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6.02 Practice Problems: Error Correcting Codes

Problem 1. For each of the following sets of codewords, please give the appropriate (n,k,d) designation where n is number of bits in each codeword, k is the number of message bits transmitted by each code word and d is the minimum Hamming distance between codewords. Also give the code rate.

A. {111, 100, 001, 010}

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B. {00000, 01111, 10100, 11011}

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C. {00000}

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Problem 2. Suppose management has decided to use 20-bit data blocks in the company's new (n,20,3) error correcting code. What's the minimum value of n that will permit the code to be used for single bit error correction?

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Problem 3. The Registrar has asked for an encoding of class year ("Freshman", "Sophomore", "Junior", "Senior") that will allow single error correction. Please give an appropriate 5-bit binary encoding for each of the four years.

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Problem 4. For any block code with minimum Hamming distance at least 2t + 1 between code words, show that:

$$2^{n-k} \geq 1 + \binom{n}{1} + \binom{n}{2} + \ldots \binom{n}{t}.$$

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Problem 5. Pairwise Communications has developed a block code with three data (D1, D2, D3) and three parity bits (P1, P2, P3):

```
P1 = D1 + D2
P2 = D2 + D3
P3 = D3 + D1
```

A. What is the (n,k,d) designation for this code.

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B. The receiver computes three syndrome bits from the (possibly corrupted) received data and parity bits:

```
E1 = D1 + D2 + P1
E2 = D2 + D3 + P2
E3 = D3 + D1 + P3.
```

The receiver performs maximum likelihood decoding using the syndrome bits. For the combinations of syndrome bits listed below, state what the maximum-likelihood decoder believes has occured: no errors, a single error in a speciï¬c bit (state which one), or multiple errors.

```
E1 E2 E3 = 000
E1 E2 E3 = 010
E1 E2 E3 = 101
E1 E2 E3 = 111
```

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Problem 6. Dos Equis Encodings, Inc. specializes in codes that use 20-bit transmit blocks. They are trying to design a (20, 16) linear block code for single error correction. Explain whether they are likely to succeed or not.

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Problem 7. Consider the following (n,k,d) block code:

```
D0  D1  D2  D3  D4   | P0
D5  D6  D7  D8  D9   | P1
D10 D11 D12 D13 D14  | P2
-------------------------
P3  P4  P5  P6  P7   |
```

where D0-D14 are data bits, P0-P2 are row parity bits and P3-P7 are column parity bits. The transmitted code word will be:

D0 D1 D2 ... D13 D14 P0 P1 ... P6 P7

A. Please give the values for n, k, d for the code above.

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B. If `D0 D1 D2 ... D13 D14 = 0 1 0 1 0, 0 1 0 0 1, 1 0 0 0 1`, please compute P0 through P7.

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C. Now we receive the four following code words:

```
M1: 0 1 0 1 0, 0 1 0 0 1, 1 0 0 0 1, 0 0 0 1 1 0 1 0
M2: 0 1 0 1 0, 0 1 0 0 1, 1 0 0 0 1, 0 0 1 1 1 0 1 0
M3: 0 1 0 1 0, 0 1 0 0 1, 1 0 0 0 1, 1 1 0 1 1 0 1 0
M4: 0 1 0 1 0, 0 1 0 0 1, 1 0 0 0 1, 1 0 0 1 1 0 1 0
```

For each of received code words, indicate the number of errors. If there are errors, indicate if they are correctable, and if they are, what the correction should be.

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Problem 8. The following matrix shows a rectangular single error correcting code consisting of 9 data bits, 3 row parity bits and 3 column parity bits. For each of the examples that follow, please indicate the correction the receiver must perform: give the position of the bit that needs correcting (e.g., D7, R1), or "no" if there are no errors, or "M" if there is a multi-bit uncorrectable error.

