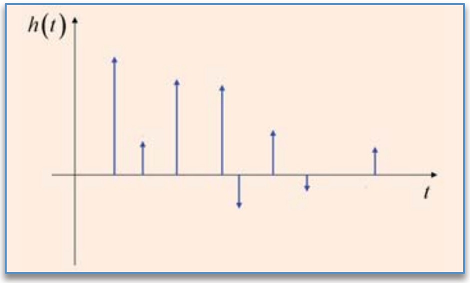
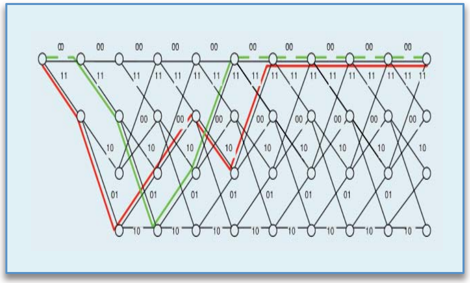
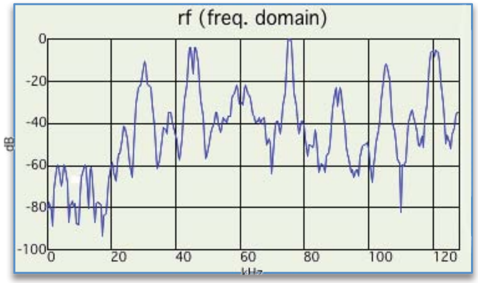
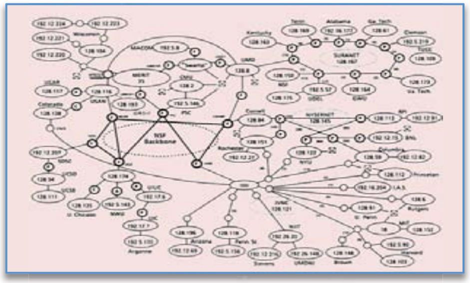




INTRODUCTION TO EECS II

DIGITAL COMMUNICATION SYSTEMS

6.02 Fall 2012
Lecture #16

- DTFT vs DTFS
- Modulation/Demodulation
- Frequency Division Multiplexing (FDM)

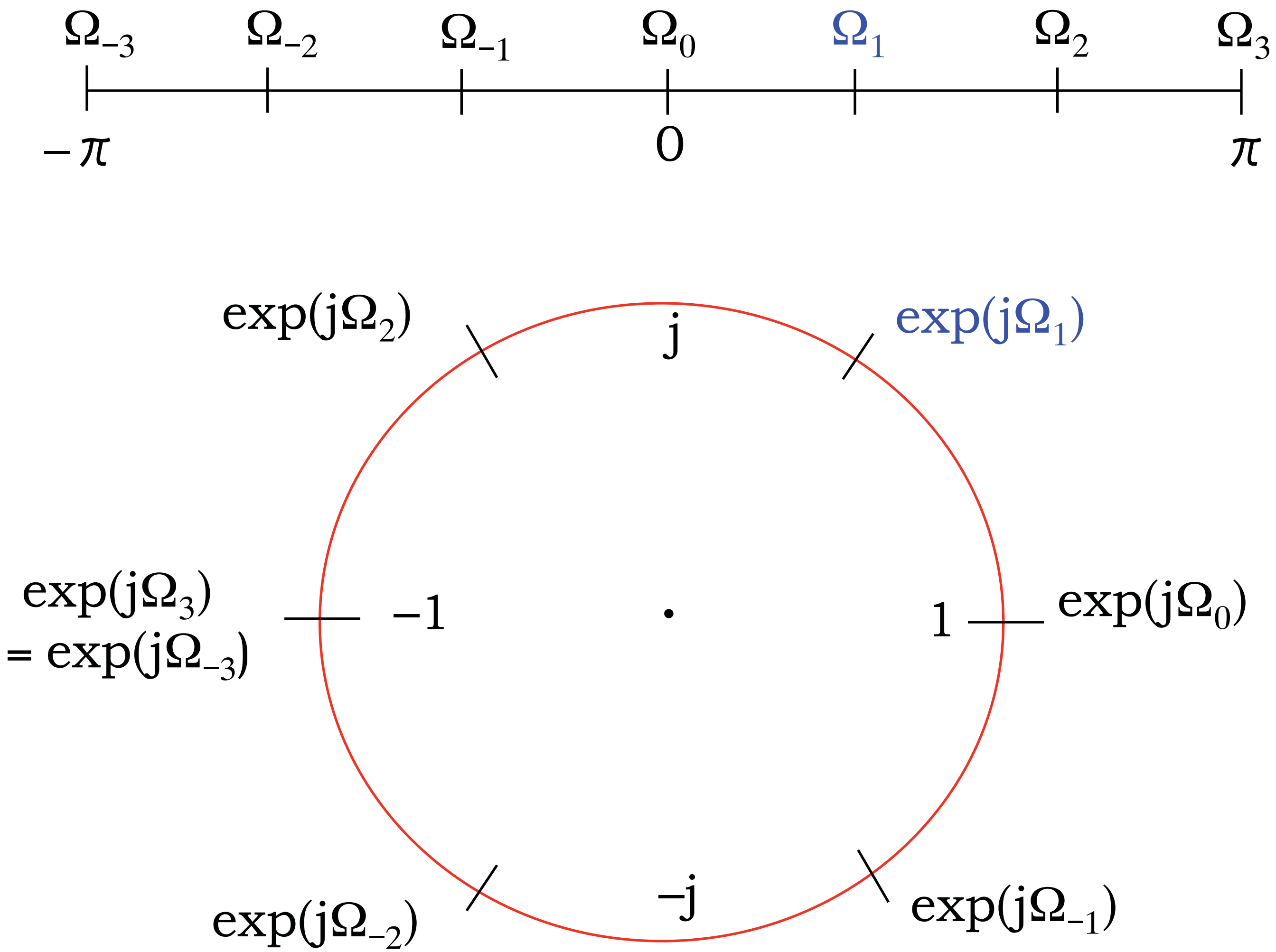
Fast Fourier Transform (FFT) to compute samples of the DTFT for signals of finite duration

