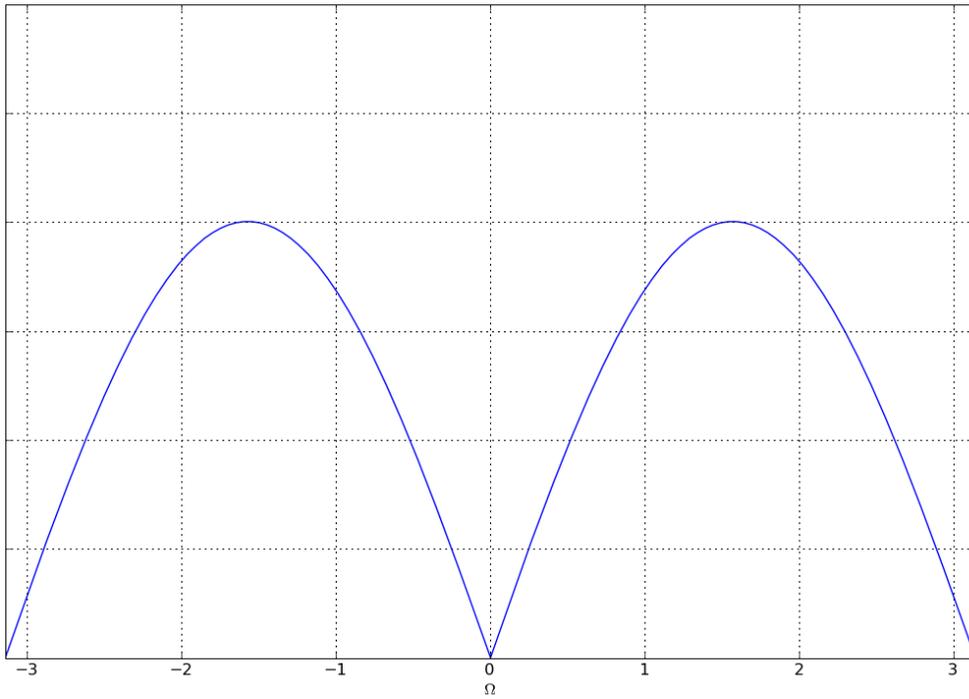


Note, the only nonzero values of unit sample response for H_2 are : $h_2[0] = 1$, $h_2[1] = -\sqrt{3}$, $h_2[2] = 1$.

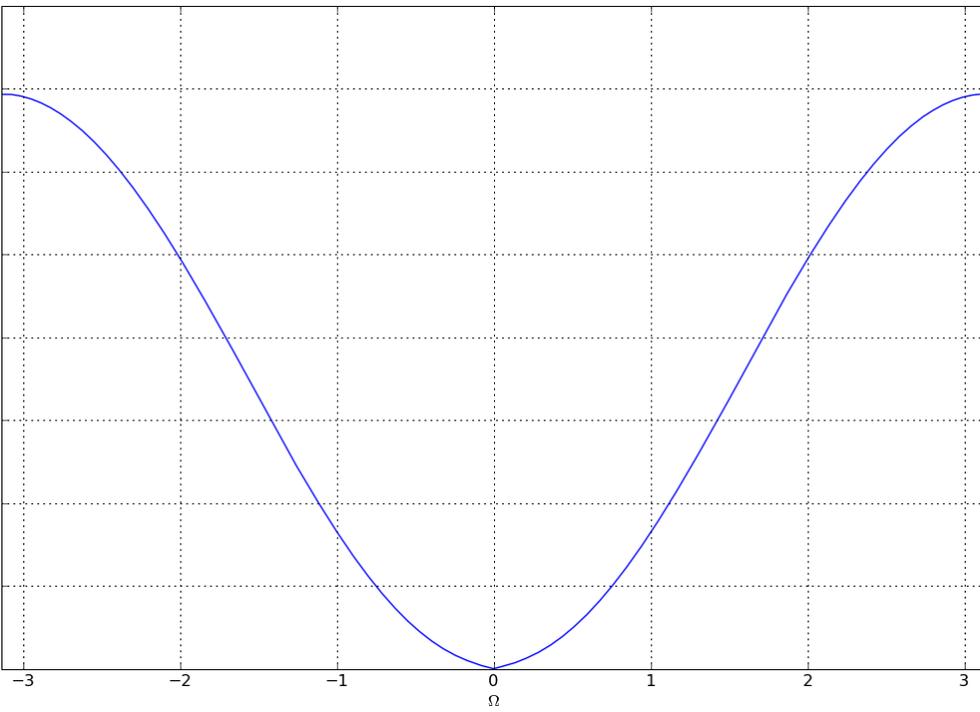
In answering the several parts of this review question consider four linear time-invariant systems, denoted A, B, C, and D, each characterized by the magnitude of its frequency response, $|H_A(e^{j\Omega})|$, $|H_B(e^{j\Omega})|$, $|H_C(e^{j\Omega})|$, and $|H_D(e^{j\Omega})|$ respectively, as given in the plots below. This is a review problem, not an actual exam question, so similar concepts are tested multiple times to give you practice