D1	D2	D3	R1
D4	D5	D6	R2
D7	D8	D9	R3
C1	C2	C3	—

1	1	1	1
1	1	0	0
0	1	1	0
0	1	1	–

0	0	1	0
0	1	1	1
0	1	1	0
0	0	1	–

1	1	0	0
1	0	0	1
0	0	1	0
0	0	1	–

1	0	1	0
1	0	0	1
1	1	0	0
1	1	1	–

1	1	1	0
1	1	1	1
1	0	0	0
0	0	1	–

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Problem 9. Consider two convolutional coding schemes - I and II. The generator polynomials for the two schemes are

Scheme I: G0 = 1101, G1 = 1110
Scheme II: G0 = 110101, G1 = 111011

Notation is follows: if the generator polynomial is, say, 1101, then the corresponding parity bit for message bit n is

(x[n] + x[n-1] + x[n-3]) mod 2

where x[n] is the message sequence.

A. Indicate TRUE or FALSE
   a. Code rate of Scheme I is 1/4.
   b. Constraint length of Scheme II is 4.
   c. Code rate of Scheme II is equal to code rate of Scheme I.
   d. Constraint length of Scheme I is 4.

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B. How many states will there be in the state diagram for Scheme I? For Scheme II?

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C. Which code will lead to a lower bit error rate? Why?

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D. Alyssa P. Hacker suggests a modification to Scheme I which involves adding a third generator polynomial G2 = 1001. What is the code rate `r` of Alyssa's coding scheme? What about constraint length `k`? Alyssa claims that her scheme is stronger than Scheme I. Based on your computations for `r` and `k`, is her statement true?

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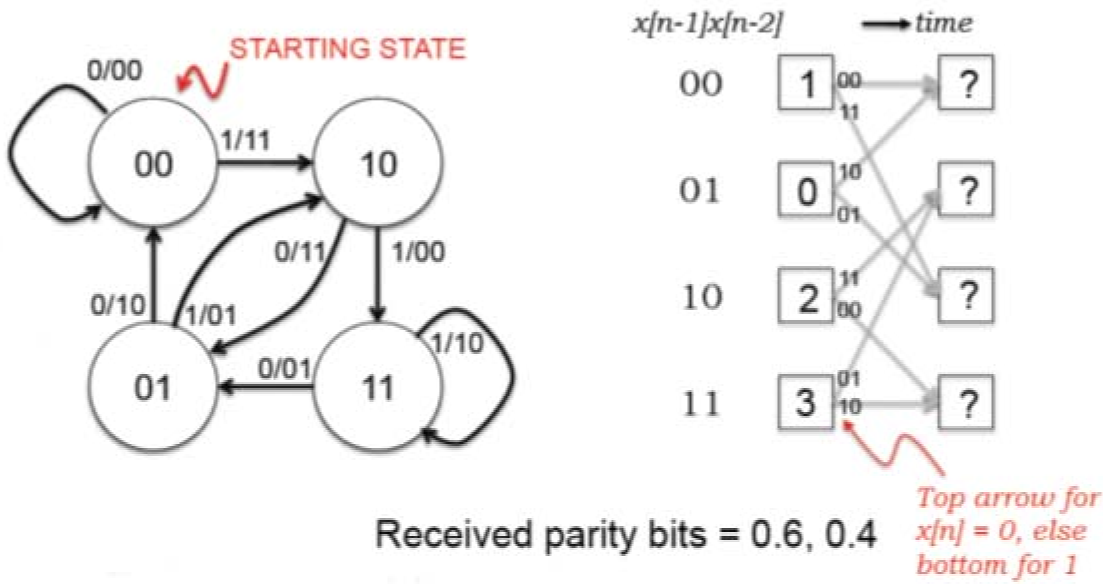
Problem 10. Consider a convolution code that uses two generator polynomials: G0 = 111 and G1 = 110. You are given a particular snapshot of the decoding trellis used to determine the most likely sequence of states visited by the transmitter while transmitting a particular message:



A. Complete the Viterbi step, i.e., fill in the question marks in the matrix, assuming a hard branch metric based on the Hamming distance between expected an received parity where the received voltages are digitized using a 0.5V threshold.

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B. Complete the Viterbi step, i.e., fill in the question marks in the matrix, assuming a soft branch metric based on the square of the Euclidean distance between expected an received parity voltages. Note that your branch and path metrics will not necessarily be integers.

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C. Does the soft metric give a different answer than the hard metric? Base your response in terms of the relative ordering of the states in the second column and the survivor paths.

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D. If the transmitted message starts with the bits "01011", what is the sequence of bits produced by the convolutional encoder?

