6.01: Introduction to EECS I

Primitives, Combination, Abstraction, and Patterns

February 8, 2011

PCAP Framework for Managing Complexity

Python has features that facilitate modular programming.

- **def** combines operations into a procedure and binds a name to it
- **lists** provide flexible and hierarchical structures for data
- **variables** associate names with data
- **classes** associate data (attributes) and procedures (methods)

	procedures	data
Primitives	`+, *, ==, !=`	numbers, booleans, strings
Combination	`if, while, f(g(x))`	lists, dictionaries, objects
Abstraction	`def`	classes
Patterns	higher-order procedures	super-classes, sub-classes

PCAP Framework for Managing Complexity

We will build on these ideas to manage complexity at higher levels.

- **Programming Styles** for dealing with complexity
- PCAP in Higher-Level Abstractions: **State Machines**

Reading: Course notes, chapters 3–4

Programming Styles for Managing Complexity

Structure of program has significant effect on its modularity.

Imperative (procedural) programming

- focus on step-by-step instructions to accomplish task
- organize program using structured conditionals and loops

Functional programming

- focus on procedures that mimic mathematical functions, producing outputs from inputs without side effects
- functions are **first-class objects** used in data structures, arguments to procedures, and can be returned by procedures

Object-oriented programming

- focus on collections of related procedures and data
- organize programs as hierarchies of related classes and instances

Example Program

Task: Find a sequence of operations (either **increment** or **square**) that transforms the integer i (initial) to the integer g (goal).

Example: applying the sequence

increment increment increment square

to **1** yields **16**

apply **increment** to **1** $\rightarrow$ **2**

apply **increment** to **2** $\rightarrow$ **3**

apply **increment** to **3** $\rightarrow$ **4**

apply **square** to **4** $\rightarrow$ **16**

Check Yourself

What is the minimum length sequence of **increment** and **square** operations needed to transform **1** to **100**?

1: **<4** 2: **4** 3: **5** 4: **6** 5: **>6**

Check Yourself

What is the minimum length sequence of `increment` and `square` operations needed to transform `1` to `100`?

Try to use as many squares (especially big ones) as possible.

apply `increment` to **1** → **2**
apply `increment` to **2** → **3**
apply `square` to **3** → **9**
apply `increment` to **9** → **10**
apply `square` to **10** → **100**

Five operations.

Check Yourself

What is the minimum length sequence of **increment** and **square** operations needed to transform **1** to **100**? **3: 5**

1: **<4**   2: **4**   3: **5**   4: **6**   5: **>6**

Imperative (Procedural) Programming

Solve the previous problem by writing an imperative program to step through all possible sequences of length 1, 2, 3, ...

```
def increment(n):
    return n+1
def square(n):
    return n**2
def findSequence(initial,goal):
    # construct list of "candidates" of form ('1 increment increment',3)
    candidates = [(str(initial),initial)]
    # loop over sequences of length "i" = 1, 2, 3, ...
    for i in range(1,goal-initial+1):
        newCandidates = []
        # construct each new candidate by adding one operation to prev candidate
        for (action,result) in candidates:
            for (a,r) in [(' increment',increment),(' square',square)]:
                newCandidates.append((action+a,r(result)))
                print i,': ',newCandidates[-1]
                if newCandidates[-1][1] == goal:
                    return newCandidates[-1]
        candidates = newCandidates

answer = findSequence(1,100)
print 'answer =',answer
```

