Show All Answers Hide All Answers

6.02 Practice Problems: Noise & Bit Errors

Problem 1.

Suppose the bit detection sample at the receiver is `V + noise` volts when the sample corresponds to a transmitted '1', and `0.0 + noise` volts when the sample corresponds to a transmitted '0', where `noise` is a zero-mean Normal(Gaussian) random variable with standard deviation σ_{NOISE}.

A. If the transmitter is equally likely to send '0"s or '1"s, and `V/2` volts is used as the threshold for deciding whether the received bit is a '0' or a '1', give an expression for the bit-error rate (BER) in terms of the erfc function and σ_{NOISE}.

Show Answer

B. Suppose the transmitter is equally likely to send zeros or ones and uses zero volt samples to represent a '0' and one volt samples to represent a '1'. If the receiver uses 0.5 volts as the threshold for deciding bit value, for what value of σ_{NOISE} is the probability of a bit error approximately equal to 1/5?

Show Answer

C. Will your answer for σ_{NOISE} in part (B) change if the threshold used by the receiver is shifted to 0.6 volts? Do not try to determine σ_{NOISE}, but justify your answer.

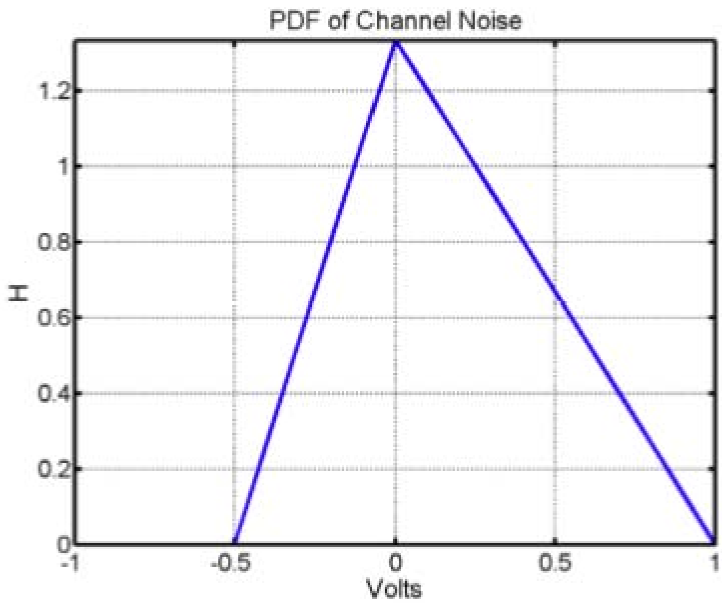
Show Answer

D. Will your answer for σ_{NOISE} in part (B) change if the transmitter is twice as likely to send ones as zeros, but the receiver still uses a threshold of 0.5 volts? Do not try to determine σ_{NOISE}, but justify your answer.

Show Answer

Problem 2.

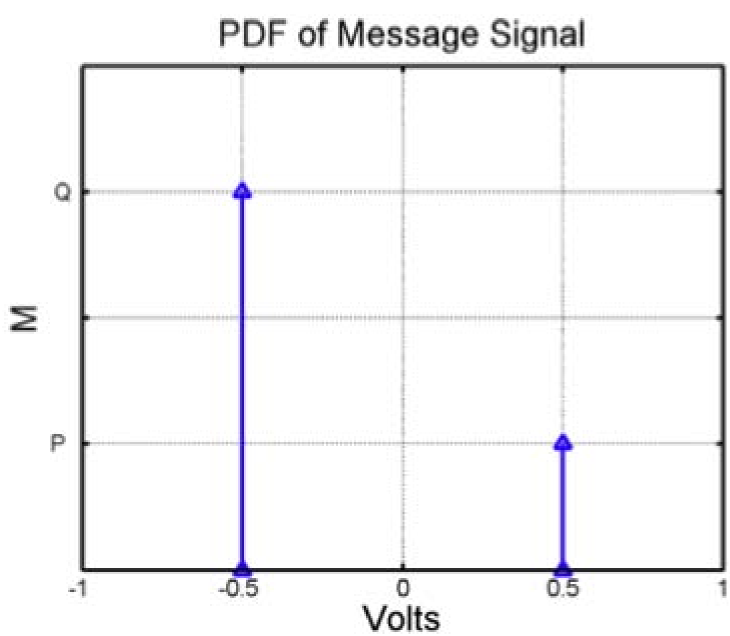
Messages are transmitted along a noisy channel using the following protocol: a "0" bit is transmitted as -0.5 Volt and a "1" bit as 0.5 Volt. The PDF of the total noise added by the channel, H, is shown below. It is not a Gaussian.



a. Compute H(0), the maximum value of H.

