Show All Answers Hide All Answers

6.02 Practice Problems: LTI Channels and Intersymbol Interference

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Problem 1.

The input sequence to a linear time-invariant (LTI) system is given by

x[0] = 0,
x[1] = 1,
x[2] = 1 and
x[n] = 0 for all other values of n

and the output of the LTI system is given by

y[0] = 1,
y[1] = 2,
y[2] = 1 and
y[n]=0 for all other values of n.

A. Is this system causal? Why or why not?

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The system is not causal because y becomes nonzero before x does, i.e., y[0]=1 but x[0]=0.

B. What are the nonzero values of the output of this LTI system when the input is

x[0] = 0,
x[1] = 1,
x[2] = 1,
x[3] = 1,
x[4] = 1 and
x[n] = 0 for all other values of n?

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The easiest approach is by superposition:

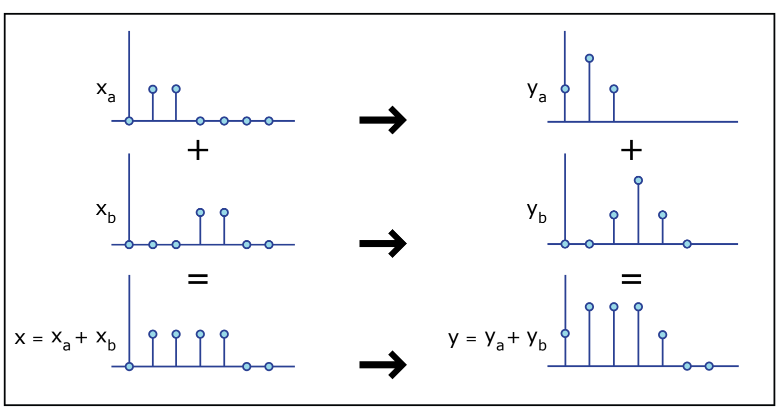
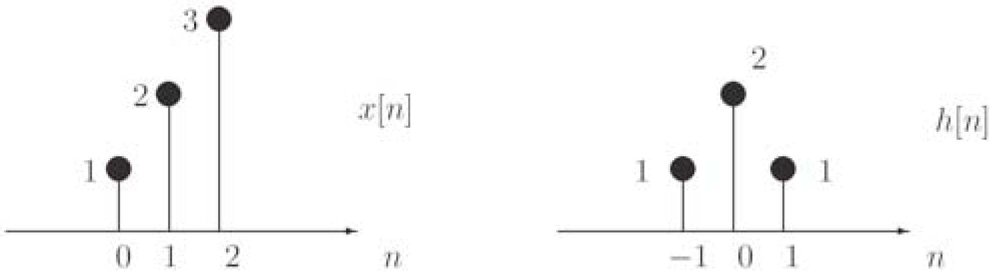


Image by MIT OpenCourseWare.

So y = 1, 2, 2, 2, 1, 0, 0, ... when n ≥ 0, 0 otherwise.

Problem 2.

Determine the output `y[n]` for a system with the input `x[n]` and unit-sample response `h[n]` shown below. Assume `h[n]=0` and `x[n]=0` for any times `n` not shown.



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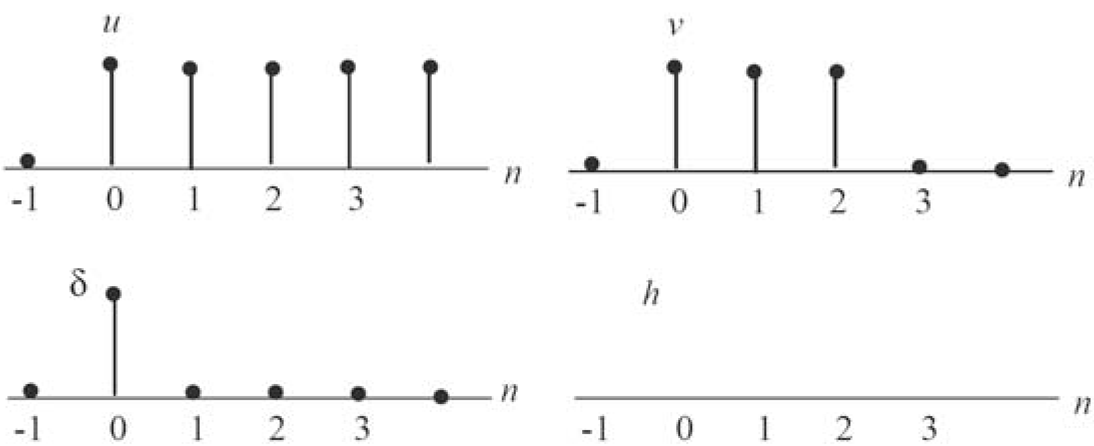
```
y[n] = Σx[k]h[n-k] = x[0]h[n] + x[1]h[n-1] + x[2]h[n-2]
     = δ[n+1] + 4δ[n] + 8δ[n-1] + 8δ[n-2] + 3δ[n-3]
```

Problem 3. A discrete-time linear system produces output `v` when the input is the unit step `u`. What is the output `h` when the input is the unit-sample δ? Assume `v[n]=0` for any times `n` not shown below.



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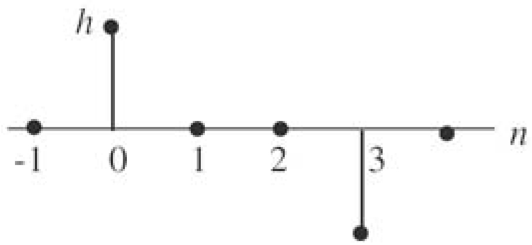
Note that

```
δ[n] = u[n] - u[n-1]
```

Since the system is linear we can compute the response of the system to the input δ[n] using the superposition of the appropriately scaled and shifted v[n]:

```
h[n] = v[n] - v[n-1]
```

The result is shown in the figure below:



Problem 4.

The output of a particular communication channel is given by

`y[n] = αx[n] + βx[n-1] where α > β`

1. Is the channel linear? Is it time invariant?

   Hide Answer

   To be linear the channel must meet two criteria:

   - if we scale the inputs `x[n]` by some factor `k`, the outputs `y[n]` should scale by the same factor.
   - if we get y_1`[n]` with inputs x_1`[n]` and y_2`[n]` with inputs x_2`[n]`, then we should get y_1`[n] +` y_2`[n]` if the input is x_1`[n] +` x_2`[n]`.

   It's easy to verify both properities given the channel response above, so the channel is linear.

   To be time invariant the channel must have the property that if we shift the input by some number of samples `s`, the output also shifts by `s` samples. Again that property is easily verified given the channel response above, so the channel is time invariant.

2. What is the channel's unit-sample response `h`?

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   The unit-sample input is `x[0]=1` and `x[n]=0` for `n≠0`.

   Using the channel response given above, the channel's unit-sample reponse can be computed as

   ```
   h[0]=α,
   h[1]=β,
   h[n]=0 for all other values of n
   ```

3. If the input is the following sequence of samples starting at time 0:

   `x[n] = [1, 0, 0, 1, 1, 0, 1, 1], followed by all 1's.`

   then what is the channel's output assuming `α=.7` and `β=.3`?