Imperative (Procedural) Programming

```
1 :  ('1 increment', 2)
1 :  ('1 square', 1)
2 :  ('1 increment increment', 3)
2 :  ('1 increment square', 4)
2 :  ('1 square increment', 2)
2 :  ('1 square square', 1)
3 :  ('1 increment increment increment', 4)
3 :  ('1 increment increment square', 9)
3 :  ('1 increment square increment', 5)
3 :  ('1 increment square square', 16)
3 :  ('1 square increment increment', 3)
3 :  ('1 square increment square', 4)
3 :  ('1 square square increment', 2)
3 :  ('1 square square square', 1)
4 :  ('1 increment increment increment increment', 5)
4 :  ('1 increment increment increment square', 16)
4 :  ('1 increment increment square increment', 10)
4 :  ('1 increment increment square square', 81)
4 :  ('1 increment square increment increment', 6)
4 :  ('1 increment square increment square', 25)
4 :  ('1 increment square square increment', 17)
4 :  ('1 increment square square square', 256)
4 :  ('1 square increment increment increment', 4)
4 :  ('1 square increment increment square', 9)
```

```
4 :  ('1 square increment square increment', 5)
4 :  ('1 square increment square square', 16)
4 :  ('1 square square increment increment', 3)
4 :  ('1 square square increment square', 4)
4 :  ('1 square square square increment', 2)
4 :  ('1 square square square square', 1)
5 :  ('1 increment increment increment increment increment', 6)
5 :  ('1 increment increment increment increment square', 25)
5 :  ('1 increment increment increment square increment', 17)
5 :  ('1 increment increment increment square square', 256)
5 :  ('1 increment increment square increment increment', 11)
```

5 : ('1 increment increment square increment square', 100)

```
answer = ('1 increment increment square increment square', 100)
```

Imperative (Procedural) Programming

This imperative version of the program has three levels of looping.

```
def findSequence(initial,goal):
    # construct list of "candidates" of form ('1 increment increment',3)
    candidates = [(str(initial),initial)]
    # loop over sequences of length "i" = 1, 2, 3, ...
    for i in range(1,goal-initial+1):
        newCandidates = []
        # construct each new candidate by adding one operation to prev candidate
        for (action,result) in candidates:
            for (a,r) in [(' increment',increment),(' square',square)]:
                newCandidates.append((action+a,r(result)))
                print i,': ',newCandidates[-1]
                if newCandidates[-1][1] == goal:
                    return newCandidates[-1]
        candidates = newCandidates
```

This approach is straightforward, but nested loops can be confusing.

Challenge is to get the indices right.

Functional Programming

This version focuses on functions as primitives.

```
def apply(opList,arg):
    if len(opList)==0:
        return arg
    else:
        return apply(opList[1:],opList[0](arg))

def addLevel(opList,fctList):
    return [x+[y] for y in fctList for x in opList]

def findSequence(initial,goal):
    opList = [[]]
    for i in range(1,goal-initial+1):
        opList = addLevel(opList,[increment,square])
        for seq in opList:
            if apply(seq,initial)==goal:
                return seq

answer = findSequence(1,100)
print 'answer =',answer
```

Functional Programming

The procedure **apply** is a “pure function.”

```
def apply(opList,arg):
    if len(opList)==0:
        return arg
    else:
        return apply(opList[1:],opList[0](arg))
```

Its first argument is a list of functions. The procedure applies these functions to the second argument **arg** and returns the result.

```
>>> apply([],7)
7
>>> apply([increment],7)
8
>>> apply([square],7)
49
>>> apply([increment,square],7)
64
```

This list of procedures uses functions as first-class objects.

Functional Programming

The procedure **addLevel** is also a pure function.

```
def addLevel(opList,fctList):
    return [x+[y] for y in fctList for x in opList]
```

The first input is a list of sequences-of-operations, each of which is a list of functions.

The second input is a list of possible next-functions.