Show Answer

b. It is known that a "0" bits 3 times as likely to be transmitted as a "1" bit. The PDF of the message signal, M, is shown below. Fill in the values P and Q.



Show Answer

c. If the digitization threshold voltage is 0V, what is the bit error rate?

Show Answer

d. What digitization threshold voltage would minimize the bit error rate?

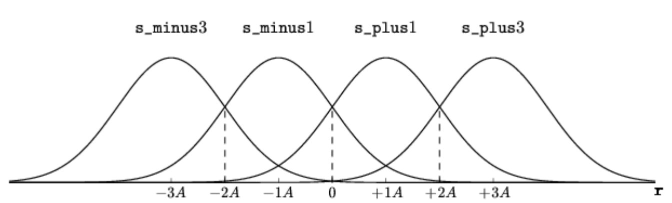
Show Answer

Problem 3.

Ben Bitdiddle studies the bipolar signaling scheme from 6.02 and decides to extend it to a **4-level signaling scheme**, which he calls Ben's Aggressive Signaling Scheme, or **BASS**. In BASS, the transmitter can send four possible signal levels, or voltages: $(-3A, -A, +A, +3A)$, where A is some positive value. To transmit bits, the sender's mapper maps consecutive pairs of bits to a fixed voltage level that is held for some fixed interval of time, creating a **symbol**. For example, we might map bits “00” to $-3A$, “01” to $-A$, “10” to $+A$, and “11” to $+3A$. Each distinct pair of bits corresponds to a unique symbol. Call these symbols s_minus3, s_minus1, s_plus1, and s_plus3. Each symbol has the same prior probability of being transmitted.

The symbols are transmitted over a channel that has no distortion but does have additive noise, and are sampled at the receiver in the usual way. Assume the samples at the receiver are perturbed from their ideal noise-free values by a zero-mean additive white Gaussian noise (AWGN) process with noise intensity $N_0 = 2\sigma^2$, where σ^2 is the variance of the Gaussian noise on each sample. In the time slot associated with each symbol, the BASS receiver digitizes a selected voltage sample, r, and returns an estimate, s, of the transmitted symbol in that slot, using the following intuitive digitizing rule (written in Python syntax):

```
def digitize(r):
    if r < -2A:
        s = sminus3
    elif r < 0:
        s = sminus1
    elif r < 2A:
         s = splus1
    else: s = splus3
    return s
```



1. The power of a symbol transmission is defined as the square of the voltage level at which the symbol is **transmitted**. What is the **average** power level, P, of a symbol **transmission** in BASS (i.e., the average power dissipated at the transmitter)?

   Show Answer

   Ben wants to calculate the **symbol error rate** for BASS, i.e., the probability that the symbol chosen by the receiver was different from the symbol transmitted. Note: we are **not** interested in the **bit** error rate here. Help Ben calculate the symbol error rate by answering the following questions below from 2 through 8

2. Suppose the sender transmits symbol **s_plus3**. What is the **conditional** symbol error rate given this information; i.e., what is **P(symbolError| s_plus3 sent)** ? Express your answer in terms of A, N_0, and the erfc function, defined as $\text{erfc}(z) = \frac{2}{\sqrt{\pi}}\int_z^{\infty} e^{-x^2}\,dx$.

Show Answer

3. Now suppose the sender transmits symbol **s_plus1**. What is the **conditional** symbol error rate given this information, in terms of A, N_0, and the erfc function? The conditional symbol error rates for the other two symbols don't need to be calculated separately.

Show Answer

4. The symbol error rate when the sender transmits symbol **s_minus3** is the same as the symbol error rate of which of these symbols?
    1. s_minus1.
    2. s_plus1.
    3. s_plus3.

Show Answer

5. The symbol error rate when the sender transmits symbol **s_minus1** is the same as the symbol error rate of which of these symbols?
    1. s_minus3.
    2. s_plus1.
    3. s_plus3.

Show Answer

6. Now suppose the sender transmits symbol **s_minus1**. What is the **conditional** symbol error rate given this information, in terms of A, N_0, and the erfc function? Do not recalculate; use symmetry and your previous answer(s).

Show Answer

7. Combining your answers to the questions above, what is the symbol error rate in terms of A, N_0, and the erfc function? Recall that all symbols are equally likely to be transmitted.

Show Answer

8. Express the answer to the above question in terms of the signal-to-noise ratio. The average signal power is in the answer to one of the questions above.

Show Answer