   Hide Answer

   Convolving `x[n]` with `h[n]` we get

   `y[n] = [.7, .3, 0, .7, 1, .3, .7, 1], followed by all 1's.`

4. Again let α=.7 and β=.3. Derive a deconvolver for this channel and compute the input sequence that produced the following output:

```
y[n] = [.7, 1, 1, .3, .7, 1, .3, 0], followed by all 0's.
```

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```
w[n] = (1/h[0])(y[n] - h[1]w[n-1]) = y[n]/.7 - (.3/.7)w[n-1]
```

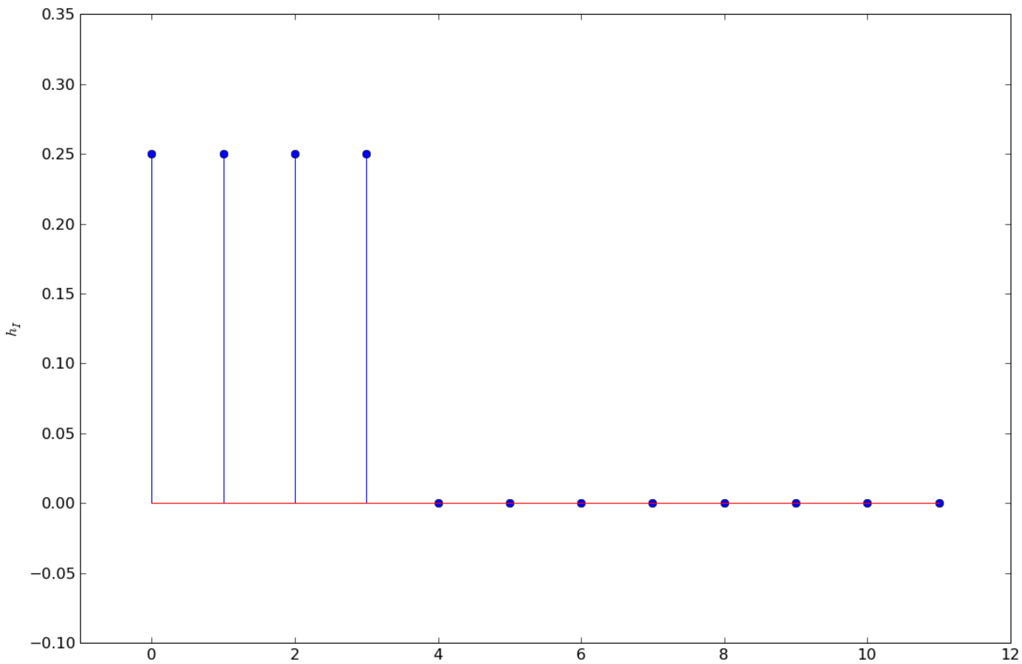
so