For an x[n] that is zero outside of the interval [0,L-1], choose P ≥ L (with P preferably a power of 2; we'll assume that it's at least a *multiple* of 2, i.e., even):

$$X(\Omega_k) = \sum_{m=0}^{P-1} x[m] e^{-j\Omega_k m}, \qquad x[n] = \frac{1}{P} \sum_{k=-P/2}^{(P/2)-1} X(\Omega_k) e^{j\Omega_k n}$$

where $\Omega_k = k(2\pi/P)$, and k ranges from –P/2 to (P/2)–1, or over any P successive integers. Simpler notation: $X(\Omega_k) = X_k$

Where do the Ω_k live? e.g., for P=6 (even)



Fast Fourier Transform (FFT) to compute samples of the DTFT for signals of finite duration

For an x[n] that is zero outside of the interval [0,L-1], choose P ≥ L (with P preferably a power of 2; we'll assume that it's at least a *multiple* of 2, i.e., even):

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Note that $X(\Omega_{P/2}) = X(\pi) = X(-\pi) = X(\Omega_{-P/2})$.

The above formulas have essentially the same structure, and are both efficiently computed, with Plog(P) computations, by the FFT.

Some further details, assuming real x[n]

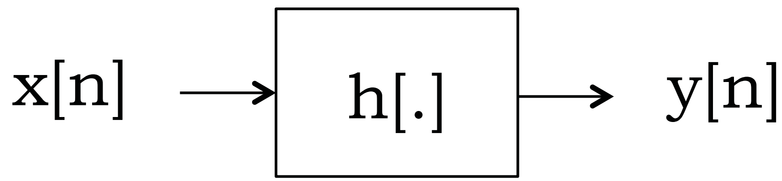
$$X(\Omega_k) = \sum_{m=0}^{P-1} x[m] e^{-j\Omega_k m}$$

- $X(0)$ = sum of x[m] = real
- $X(\pi) = X(-\pi)$ = alternating sum of x[m] = real
- In general P-2 other complex values, but $X(-\Omega_k) = X^*(\Omega_k)$
- So: total of P numbers to be determined, given P values of x[m]

$$x[n] = \frac{1}{P} \sum_{k=-P/2}^{(P/2)-1} X(\Omega_k) e^{j\Omega_k n}$$

- Evaluating this eqn. for n in [0,P-1] recovers the original x[n] in this interval
- Evaluating it for n outside this interval results in periodic replication of the values in [0,P-1], producing a periodic signal $\boldsymbol{x}$[n]
- So this eqn. is also called a DT Fourier Series (DTFS) for the periodic signal $\boldsymbol{x}$[n]. Notation: $A_k = X(\Omega_k)/P = X_k/P$, Fourier *coefficient*.

Why the periodicity of *x*[n] is irrelevant in many applications



Suppose x[n] is nonzero only over the time interval $[0, n_x]$, and h[n] is nonzero only over the time interval $[0, n_h]$.

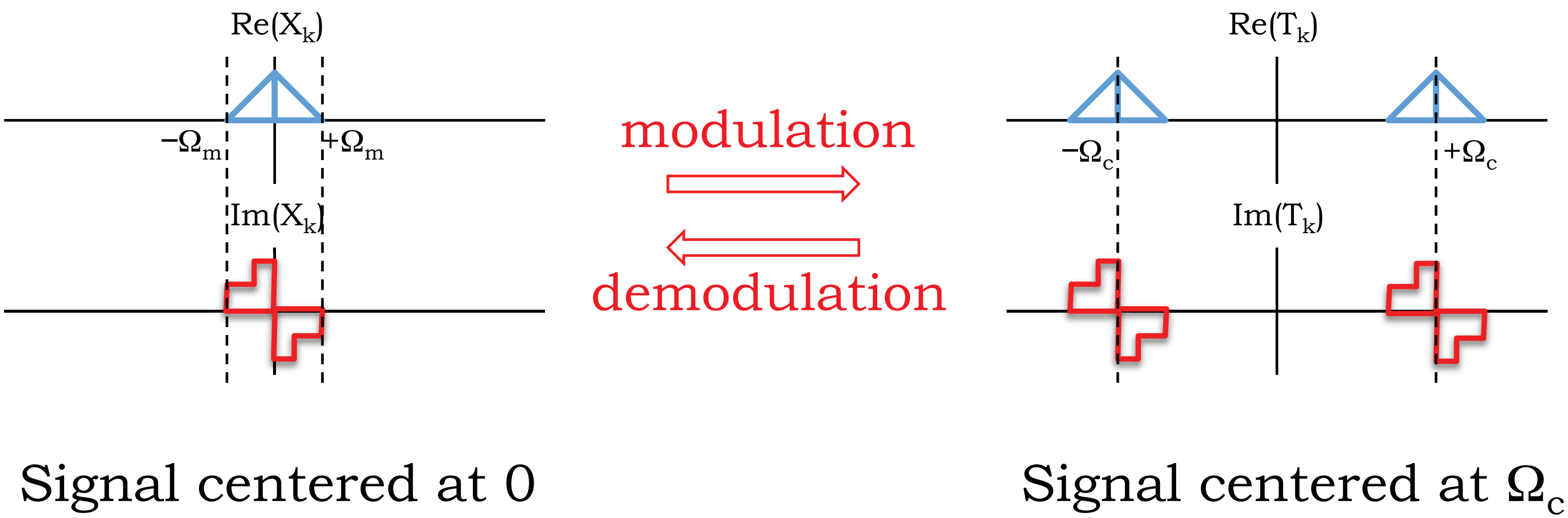
In what time interval can the non-zero values of y[n] be guaranteed to lie? The interval $[0, n_x + n_h]$.

Since all the action we are interested in is confined to this interval, choose $P - 1 \geq n_x + n_h$.