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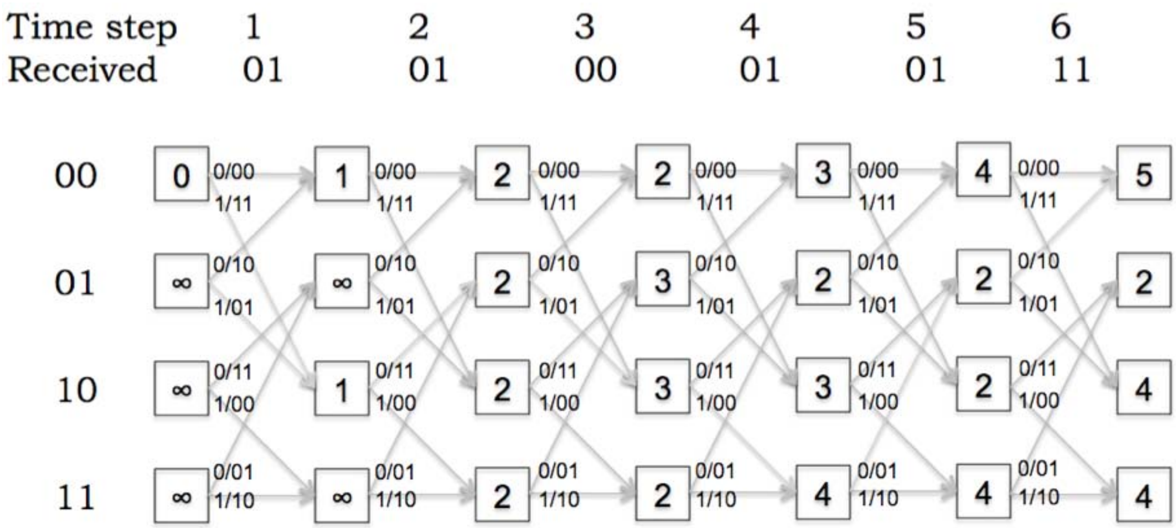
The receiver determines the most-likely transmitted message by using the Viterbi algorithm to process the (possibly corrupted) received parity bits. The path metric trellis generated from a particular set of received parity bits is shown below. The boxes in the trellis contain the minimum path metric as computed by the

Viterbi algorithm.



E. Referring to the trellis above, what is the receiver's estimate of the most-likely transmitter state after processing the bits received at time step 6?

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F. Referring to the trellis above, show the most-likely path through the trellis by placing a circle around the appropriate state box at each time step and darkening the appropriate arcs. What is the receiver's estimate of the most-likely transmitted message?

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G. Referring to the trellis above, and given the receiver's estimate of the most-likely transmitted message, at what time step(s) were errors detected by the receiver? Briefly explain your reason- ing.