```
w[n] = [1, 1, 1, 0, 1, 1, 0, 0], followed by all 0's
```

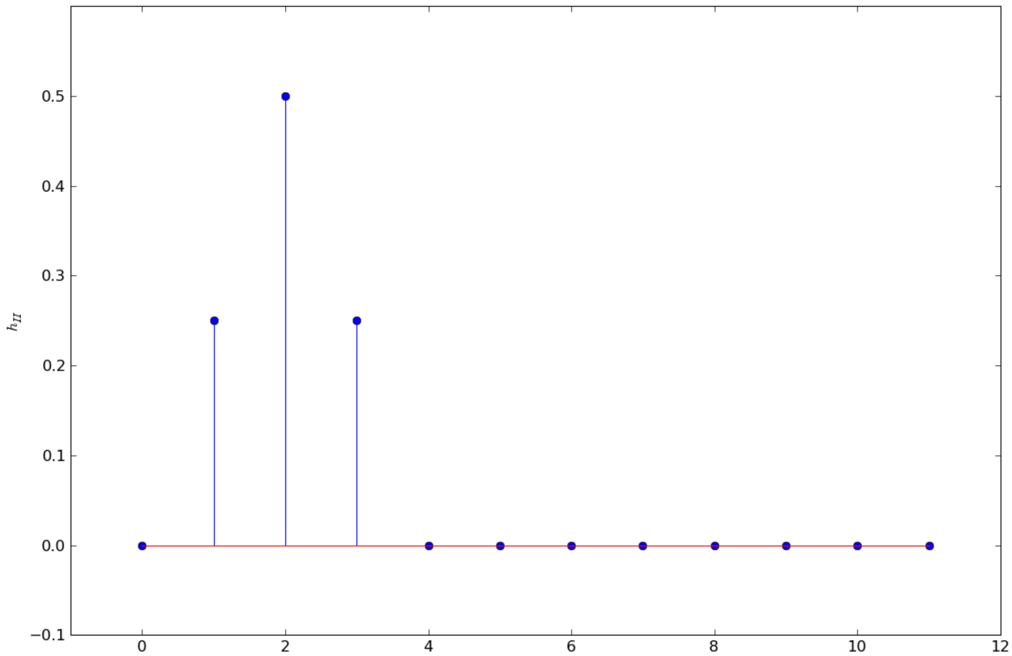
Problem 5.

Suppose four different channels {I,II,III,IIII} have four different unit sample responses:

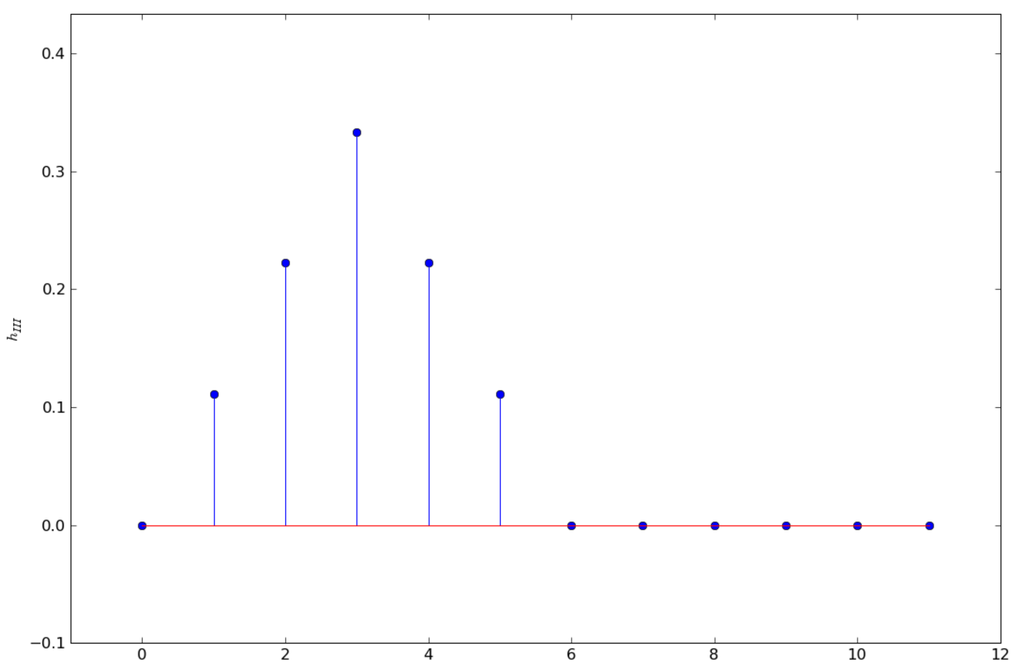
h1 = .25, .25, .25, .25, 0, ...



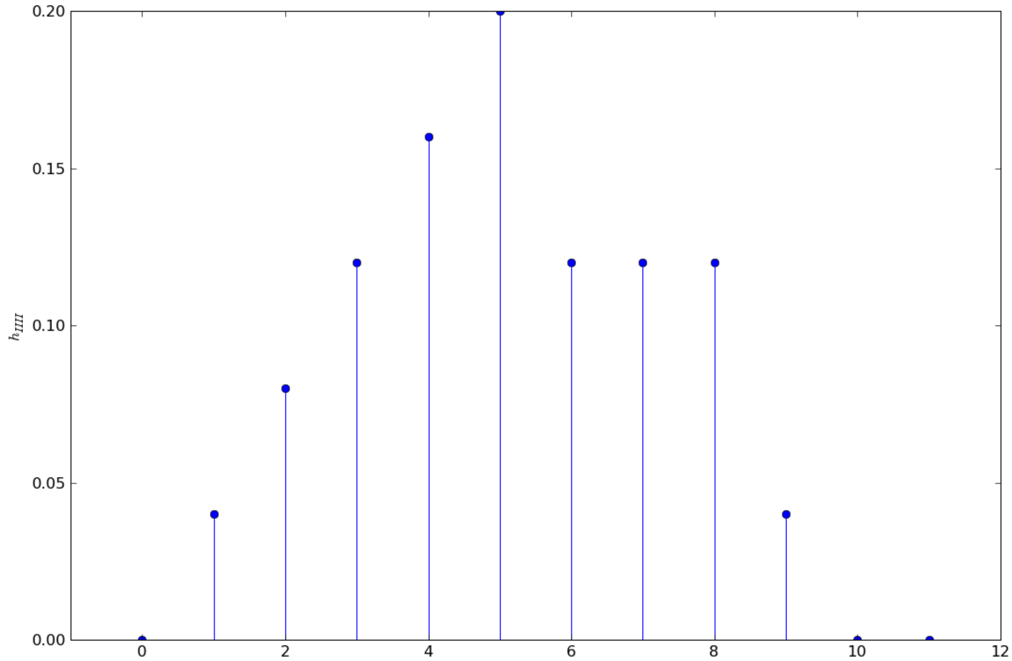
h2 = 0, .25, .5, .25, 0, ...



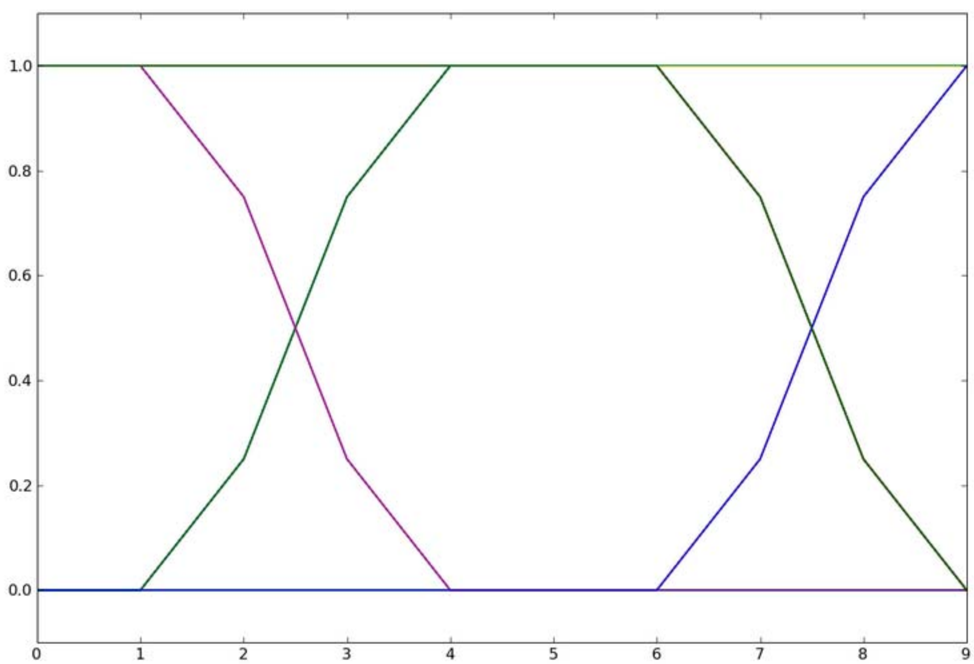
h3 = .11, .22, .33, .22, .11, 0, ...



h4 = .04, .08, .12, .16, .20, .12, .12, .12, .04, 0, ...



Each of the following eye diagrams is associated with transmitting bits using one of the four channels, where five samples were used per bit. That is, a one bit is five one-volt samples and a zero bit is five zero-volt samples. Please determine which channel was used in each case.



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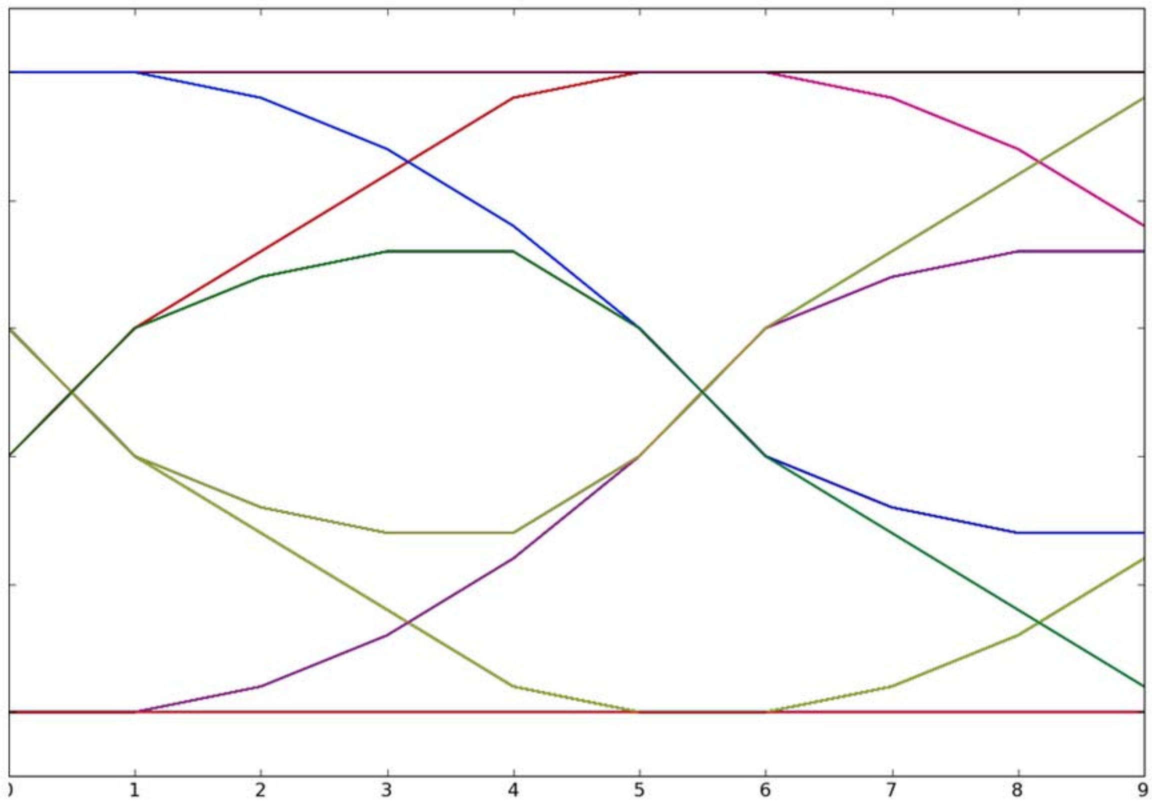
The eye diagram is from h2. Note that the signal transitions take three samples to complete and that the transitions occur in 3 steps with a larger slope in the middle step, an indicator of a response with 3 taps with a larger middle tap.



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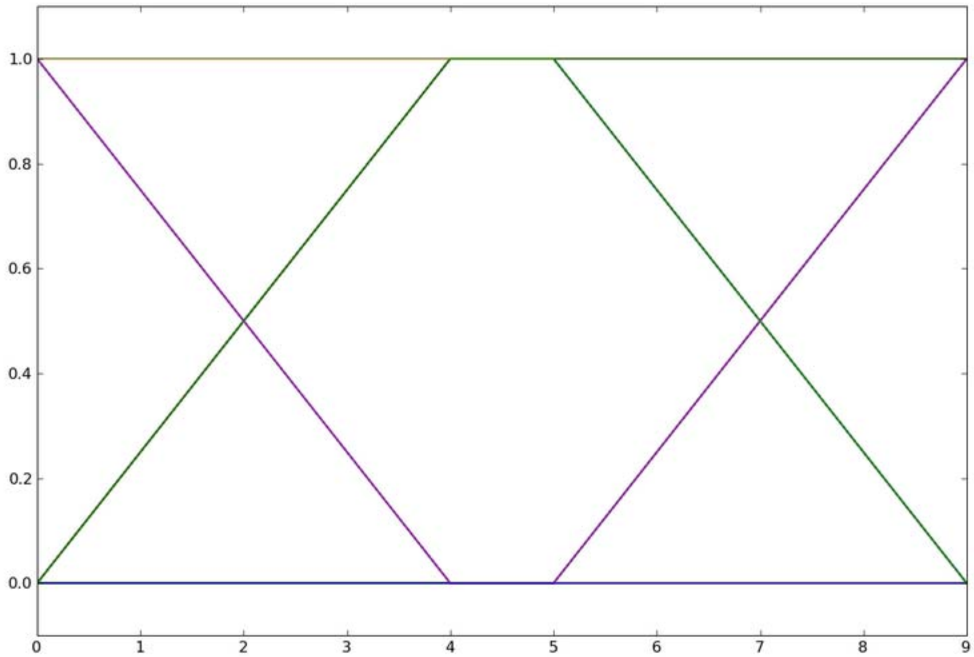
The eye diagram is from h4. Note that the signal transitions take more than five samples to complete and hence result in considerable inter-symbol interference. Response h4 is the only response that's non-zero for more than 5 taps.



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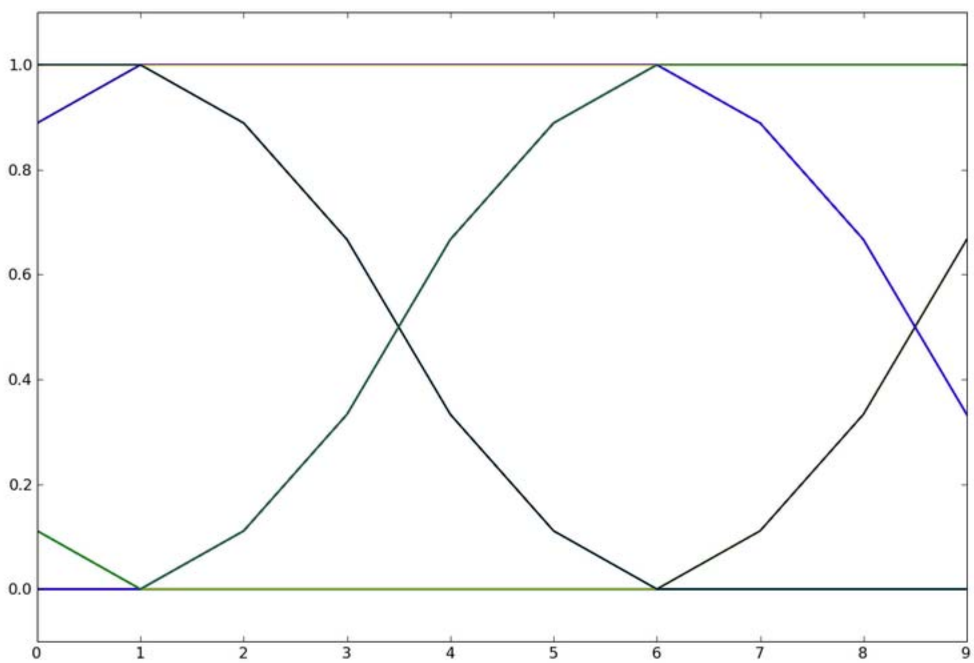