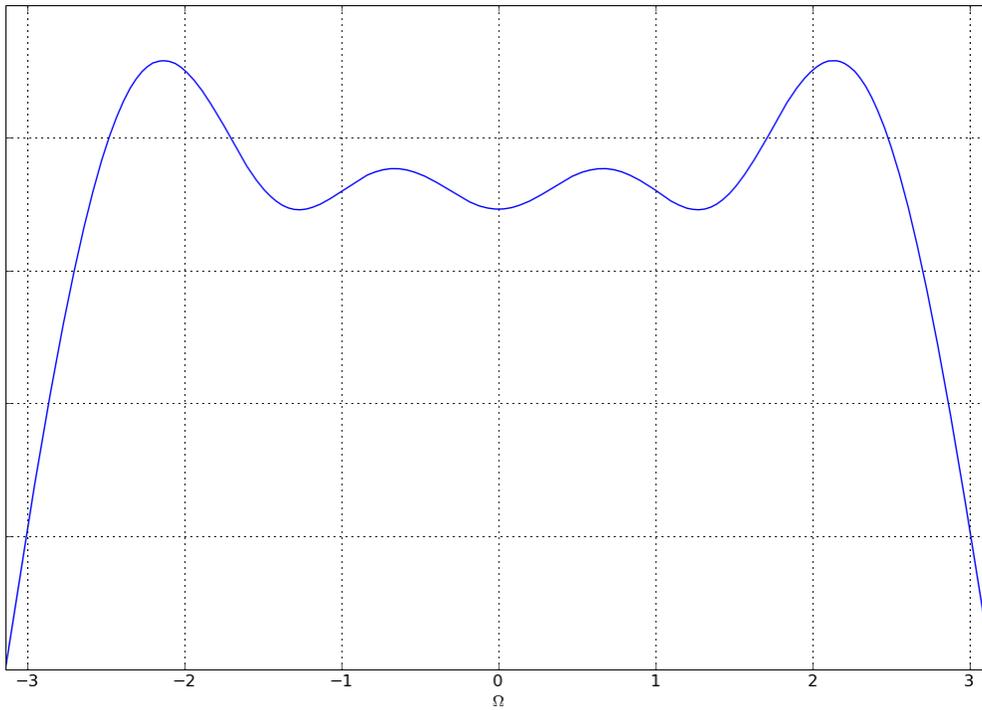
Magnitude of $|H(e^{j\Omega})|$ for H_B



Magnitude of $|H(e^{j\Omega})|$ for H_C



Magnitude of $|H(e^{j\Omega})|$ for H_D



A. Which frequency response (A, B, C or D) corresponds to a unit sample response given by

$$h[n] = \alpha \delta[n] - h_1[n]$$

and what is the numerical value of $|\alpha|$?