It returns a new list of sequences.

```
>>> addLevel([[increment]],[increment,square])
[[<function increment at 0xb7480aac>, <function increment at 0xb7480aac>],
[<function increment at 0xb7480aac>, <function square at 0xb747b25c>]]
```

Functional Programming

The answer is now a list of functions.

```
def apply(opList,arg):
    if len(opList)==0:
        return arg
    else:
        return apply(opList[1:],opList[0](arg))

def addLevel(opList,fctList):
    return [x+[y] for y in fctList for x in opList]

def findSequence(initial,goal):
    opList = [[]]
    for i in range(1,goal-initial+1):
        opList = addLevel(opList,[increment,square])
        for seq in opList:
            if apply(seq,initial)==goal:
                return seq

answer = findSequence(1,100)
print 'answer =',answer
```

answer = [<function increment at 0xb777ea74>, <function increment at 0xb777ea74>, <function square at 0xb7779224>, <function increment at 0xb777ea74>, <function square at 0xb7779224>]

Functional Programming

The functions **apply** and **addLevel** are easy to check.

```
def apply(opList,arg):
    if len(opList)==0:
        return arg
    else:
        return apply(opList[1:],opList[0](arg))

def addLevel(opList,fctList):
    return [x+[y] for y in fctList for x in opList]

>>> apply([],7)
7
>>> apply([increment],7)
8
>>> apply([square],7)
49
>>> apply([increment,square],7)
64
>>> addLevel([[increment]],[increment,square])
[[<function increment at 0xb7480aac>, <function increment at 0xb7480aac>],
[<function increment at 0xb7480aac>, <function square at 0xb747b25c>]]
```

Greater modularity reduces complexity and simplifies debugging.

Functional Programming

Also notice that the definition of **apply** is **recursive**:
the definition of **apply** calls **apply**.

```
>>> def apply(oplist,arg):
...     if len(opList)==0:
...         return arg
...     else:
...         return apply(opList[1:],opList[0](arg))
```

Recursion is

- an alternative way to implement iteration (looping)
- a natural generalization of functional programming
- powerful way to think about PCAP

Recursion

Express solution to problem in terms of simpler version of problem.

Example: raising a number to a non-negative integer power

$$b^n = \begin{cases} 1 & \text{if } n = 0 \\ b \cdot b^{n-1} & \text{if } n > 0 \end{cases}$$

functional notation:

$$f(n) = \begin{cases} 1 & \text{if } n = 0 \\ b\, f(n-1) & \text{if } n > 0 \end{cases}$$

Python implementation:

```
def exponent(b,n):
    if n==0:
        return 1
    else:
        return b*exponent(b,n-1)
```

Recursive Exponentiation

Invoking **exponent(2,6)** generates 6 more invocations of **exponent**.

```
def exponent(b,n):
    if n==0:
        return 1
    else:
        return b*exponent(b,n-1)

exponent(2,6)
    calls exponent(2,5)
        calls exponent(2,4)
            calls exponent(2,3)
                calls exponent(2,2)
                    calls exponent(2,1)
                        calls exponent(2,0)
                        returns 1
                    returns 2
                returns 4
            returns 8
        returns 16
    returns 32
returns 64
64
```

Number of invocations increases in proportion to **n** (i.e., linearly).

Fast Exponentiation

There is a straightforward way to speed this process:

If **`n`** is even, then square the result of raising **`b`** to the **`n/2`** power.

$$b^n = \begin{cases} 1 & \text{if } n = 0 \\ b \cdot b^{n-1} & \text{if } n \text{ odd} \\ \left(b^{n/2}\right)^2 & \text{otherwise} \end{cases}$$

functional notation:

$$f(n) = \begin{cases} 1 & \text{if } n = 0 \\ bf(n-1) & \text{if } n \text{ odd} \\ (f(n/2))^2 & \text{otherwise} \end{cases}$$

Fast Exponentiation

Implement in Python.

```
def fastExponent(b,n):
    if n==0:
        return 1
    elif n%2==1:
        return b*fastExponent(b,n-1)
    else:
        return fastExponent(b,n/2)**2
```

Check Yourself

```
def fastExponent(b,n):
    if n==0:
        return 1
    elif n%2==1:
        return b*fastExponent(b,n-1)
    else:
        return fastExponent(b,n/2)**2
```

How many invocations of **fastExponent** is generated by **fastExponent(2,10)**?