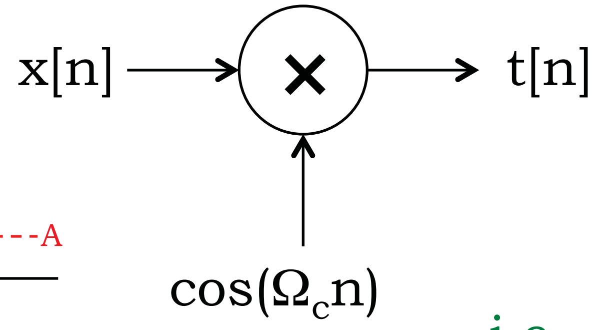
It's now irrelevant what happens outside [0,P–1]. So we can use the FFT to go back and forth between samples of $X(\Omega)$, $H(\Omega)$, $Y(\Omega)$ in the frequency domain and time-domain behavior in [0,P-1].

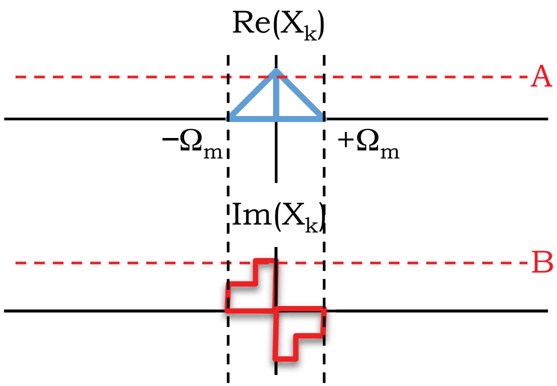
Back to Modulation/Demodulation

- You have: a signal x[n] at *baseband* (i.e., centered around 0 frequency)
- You want: the same signal, but centered around some specific frequency Ω_c
- Modulation: convert from baseband up to Ω_c , to get t[n]
- Demodulation: convert from Ω_c down to baseband

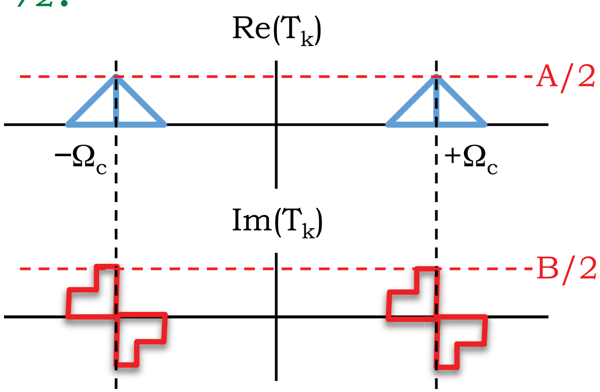


Modulation by Heterodyning or Amplitude Modulation (AM)

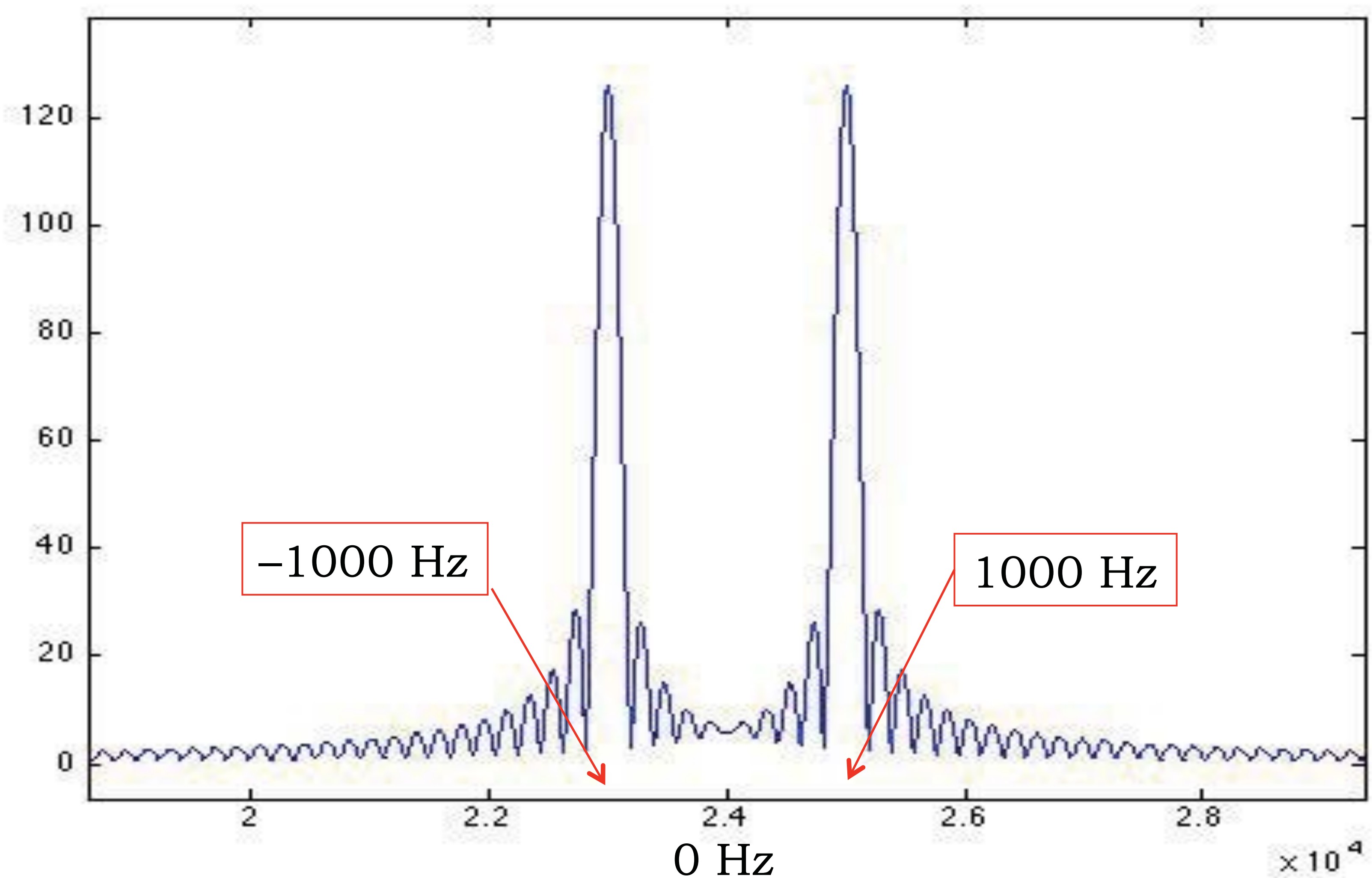




i.e., just replicate baseband signal at $\pm\Omega_c$, and scale by ½.