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B. Which frequency response (A, B, C or D) corresponds to a unit sample response given by

$$h[n] = \sum_m h_1[m] h_2[n-m] \text{ for } m = 0 \text{ to } n$$

and what are the numerical values of $h[2]$, $h[3]$ and $H(e^{j0})$?

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C. Which frequency response (A, B, C or D) corresponds to a unit sample response given by

$$h[n] = \alpha \delta[n] - \sum_m h_1[m] h_2[n-m] \text{ for } m = 0 \text{ to } n$$

and what is the numerical value of $|\alpha|$?

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D. Which frequency response (A, B, C or D) corresponds to a unit sample response given by

$$h[n] = \alpha \delta[n] - h_2[n]$$

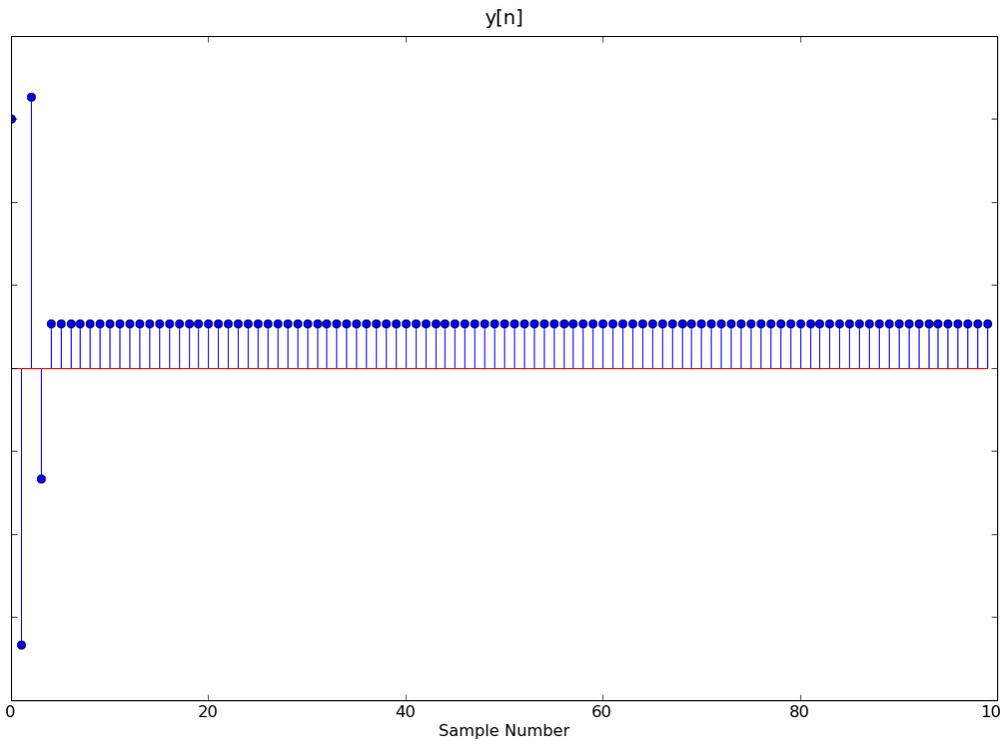
and what is the numerical value of $|\alpha|$?

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E. Suppose the input to each of the above four systems is

$x[n]=0$ for $n < 0$ and
 $x[n] = \cos(n\pi/6) + \cos(n\pi/2) + 1.0$ for $n \geq 0$

Which system (A, B, C or D) produced an output, $y[n]$ below, and what is the value of $y[n]$ for $n > 10$?