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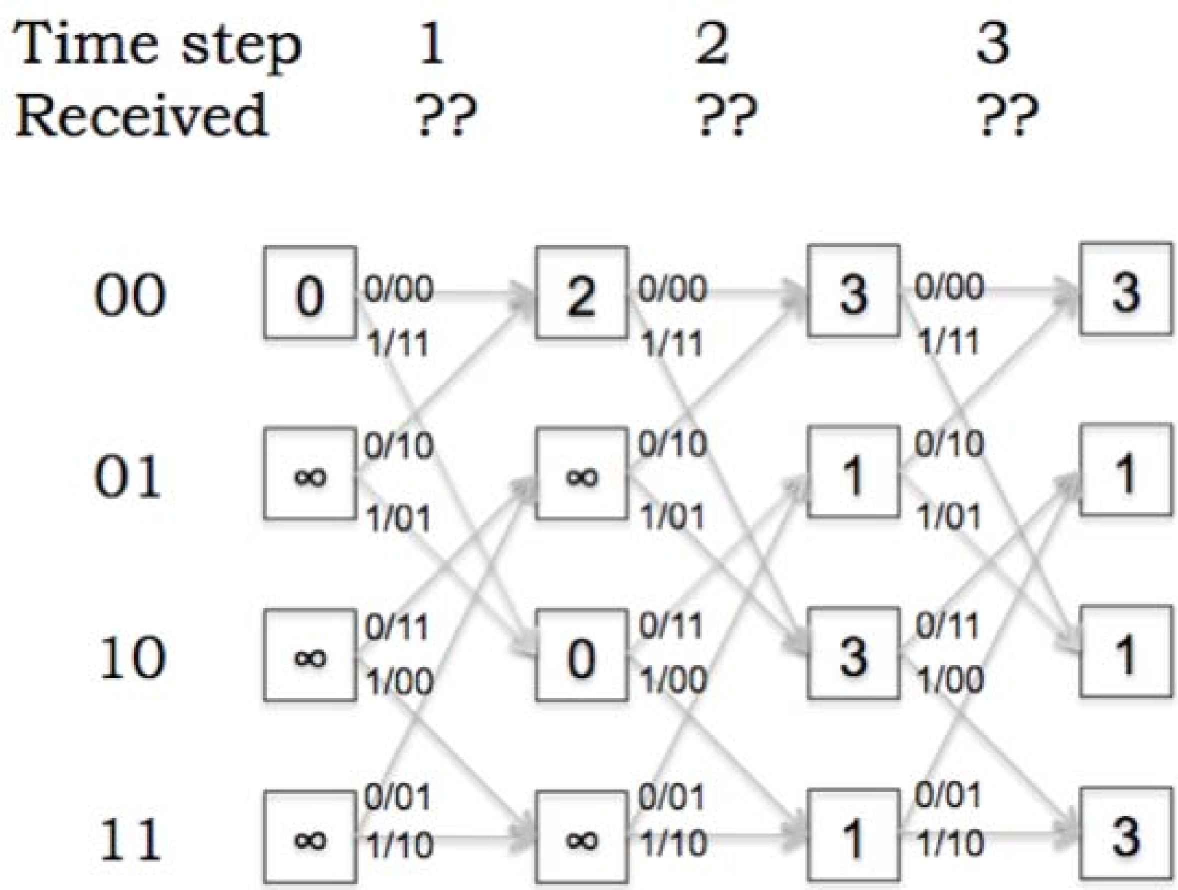
H. Now consider the path metric trellis generated from a *different* set of received parity bits.



Referring to the trellis above, determine which pair(s) of parity bits could have been been received at time steps 1, 2 and 3. Briefly explain your reasoning.

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Problem 11. Consider a binary convolutional code specified by the generators (1011, 1101, 1111).

A. What are the values of
   a. constraint length of the code
   b. rate of the code
   c. number of states at each time step of the trellis
   d. number of branches transitioning into each state
   e. number of branches transitioning out of each state
   f. number of expected parity bits on each branch

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A 10000-bit message is encoded with the above code and transmitted over a noisy channel. During Viterbi decoding at the receiver, the state 010 had the lowest path metric (a value of 621) in the final time step, and the survivor path from that state was traced back to recover the original message.

B. What is the likely number of bit errors that are corrected by the decoder? How many errors are likely left uncorrected in the decoded message?

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C. If you are told that the decoded message had no uncorrected errors, can you guess the approximate number of bit errors that would have occured had the 10000 bit message been transmitted without any coding on the same channel?

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D. From knowing the final state of the trellis (010, as given above), can you infer what the last bit of the original message was? What about the last-but-one bit? The last 4 bits?

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Consider a transition branch between two states on the trellis that has 000 as the expected set of parity bits. Assume that 0V and 1V are used as the signaling voltages to transmit a 0 and 1 respectively, and 0.5V is used as the digitization threshold.

E. Assuming hard decision decoding, which of the two set of received voltages will be considered more likely to correspond to the expected parity bits on the transition: (0V, 0.501V, 0.501V) or (0V, 0V, 0.9V)? What if one is using soft decision decoding?

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Problem 12. Indicate whether each of the statements below is true or false, and a brief reason why you think so.

A. If the number states in the trellis of a convolutional code is S, then the number of survivor paths at any point of time is S. Remember that if there is "tie" between to incoming branches (i.e., they both result in the same path metric), we arbitrarilly choose only one as the predecessor.

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The path metric of a state s1 in the trellis indicates the number of residual uncorrected errors left along the trellis path from the start state to s1.

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B. Among the survivor paths left at any point during the decoding, no two can be leaving the same state at any stage of the trellis.