1. 10
2. 8
3. 7
4. 6
5. 5

Recursive Exponentiation

Implement recursion in Python.

```
def fastExponent(b,n):
    if n==0:
        return 1
    elif n%2==1:
        return b*fastExponent(b,n-1)
    else:
        return fastExponent(b,n/2)**2

fastExponent(2,10)
    calls fastExponent(2,5)
        calls fastExponent(2,4)
            calls fastExponent(2,2)
                calls fastExponent(2,1)
                    calls fastExponent(2,0)
                    returns 1
                returns 2
            returns 4
        returns 16
    returns 32
returns 1024
1024
```

The number of calls increases in proportion to **log n** (for large **n**).

Check Yourself

```
def fastExponent(b,n):
    if n==0:
        return 1
    elif n%2==1:
        return b*fastExponent(b,n-1)
    else:
        return fastExponent(b,n/2)**2
```

How many invocations of **fastExponent** is generated by **fastExponent(2,10)**? 5

1. 10
2. 8
3. 7
4. 6
5. 5

Recursive Exponentiation

Functional approach makes this simplification easy to spot.

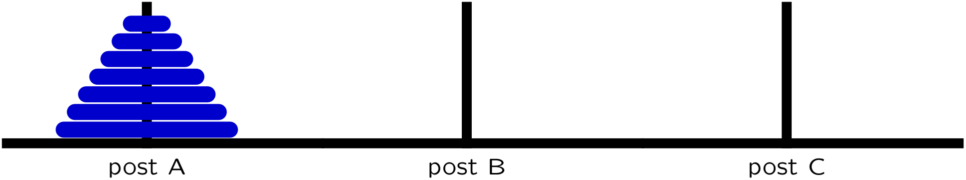
```
def fastExponent(b,n):
    if n==0:
        return 1
    elif n%2==1:
        return b*fastExponent(b,n-1)
    else:
        return fastExponent(b,n/2)**2

fastExponent(2,10)
    calls fastExponent(2,5)
        calls fastExponent(2,4)
            calls fastExponent(2,2)
                calls fastExponent(2,1)
                    calls fastExponent(2,0)
                    returns 1
                returns 2
            returns 4
        returns 16
    returns 32
returns 1024
1024
```

Functional approach is “expressive.”

Towers of Hanoi

Transfer a stack of disks from post A to post B by moving the disks one-at-a-time, without placing any disk on a smaller disk.




```
def Hanoi(n,A,B,C):
  if n==1:
      print 'move from ' + A + ' to ' + B
  else:
      Hanoi(n-1,A,C,B)
      Hanoi(1,A,B,C)
      Hanoi(n-1,C,B,A)
```

Towers of Hanoi

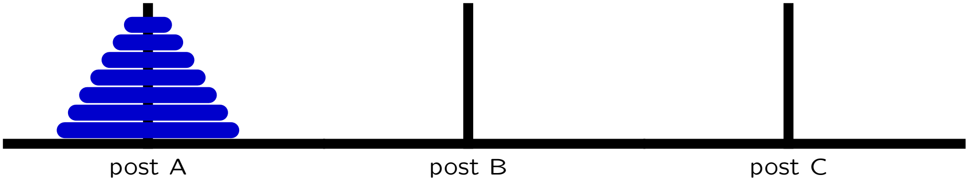
Towers of height 3 and 4.

```
> > > Hanoi(3,'a','b','c')
move from a to b
move from a to c
move from b to c
move from a to b
move from c to a
move from c to b
move from a to b

> > > Hanoi(4,'a','b','c')
move from a to c
move from a to b
move from c to b
move from a to c
move from b to a
move from b to c
move from a to c
move from a to b
move from c to b
move from c to a
move from b to a
move from c to b
move from a to c
move from a to b
move from c to b
```

Towers of Hanoi

Transfer a stack of disks from post A to post B by moving the disks one-at-a-time, without placing any disk on a smaller disk.




```
def Hanoi(n,A,B,C):
  if n==1:
       print 'move from ' + A + ' to ' + B
  else:
       Hanoi(n-1,A,C,B)
       Hanoi(1,A,B,C)
       Hanoi(n-1,C,B,A)
```

Recursive solution is “expressive” (also simple and elegant).