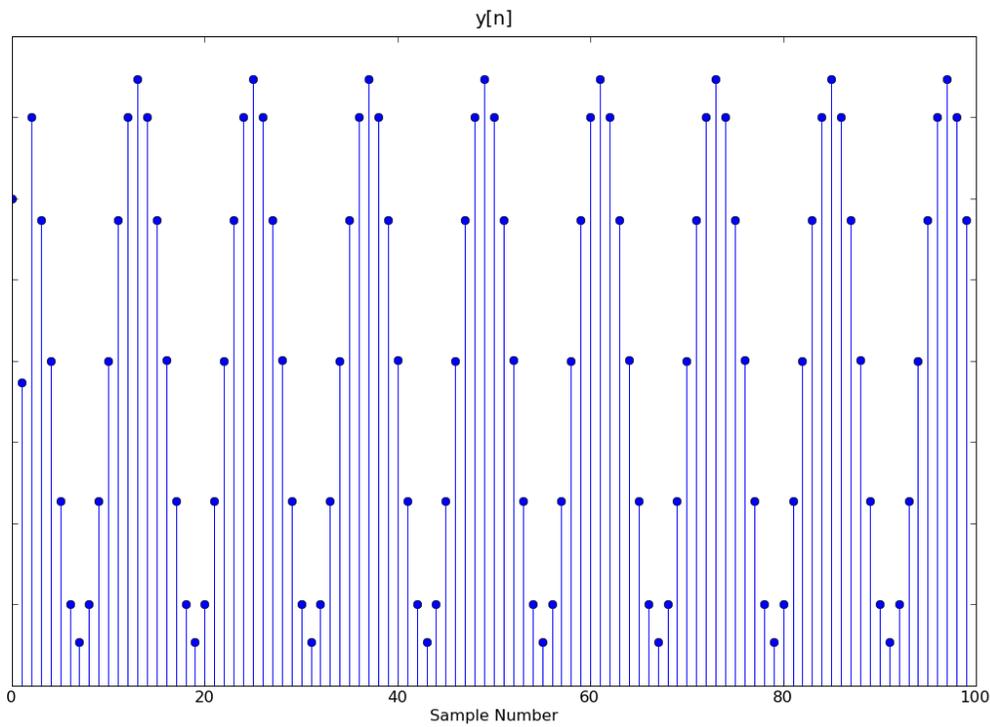


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F. Suppose the input to each of the above four systems is

$x[n]=0$ for $n < 0$ and
 $x[n] = \cos(n\pi/6) + \cos(n\pi/2) + 1.0$ for $n \geq 0$

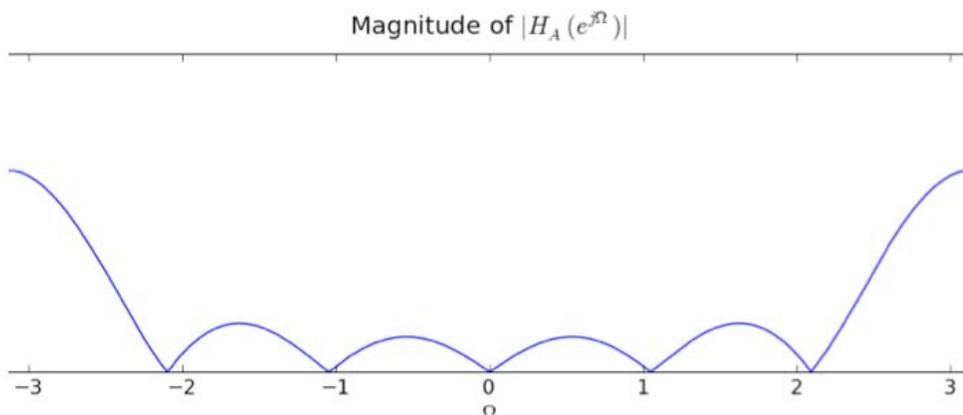
Which system (H_1 or H_2) produced an output, $y[n]$ below, and what is the value of $y[22]$?