To get this nice picture, the baseband signal needs to be band-limited to some range of frequencies $[-\Omega_m, \Omega_m]$, where $\Omega_m \leq \Omega_c$



Not great band-limiting, but maybe we can get away with it!

At the Receiver: Demodulation

- In principle, this is (as easy as) modulation again:

If the received signal is

$$r[n] = x[n]\cos(\Omega_c n) = t[n],$$

(no distortion or noise) then simply compute

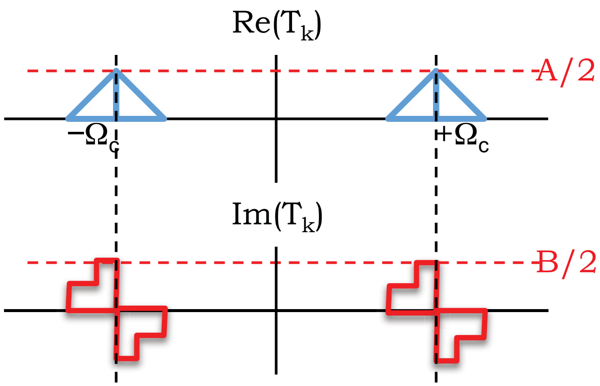
$$\begin{aligned} d[n] &= r[n]\cos(\Omega_c n) \\ &= x[n]\cos^2(\Omega_c n) \\ &= 0.5\,\{x[n] + x[n]\cos(2\Omega_c n)\} \end{aligned}$$

If there is distortion (i.e., $r[n] \neq t[n]$), then write $y[n]$ instead of $x[n]$ (and hope that in the noise-free case $y[.]$ is related to $x[.]$ by an approximately LTI relationship!)

- What does the spectrum of $d[n]$, i.e., $D(\Omega)$, look like?
- What constraint on the bandwidth of $x[n]$ is needed for perfect recovery of $x[n]$?

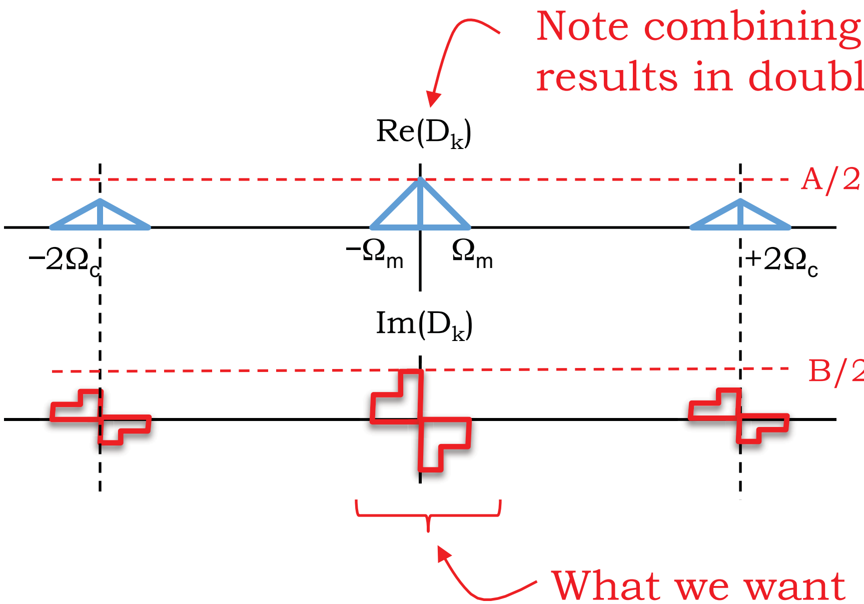
Demodulation Frequency Diagram

$R(\Omega) = T(\Omega)$



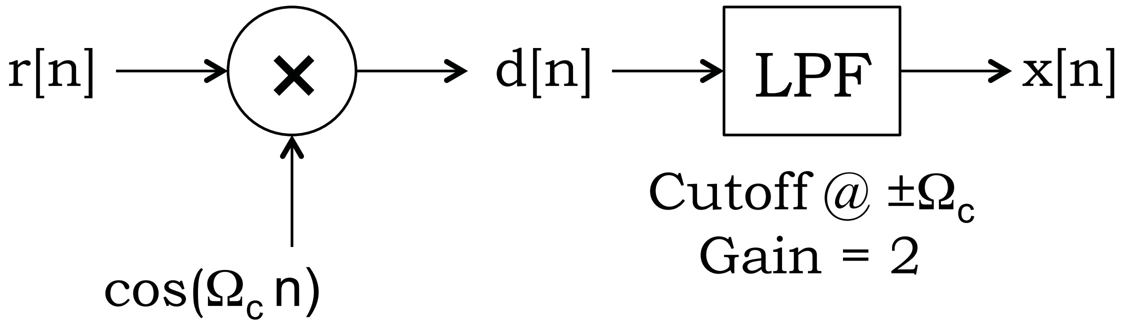
Note combining of signals around 0 results in doubling of amplitude