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C. Among the survivor paths left at any point during the decoding, no two can be entering the same state at any stage of the trellis. Remember that if there is "tie" between to incoming branches (i.e., they both result in the same path metric), we arbitrarilly choose only one as the predecessor.

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D. For a given state machine of a convolutional code, a particular input message bit stream always produces the same output parity bits.

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Problem 13. Consider a convolution code with two generator polynomials: G0=101 and G1=110.

A. What is code rate `r` and constraint length `k` for this code?

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B. Draw the state transition diagram for a transmitter that uses this convolutional code. The states should be labeled with the binary string $x_{n-1}...x_{n-k+1}$ and the arcs labeled with x_n/p_0p_1 where `x[n]` is the next message bit and p_0 and p_1 are the two parity bits computed from G0 and G1 respectively.

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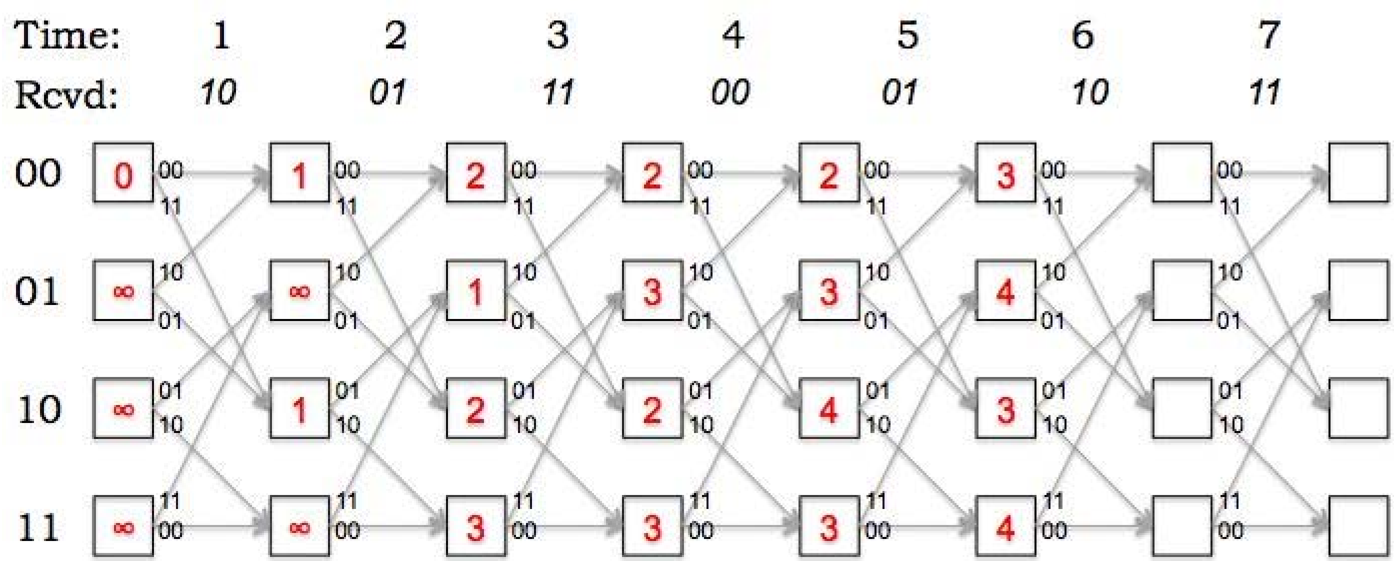
The figure below is a snapshot of the decoding trellis showing a particular state of a maximum likelihood decoder implemented using the Viterbi algorithm. The labels in the boxes show the path metrics computed for each state after receiving the incoming parity bits at time `t`. The labels on the arcs show the expected parity bits for each transition; the actual received bits at each time are shown above the trellis.



C. Fill in the path metrics in the empty boxes in the diagram above (corresponding to the Viterbi calculations for times 6 and 7).

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D. Based on the updated trellis, what is the most-likely final state of the transmitter? How many errors were detected along the most-likely path to the most-likely final state?

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E. What's the most-likely path through the trellis (i.e., what's the most-likely sequence of states for the transmitter)? What's the decoded message?

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F. Based on your choice of the most-likely path through the trellis, at what times did the errors occur?

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