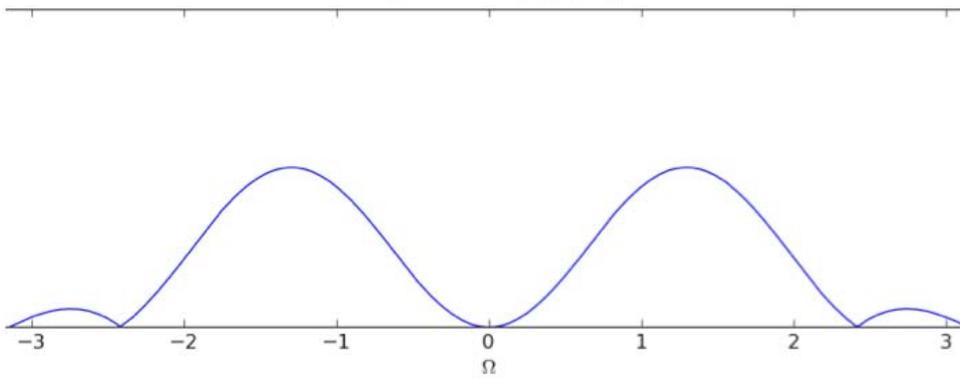
Show Answer

Problem 5.

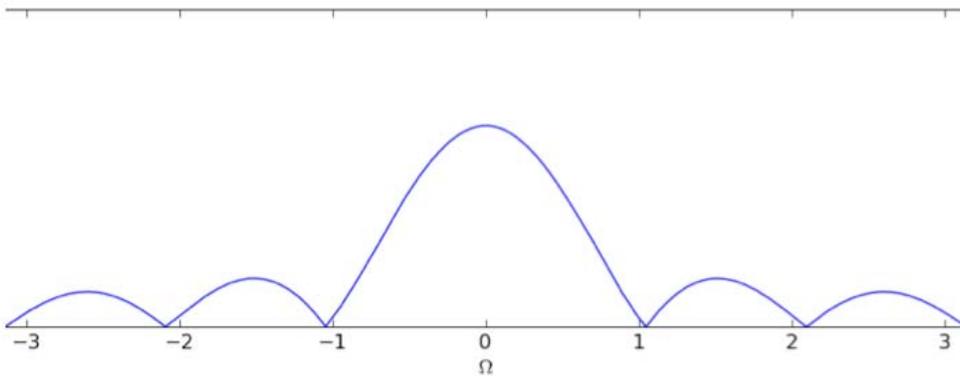
In answering the several parts of this question, consider three linear time-invariant filters, denoted A, B, and C, each characterized by the magnitude of their frequency responses, $|H_A(e^{j\Omega})|$, $|H_B(e^{j\Omega})|$, $|H_C(e^{j\Omega})|$, respectively, as given in the plots below.