The eye diagram is from h1. Note that the signal transitions take four samples to complete and that the transitions have constant slope, an indicator of a response with 4 equal taps.



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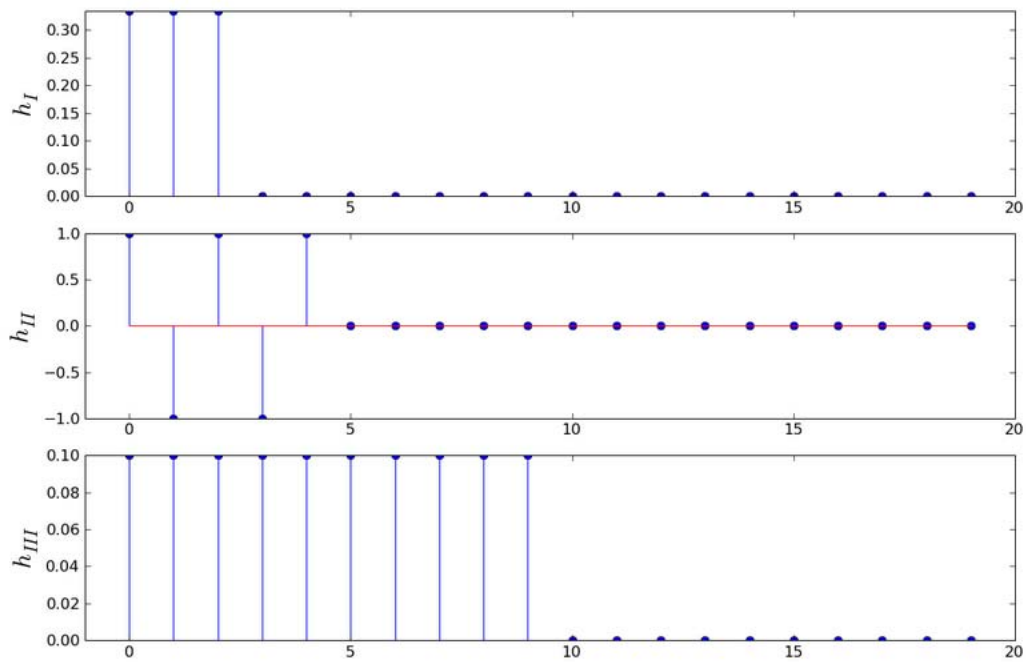
The eye diagram is from h3. Note that the signal transitions take five samples to complete and that the transitions occur in 5 steps with larger slopes in the middle of the transition, an indicator of a response with 5 taps with larger middle taps.

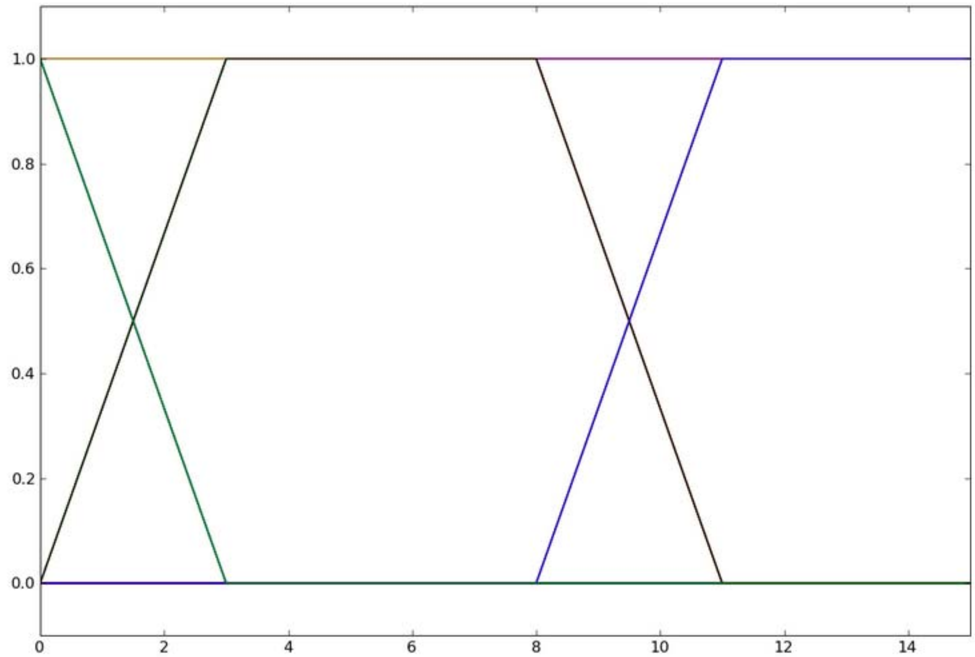
Problem 6.

This question refers to the LTI systems, I, II and III, whose unit-sample responses are shown below:



In this question, the input to these systems are bit streams with eight voltage samples per bit, with eight one-volt samples representing a one bit and eight zero-volt samples representing a zero bit.

A. Which system (I, II or III) generated the following eye diagram? To ensure at least partial credit for your answer, explain what led you to rule out the systems you did not select.



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The rise or fall time of a transition as seen in the eye diagram is only 3 samples, so it can only be System I since it's the only system with a 3-sample unit-sample response. Note that all the systems have some ISI, but System I's ISI is limited to only 3 samples after which the received signal is stable for the remainder of the bit cell.

Problem 7.

Suppose a linear time-variant channel has a unit sample response given by