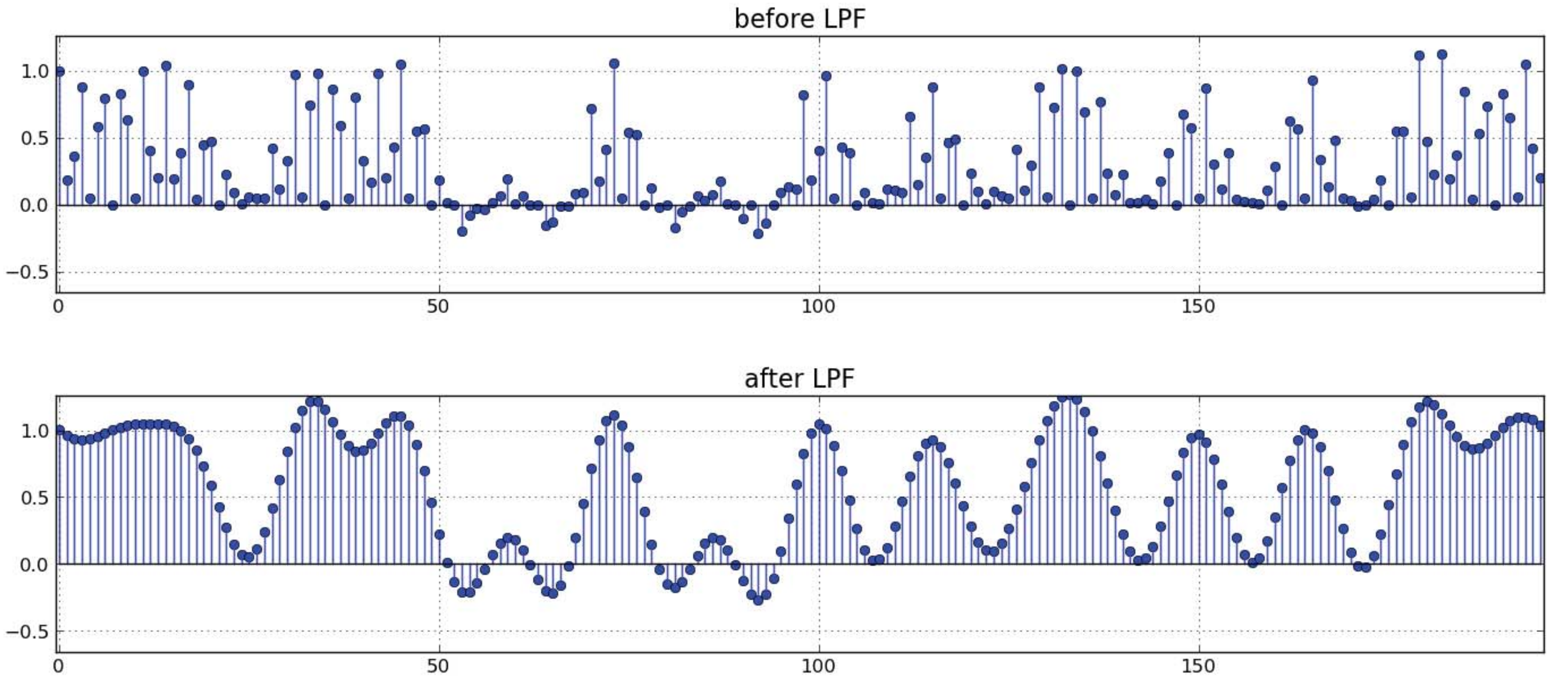
$D(\Omega)$



What we want

Demodulation + LPF





Phase Error In Demodulation

When the receiver oscillator is out of phase with the transmitter:

$$d[n] = r[n] \cdot \cos(\Omega_c n - \varphi) = x[n] \cdot \cos(\Omega_c n) \cdot \cos(\Omega_c n - \varphi)$$

But

$$\cos(\Omega_c n).\cos(\Omega_c n - \varphi) = 0.5\{\cos(\varphi) + \cos(2\Omega_c n - \varphi)\}$$

It follows that the demodulated output, after the LPF of gain 2, is

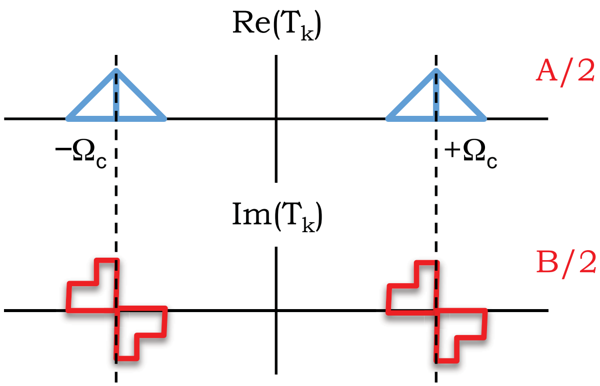
$$y[n] = x[n].\cos(\varphi)$$

So a phase error of φ results in amplitude scaling by $\cos(\varphi)$.

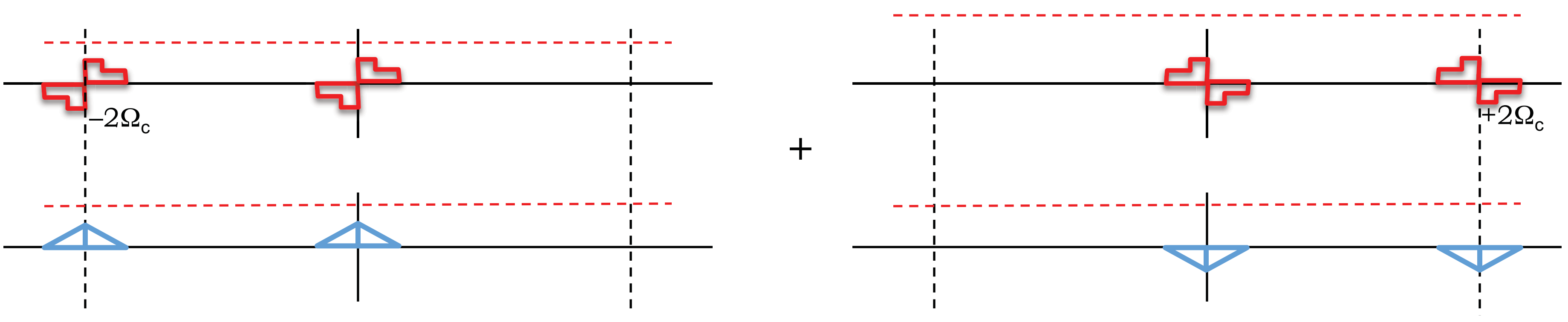
Note: in the extreme case where $\varphi = \pi/2$, we are demodulating by a sine rather than a cosine, and we get y[n]=0 .