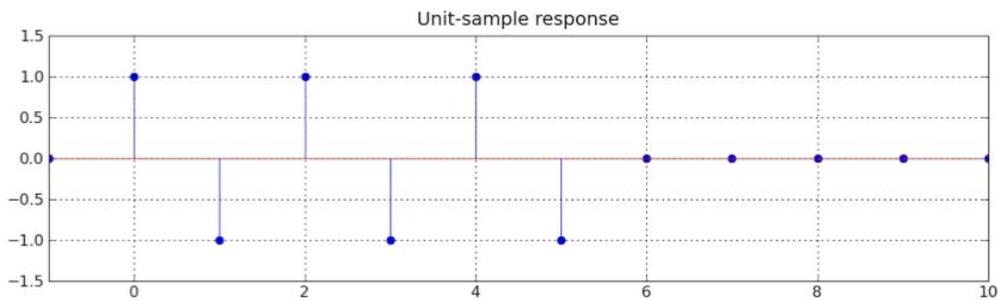
Magnitude of $|H_B(e^{j\Omega})|$



Magnitude of $|H_C(e^{j\Omega})|$

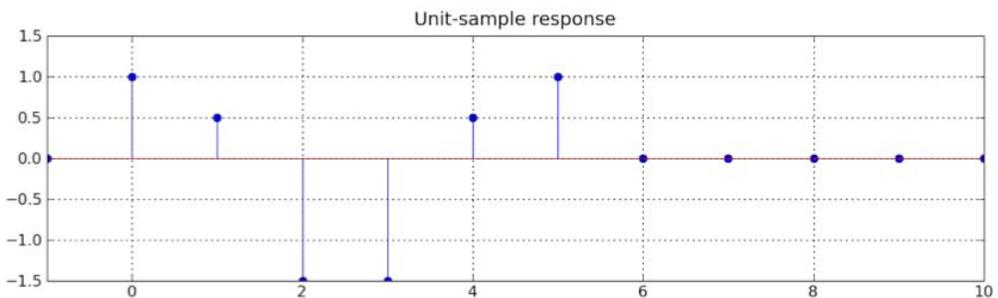


- A. Which frequency response (A, B, or C) corresponds to the following unit sample response, and what is $\max_{\Omega} |H(e^{j\Omega})|$ for your selected filter? Please justify your selection.



Show Answer

- B. Which frequency response (A, B, or C) corresponds to the following unit sample response, and is $\max_{\Omega} |H(e^{j\Omega})| > 6$ for your selected filter? Please justify your answers.

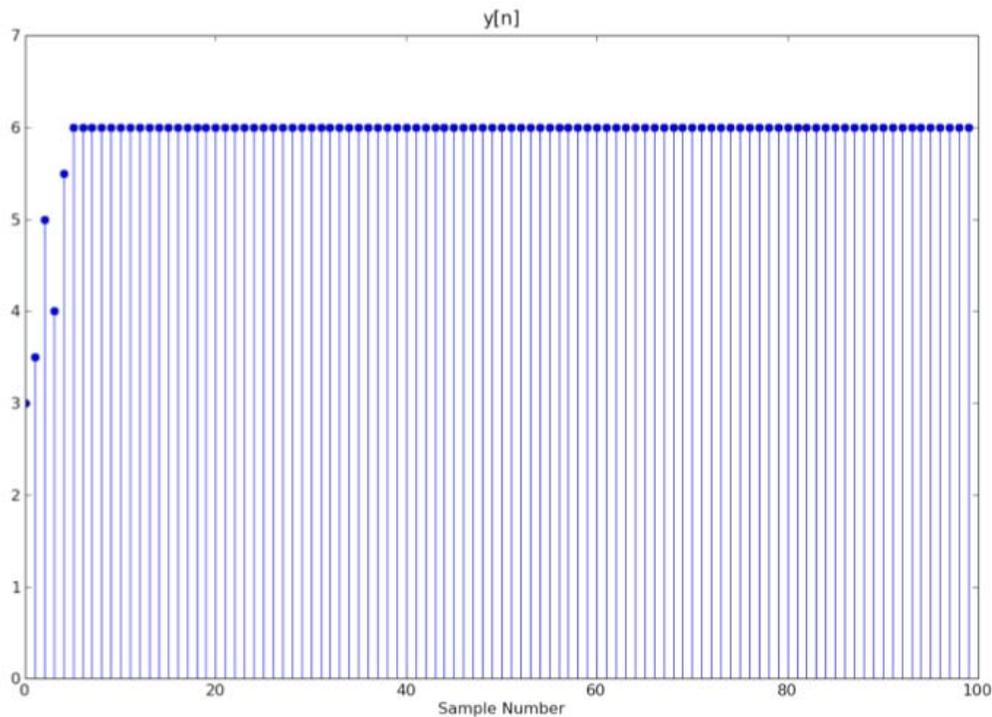


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C. Suppose the input to each of the above three filters is $x[n] = 0$ for $n < 0$ and for $n \geq 0$ is

$$x[n] = \cos((\pi/3)n) + \cos(\pi n) + 1.0$$

Which filter (A, B, or C) produced the output, $y[n]$ below, and what is $\max_{\Omega} |H(e^{j\Omega})|$ for your selected system?