```
h[n] = 1/2  n = 0, 1, 2
h[n] = 0    otherwise
```

If the input to the channel is

```
x[n] = 3/2  n = 2, 3, 4
x[n] = 0    otherwise
```

please determine the maximum value of the output of the channel and the index at which that maximum occurs.

Hide Answer

In this example, there are three non-zero `h[n]` values, so

```
y[n] = x[n]h[0] + x[n-1]h[1] + x[n-2]h[2]
```

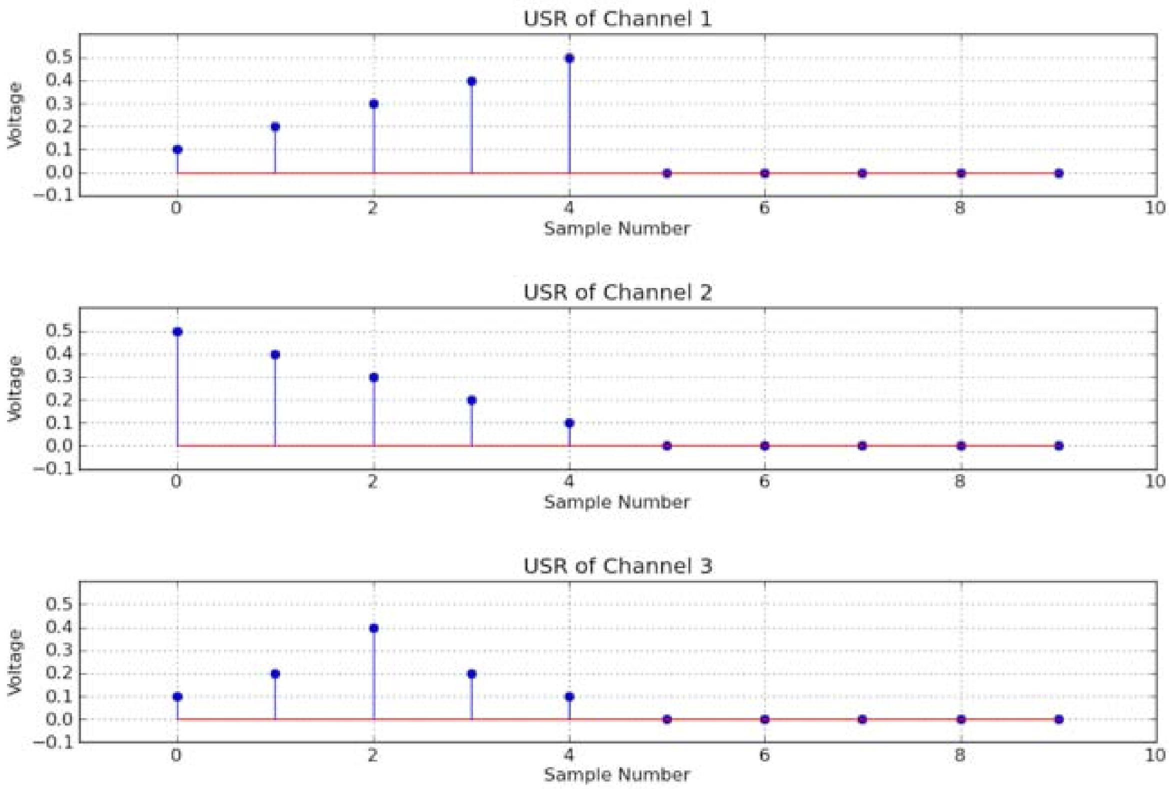
and three non-zero `x[n]` values, all positive. So the maximum value of `y[n]` occurs when using all three `x[n]` values:

```
y[4] = x[4]h[0] + x[3]h[1] + x[2]h[2]
```

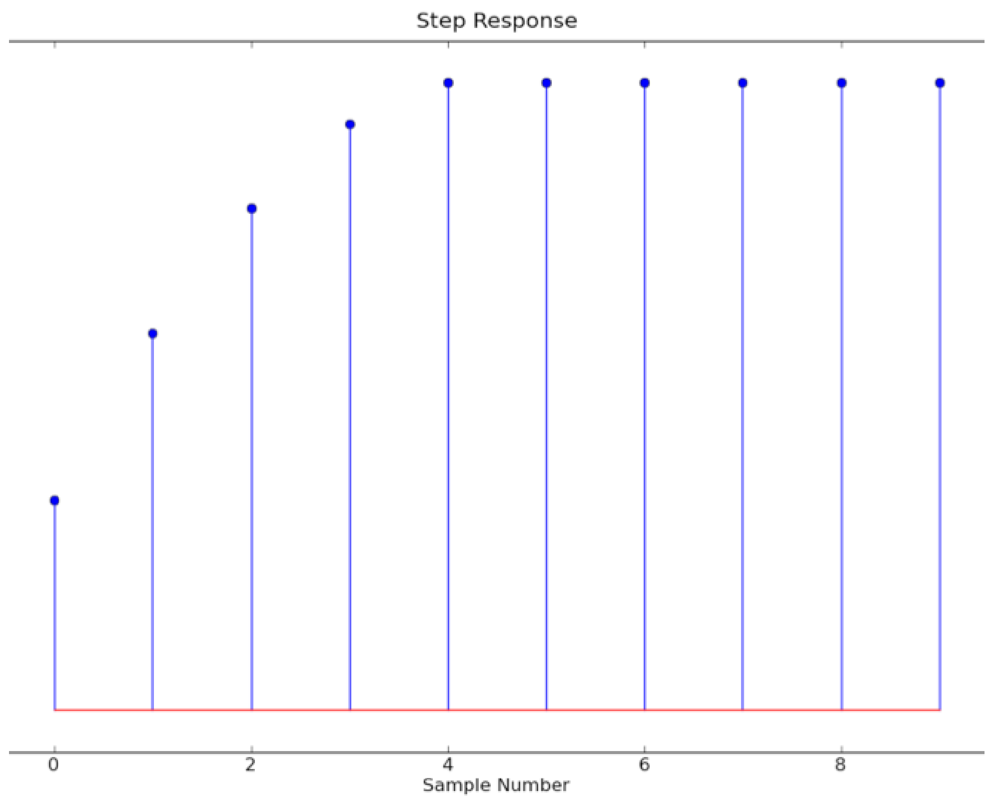
```
= (3/2)(1/2) + (3/2)(1/2) + (3/2)(1/2)
= 9/4
```

Problem 8.

For this problem, please consider three linear and time-invariant channels, *channel one*, *channel two*, and *channel three*. The unit sample response for each of these three channels are plotted below. Please use these plots to answer all the parts of this question.



A. Which channel (1, 2, or 3) has the following step response, and what is the value of maximum value of the step response?



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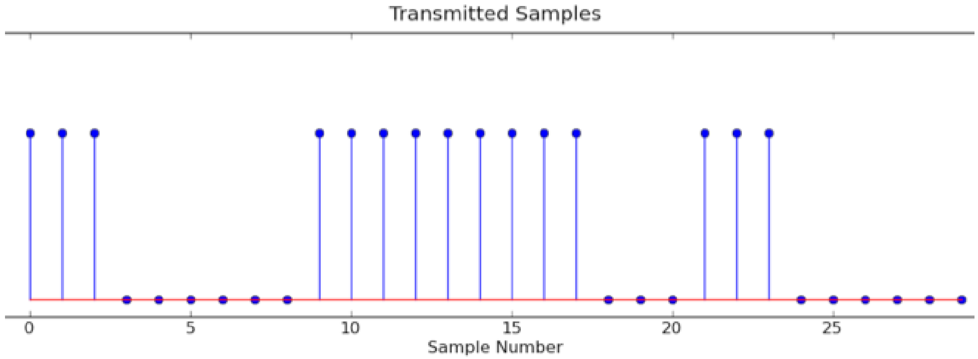
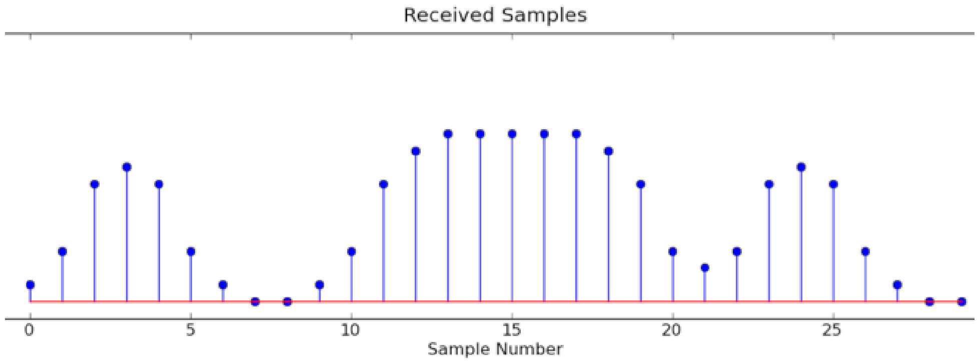
In this step response, the largest change in value happens on the first sample, with successively smaller steps in subsequent samples. This corresponds to the unit sample response for Channel 2.

The maximum value happens after the transition is complete, so we just need to sum all the `h[n]` values:

`Σh[n] = 0.5 + 0.4 + 0.3 + 0.2 + 0.1 = 1.5`

B. Which channel (1, 2, or 3) produced the pair of transmitted and received samples in the graph below, and what is the value of voltage sample number 24 (assuming the transmitted samples have the value of either one volt or zero volts)?





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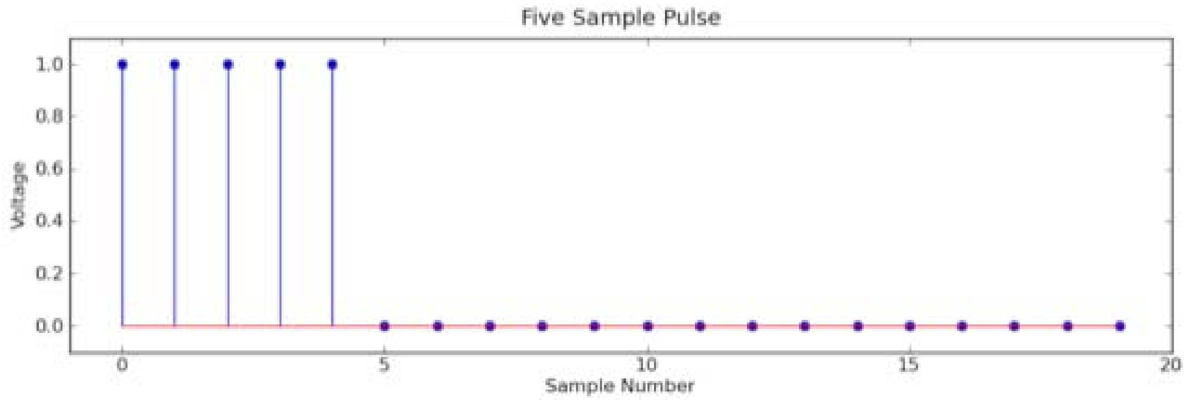
Looking at the transitions in the response, the largest change in value happens in the middle of the transition, corresponding to the unit sample response for Channel 3.

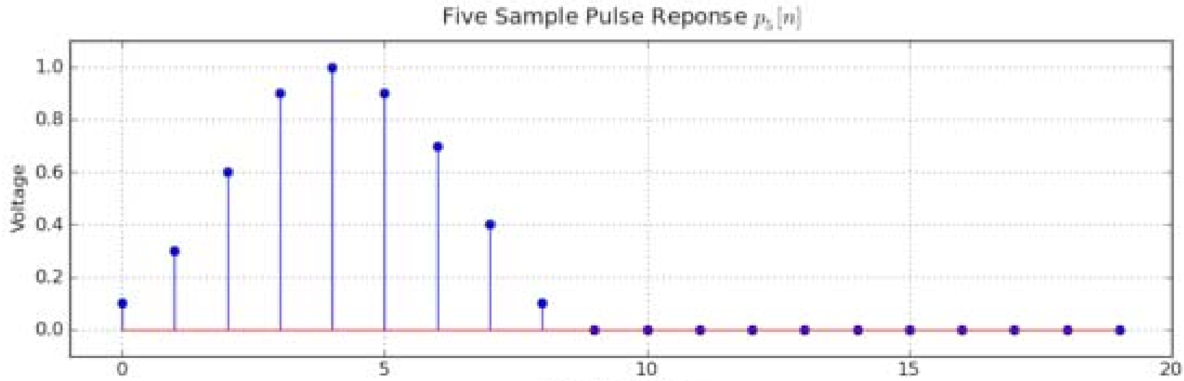
Using the convolution sum and noticing that `h[m] = 0, m ≥ 5`:

```
y[24] = x[24]h[0] + x[23]h[1] + x[22]h[2] + x[21]h[3] + x[20]h[4]
      = (0)(0.1) + (1)(0.2) + (1)(0.4) + (1)(0.2) + (0)(0.2)
      = 0.2 + 0.4 + 0.2
      = 0.8
```

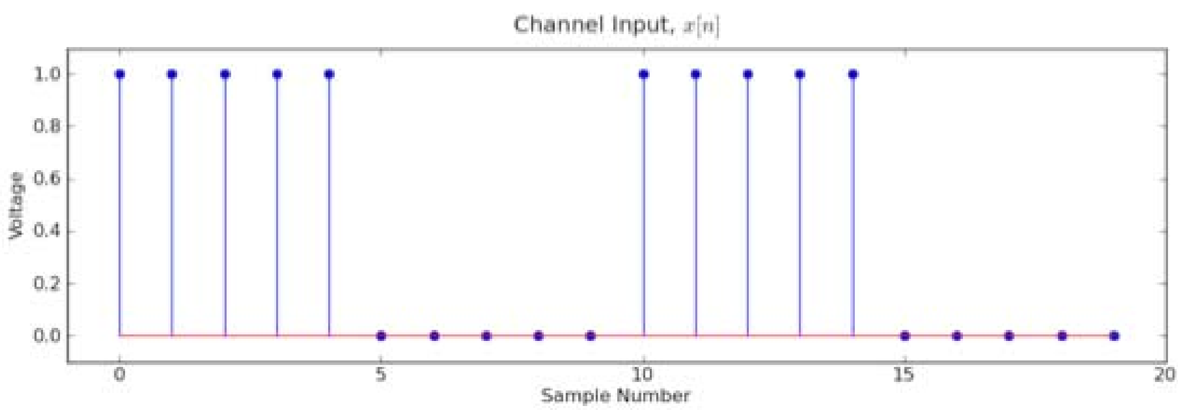
Problem 9.

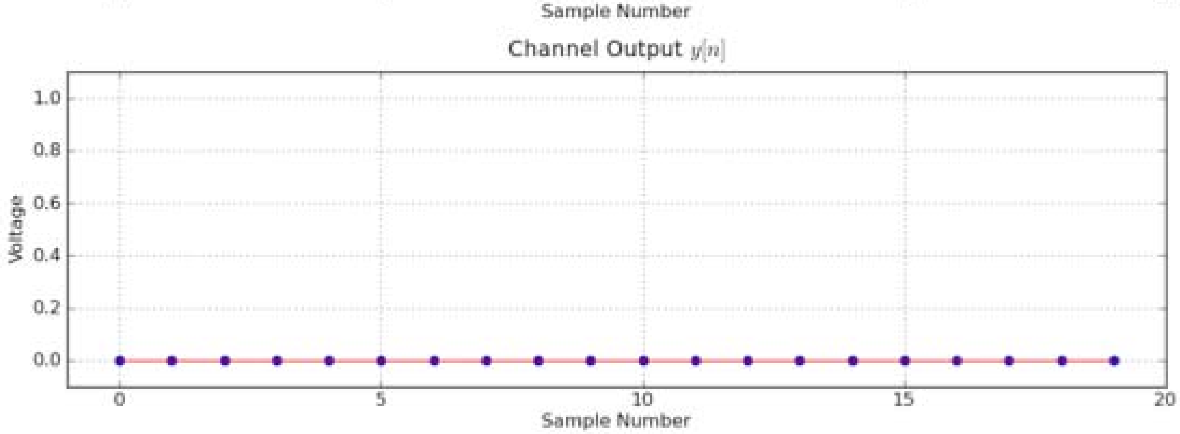
In this problem you will be answering questions about a causal linear time-invariant channel characterized by its response to a five-sample pulse, denoted $p_5[n]$.





A. Suppose the input to the channel is as plotted below. Plot the output of the channel on the axes provided beneath the input.





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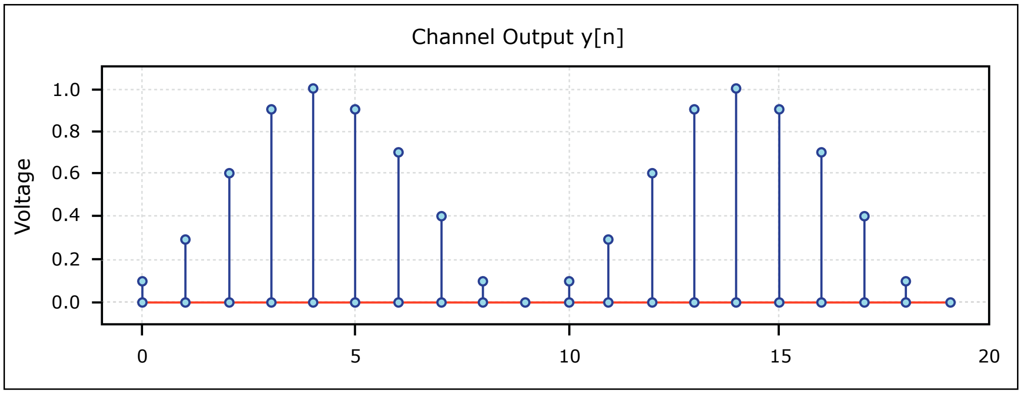
Noticing that