Demodulation with sin(Ω_cn)

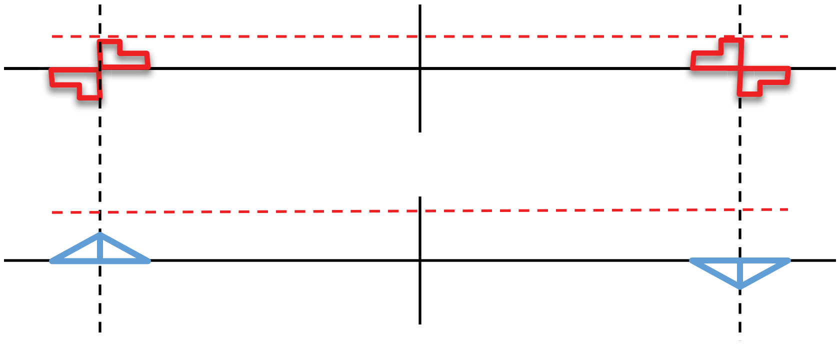
$R(\Omega)$



$D(\Omega)$

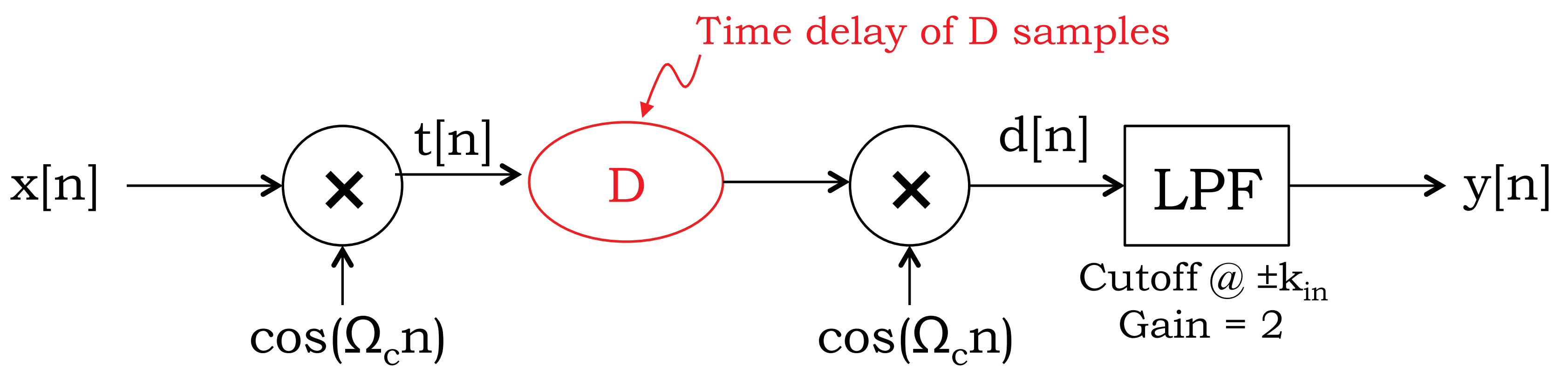


… produces



Note combining of signals around 0 results in cancellation!

Channel Delay



Very similar math to the previous "phase error" case:

$$d[n] = t[n-D]\cdot\cos(\Omega_c n)$$
$$= x[n-D]\cdot\cos[\Omega_c(n-D)]\cdot\cos(\Omega_c n)$$

Passing this through the LPF:

$$y[n] = x[n-D]\cdot\cos(\Omega_c D)$$

Looks like a phase error of $\Omega_c D$

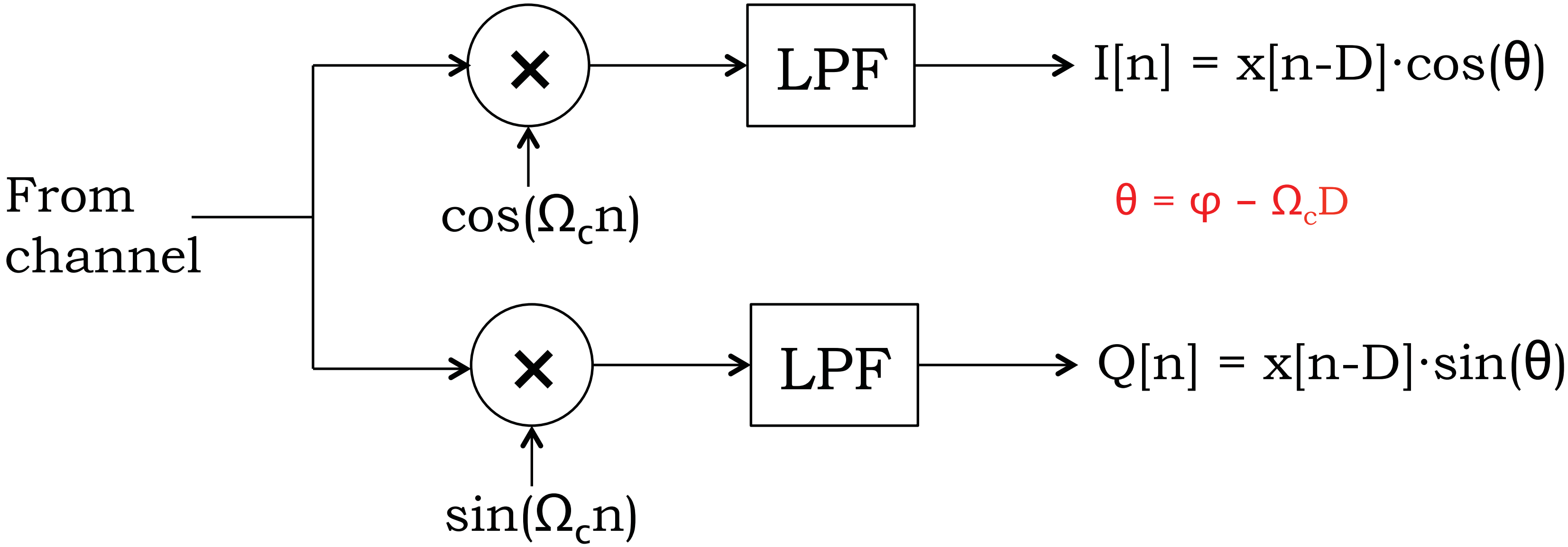
If $\Omega_c D$ is an odd multiple of $\pi/2$, then y[n]=0 !!