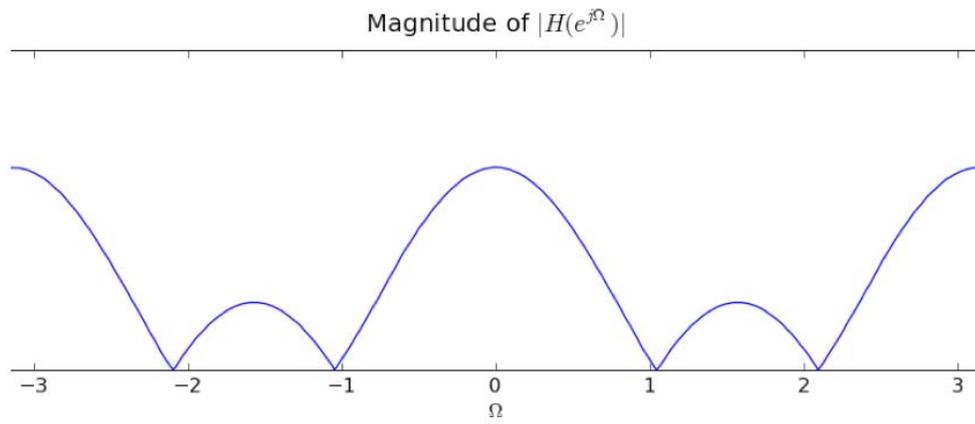


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D. Six new filters were generated using the unit sample responses of filters A, B and C, denoted $h_A[n]$, $h_B[n]$, and $h_C[n]$ respectively. The unit sample responses of the new filters were generated in the following way:

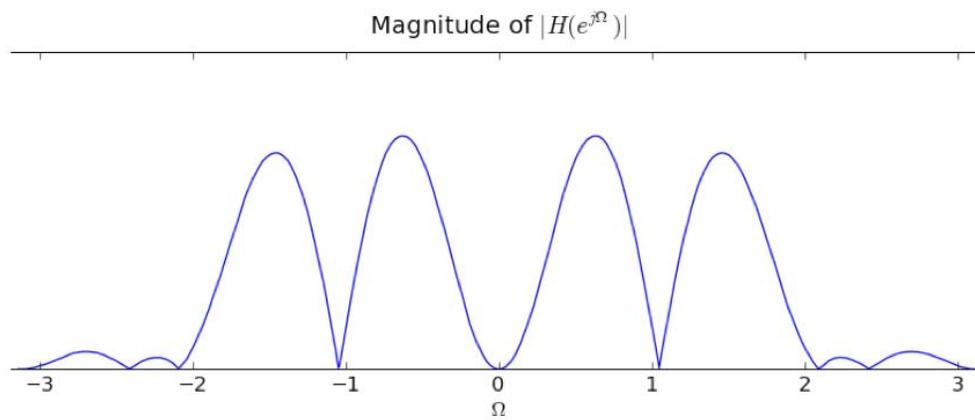
- $h_1[n] = h_A[n] + h_B[n]$. Filters A and B in parallel.
- $h_2[n] = h_A[n] + h_C[n]$. Filters A and C in parallel.
- $h_3[n] = h_B[n] + h_C[n]$. Filters B and C in parallel.
- $h_4[n] = \text{convolve}(h_A, h_B)$. Filters A and B in series.
- $h_5[n] = \text{convolve}(h_A, h_C)$. Filters A and C in series.
- $h_6[n] = \text{convolve}(h_B, h_C)$. Filters B and C in series.

Which of the six new filters has the frequency response plotted below?



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Which of the six new filters from the previous question has the frequency response plotted below?



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