```
x[n] = pulse[n] + pulse[n-10]
```

We can use superposition to determine the response:

$y[n] = p_5[n] + p_5[n-10]$

which gives us the following picture:



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B. The unit sample response, `h[n]`, can be related to the step response, `s[n]` by the formula `h[n] = s[n] - s[n-1]`. Please derive a similar formula for `h[n]` in terms of the five-sample pulse response $p_5[n]$ (an infinite series is an acceptable form for the answer).

Hide Answer

We can construct a unit step waveform by adding together copies of the 5-sample pulse spaced at offsets of 5 samples:

$u[n] = \Sigma_k \text{pulse}[n - 5k] \quad \text{for } k = 0, 1, ..., \infty$

So that means the unit step response is a scaled, time-shifted sum of $p_5[n]$:

$s[n] = \Sigma_k p_5[n - 5k] \quad \text{for } k = 0, 1, ..., \infty$

Now we can derive an expression for `h[n]`:

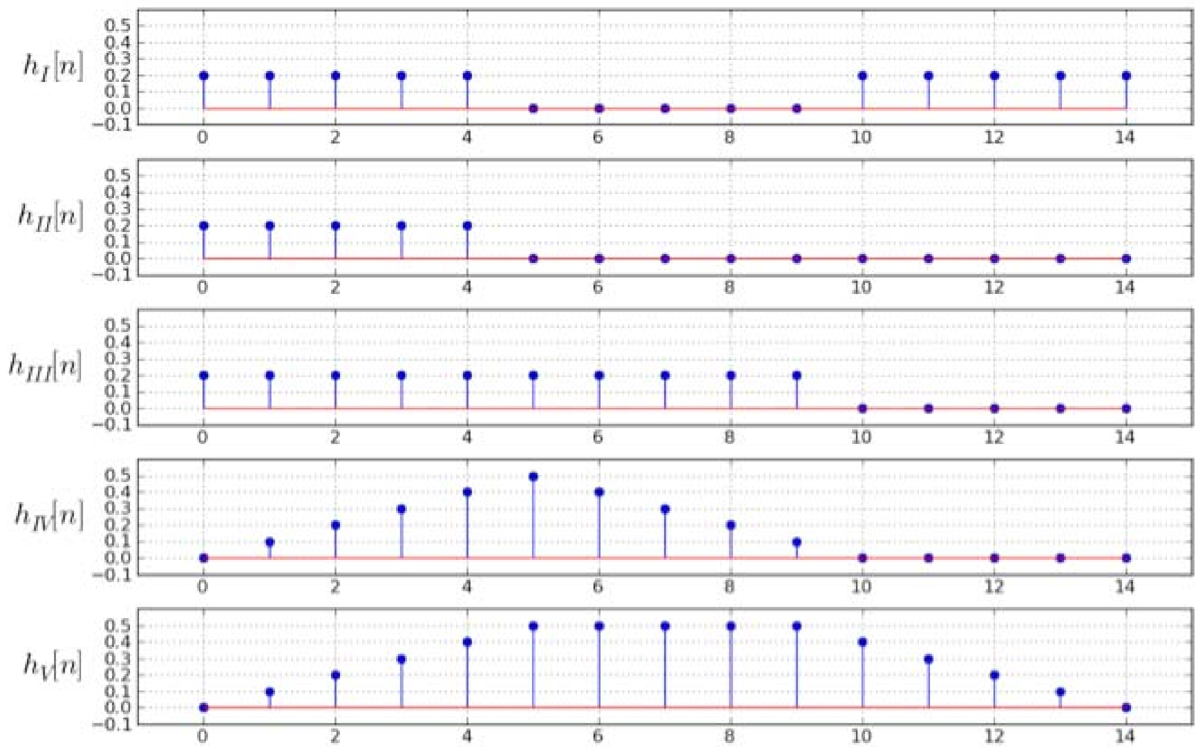
$h[n] = s[n] - s[n-1] = \Sigma_k (p_5[n - 5k] - p_5[n - 5k - 1])$

Problem 10.

For all parts of this problem, please consider five linear and time-invariant channels, cleverly titled channel I, channelII, channel III, channel IV and channel V. The unit sample response for each of these five channels is plotted below, with the values outside the interval 0 to 14 being zero. Please use these plots to answer all the parts of this problem.

Please note:

- **All the voltage values in the five plots are integer multiples of 0.1 volt.**
- **A particular channel can be the answer to more than one part.**



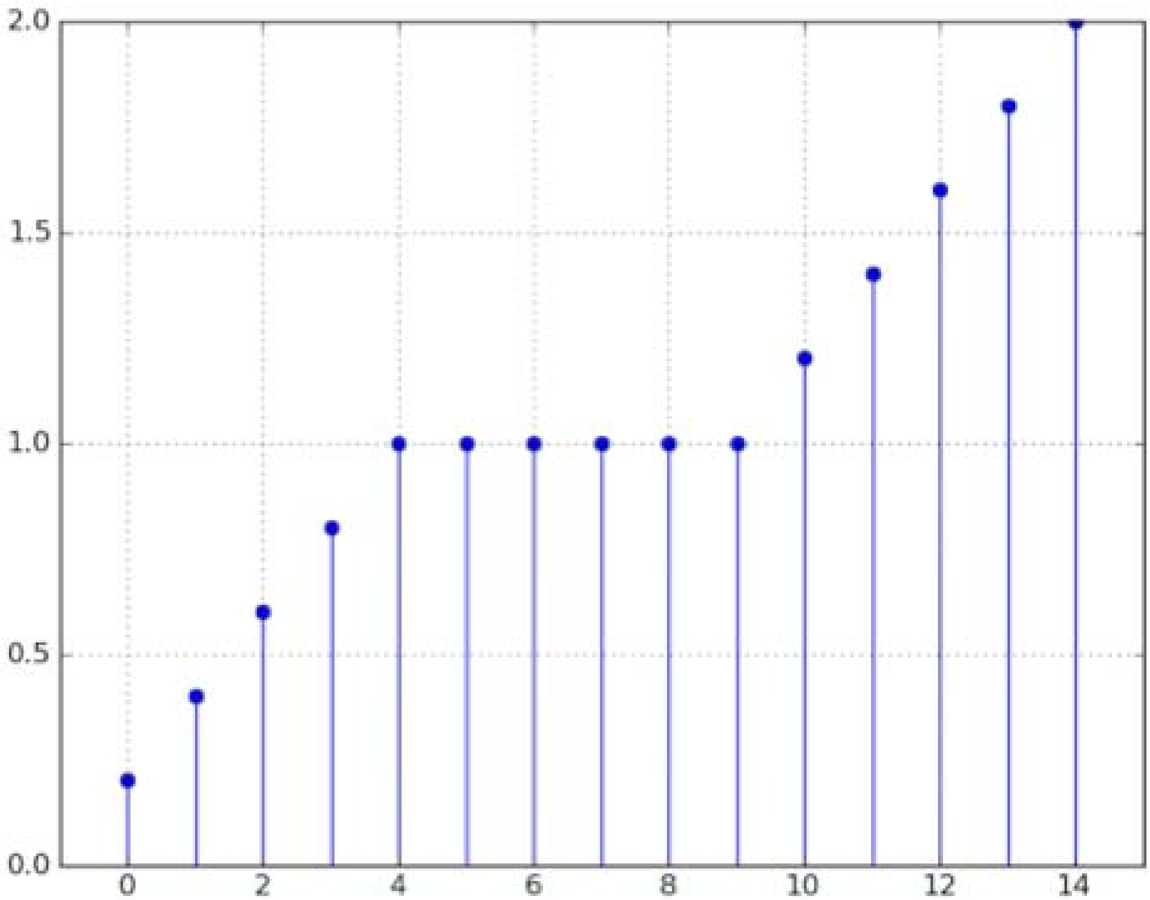
A. Plot the unit step response `s[n]` for Channel I for $0 \le n \le 14$.

Hide Answer

Recall that the unit step response is the cumulative sum of the unit sample response:

`s[n] = Σk h[k]    for k = 0, 1, ..., n`

This gives the following plot:



B. Which two channels have step responses, `s[n]`, that approach the same value as $n \to \infty$ and what is that value?

Hide Answer

Recall that the unit step response is the cumulative sum of the unit sample response:

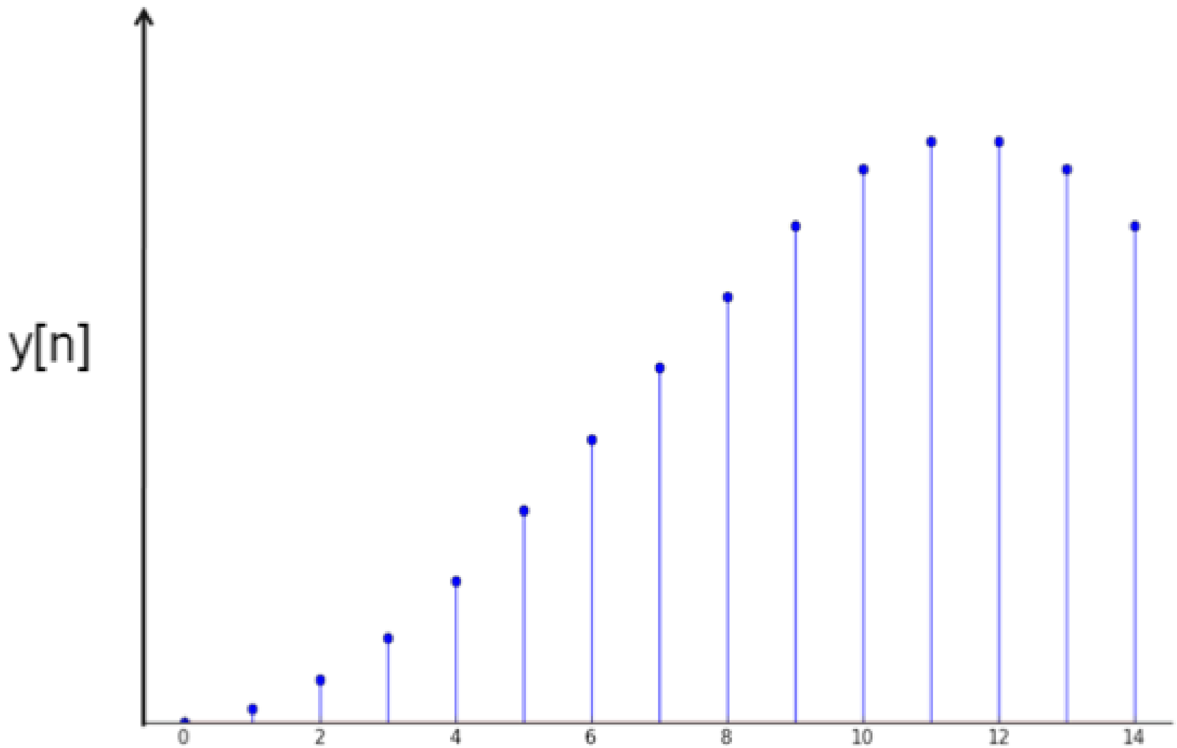
$s[n] = \Sigma_k h[k]$ for $k = 0, 1, ..., n$

As $n \to \infty$, we'll add up all the non-zero `h[n]` for each channel to get the value for $s[\infty]$:

$s_I[\infty] = 2$
$s_{II}[\infty] = 1$
$s_{III}[\infty] = 2$
$s_{IV}[\infty] = 2.5$
$s_V[\infty] = 4.5$

So the channels are I and III, which both have the final value of 2.

C. Suppose the input to each of the channels is $x[n] = 1$ for $0 \le n \le 9$ and zero otherwise. Which channel has the output `y[n]` plotted below, and what is value of the $n = 15$ output sample (not plotted)?



Hide Answer

Answer: channel V, with y[15]=3.

Explanation: since all five channels under consideration have unit sample responses `h[n]` satisfying the condition `h[n] = 0` for $n < 0$, the channels are causal. Since `x[n]` equals `u[n]` (the unit step sequence) for `n < 10`, `y[n]` equals the unit step response of the channel for `n < 10`. Since the *increments* of the unit step response are the values of the unit sample response, and since the *increments* of `x[n]`, as shown, are first increasing (until $n = 5$), and then not decreasing much (until $n = 10$), the unit sample response of the channel is increasing until $n = 5$ and then not decreasing much until $n = 10$, which leaves channel V as the only option.

For channel V, we have

```
y[15] = h[0]x[15] + h[1]x[14] + ... + h[15]x[0]
      = h[6] + h[7] + ... + h[15]
      = 0.5 + 0.5 + 0.5 + 0.5 + 0.4 + 0.3 + 0.2 + 0.1
      = 3
```