Fixing Phase Problems in the Receiver

So phase errors and channel delay both result in a scaling of the output amplitude, where the magnitude of the scaling can't necessarily be determined at system design time:

- channel delay varies on mobile devices
- phase difference between transmitter and receiver is arbitrary

One solution: *quadrature demodulation*



Quadrature Demodulation

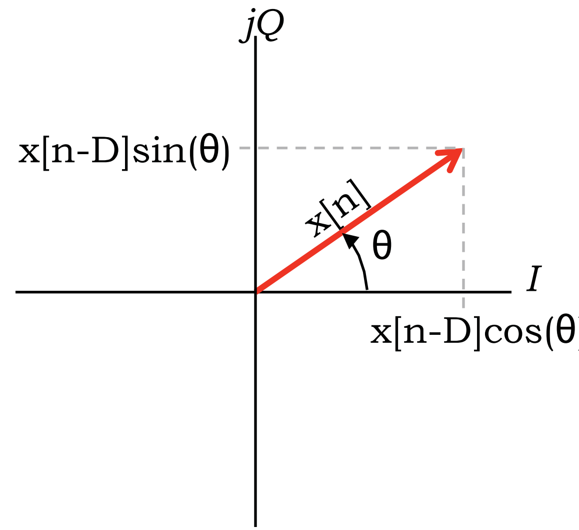
If we let

$$w[n] = I[n] + jQ[n]$$

then

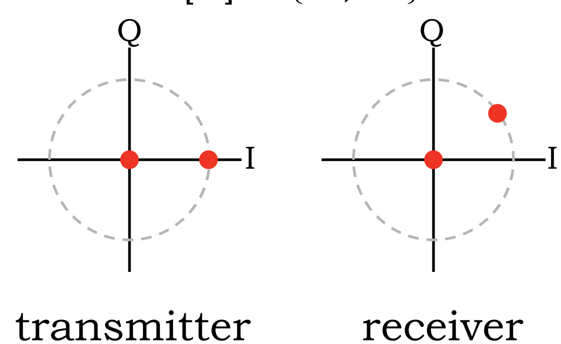
$$|w[n]| = \sqrt{I[n]^2 + Q[n]^2}$$
$$= |x[n-D]| \sqrt{\cos^2\theta + \sin^2\theta}$$
$$= |x[n-D]|$$

OK for recovering x[n] if it never goes negative, as in on-off keying

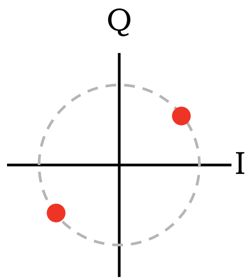


Constellation diagrams:

x[n] = { 0, 1 }



Dealing With Phase Ambiguity in Bipolar Modulation



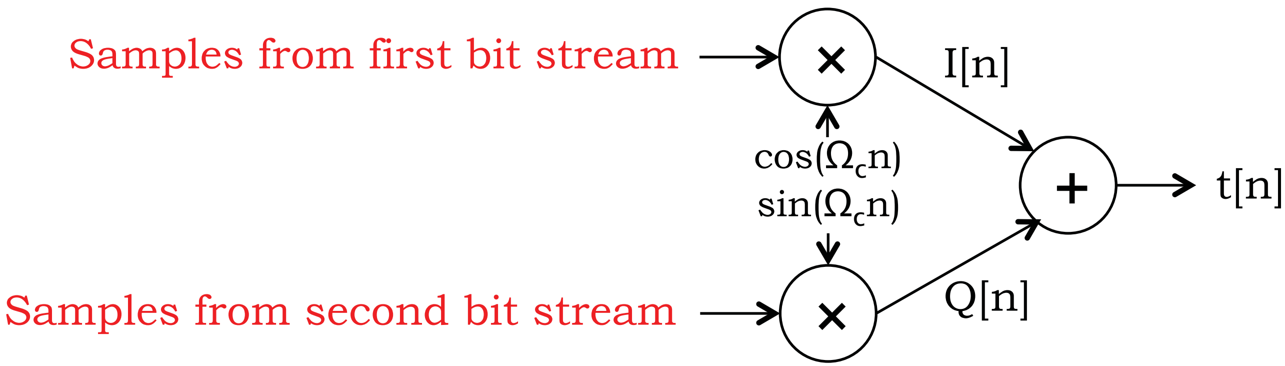
In bipolar modulation ($x[n]=\pm 1$), also called Binary Phase Shift Keying (BPSK) since the modulated carrier changes phase by $\pi/2$ when x[n] switches levels, the received constellation will be rotated with respect to the transmitter's constellation. Which phase corresponds to which bit?

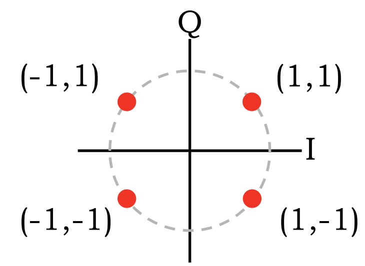
Different fixes:

1. Send an agreed-on sign-definite preamble
2. Transmit differentially encoded bits, e.g., transmit a "1" by stepping the phase by π, transmit a "0" by not changing the phase

QPSK Modulation

We can use the quadrature scheme at the transmitter too:





Phase Shift Keying underlies many familiar modulation schemes

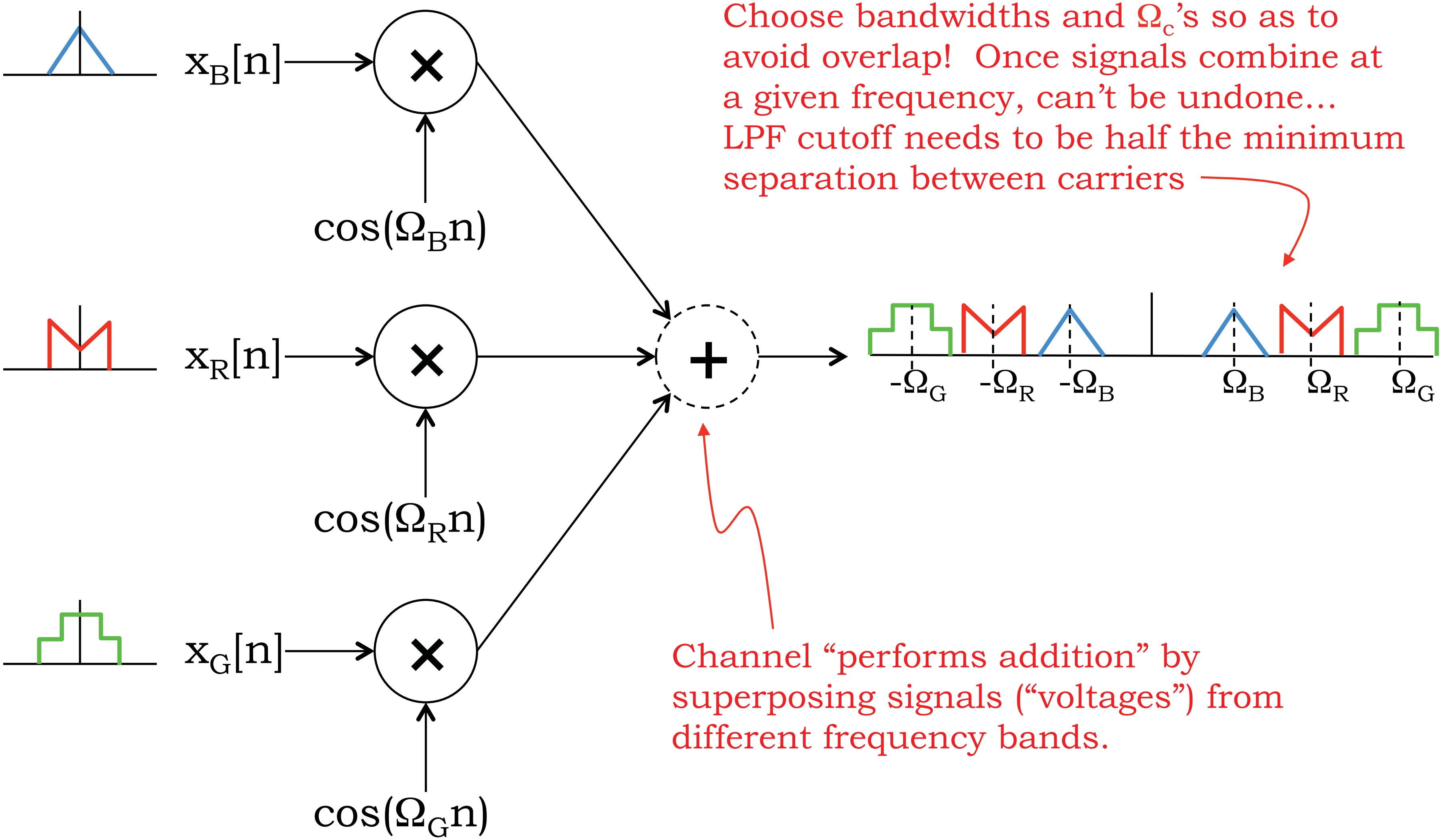
The wireless LAN standard, IEEE 802.11b-1999, uses a variety of different PSKs depending on the data-rate required. At the basic-rate of 1 Mbit/s, it uses DBPSK (differential BPSK). To provide the extended-rate of 2 Mbit/s, DQPSK is used. In reaching 5.5 Mbit/s and the full-rate of 11 Mbit/s, QPSK is employed, but has to be coupled with complementary code keying. The higher-speed wireless LAN standard, IEEE 802.11g-2003 has eight data rates: 6, 9, 12, 18, 24, 36, 48 and 54 Mbit/s. The 6 and 9 Mbit/s modes use OFDM modulation where each sub-carrier is BPSK modulated. The 12 and 18 Mbit/s modes use OFDM with QPSK. The fastest four modes use OFDM with forms of quadrature amplitude modulation.

Because of its simplicity BPSK is appropriate for low-cost passive transmitters, and is used in RFID standards such as ISO/IEC 14443 which has been adopted for biometric passports, credit cards such as American Express's ExpressPay, and many other applications.

Bluetooth 2 will use (p/4)-DQPSK at its lower rate (2 Mbit/s) and 8-DPSK at its higher rate (3 Mbit/s) when the link between the two devices is sufficiently robust. Bluetooth 1 modulates with Gaussian minimum-shift keying, a binary scheme, so either modulation choice in version 2 will yield a higher data-rate. A similar technology, IEEE 802.15.4 (the wireless standard used by ZigBee) also relies on PSK. IEEE 802.15.4 allows the use of two frequency bands: 868–915 MHz using BPSK and at 2.4 GHz using OQPSK.

http://en.wikipedia.org/wiki/Phase-shift_keying

Multiple Transmitters: *Frequency Division Multiplexing (FDM)*



Choose bandwidths and Ω_c's so as to avoid overlap! Once signals combine at a given frequency, can't be undone… LPF cutoff needs to be half the minimum separation between carriers

Channel "performs addition" by superposing signals ("voltages") from different frequency bands.

AM Radio

AM radio stations are on 520 – 1610 kHz ('medium wave") in the US, with carrier frequencies of different stations spaced 10 kHz apart.

Physical effects very much affect operation. e.g., EM signals at these frequencies propagate much further at night (by "skywave" through the ionosphere) than during the day (100's of miles by "groundwave" diffracting around the earth's surface), so transmit power may have to be lowered at night!

6.02 Introduction to EECS II: Digital Communication Systems
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