Back to the Earlier Example

Task: Find a sequence of operations (either **`increment`** or **`square`**) that transforms the integer i (initial) to the integer g (goal).

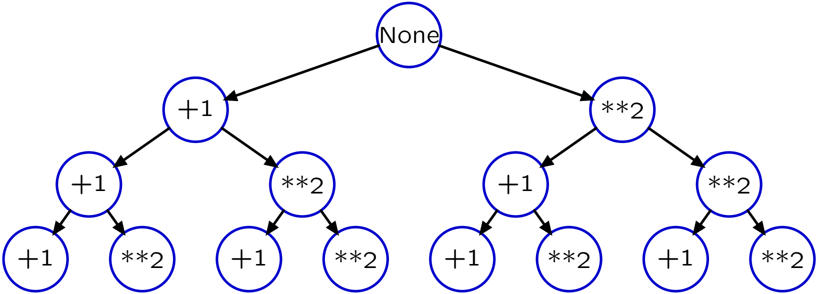
Imperative (procedural) approach $\surd$

Functional approach $\surd$

Object-oriented approach

OOP

Represent all possible sequences in a tree.



Define an object to repesent each of these "nodes":

```
class Node:
    def __init__(self,parent,action,answer):
        self.parent = parent
        self.action = action
        self.answer = answer
    def path(self):
        if self.parent == None:
            return [(self.action, self.answer)]
        else:
            return self.parent.path() + [(self.action,self.answer)]
```

OOP

Systematically create and search through all possible **Nodes**

```
def findSequence(initial,goal):
    q = [Node(None,None,1)]
    while q:
        parent = q.pop(0)
        for (a,r) in [('increment',increment),('square',square)]:
            newNode = Node(parent,a,r(parent.answer))
            if newNode.answer==goal:
                return newNode.path()
            else:
                q.append(newNode)
    return None

answer = findSequence(1,100)
print 'answer =',answer
```

answer = [(None, 1), ('increment', 2), ('increment', 3), ('square', 9), ('increment', 10), ('square', 100)]

Focus on constructing objects that represent pieces of the solution.

More later, when we focus on effective **search** strategies.

Programming Styles for Managing Complexity

Task: Find a sequence of operations (either `increment` or `square`) that transforms the integer i (initial) to the integer g (goal).

Imperative (procedural) approach

- structure of search was embedded in loops

Functional approach

- structure of search was constructed in lists of functions

Object-oriented approach

- structure of search was constructed from objects

Structure of program has significant effect on its modularity.

Now consider abstractions at even higher levels.

Controlling Processes

Programs that control the evolution of processes are different.

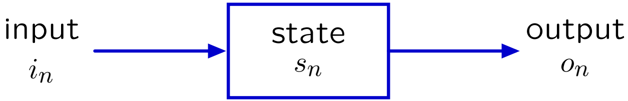
Examples:

- bank accounts
- graphical user interfaces
- controllers (robotic steering)

We need a different kind of abstraction.

State Machines

Organizing computations that evolve with time.



On the n^{th} **step**, the system

- gets **input** i_n
- generates **output** o_n and
- moves to a new **state** s_{n+1}

Output and next state depend on input and current state

Explicit representation of stepwise nature of required computation.

State Machines

Example: Turnstile

$$\text{Inputs} = \{\text{coin}, \text{turn}, \text{none}\}$$

$$\text{Outputs} = \{\text{enter}, \text{pay}\}$$

$$\text{States} = \{\text{locked}, \text{unlocked}\}$$

$$\text{nextState}(s, i) = \begin{cases} \text{unlocked} & \text{if } i = \text{coin} \\ \text{locked} & \text{if } i = \text{turn} \\ s & \text{otherwise} \end{cases}$$

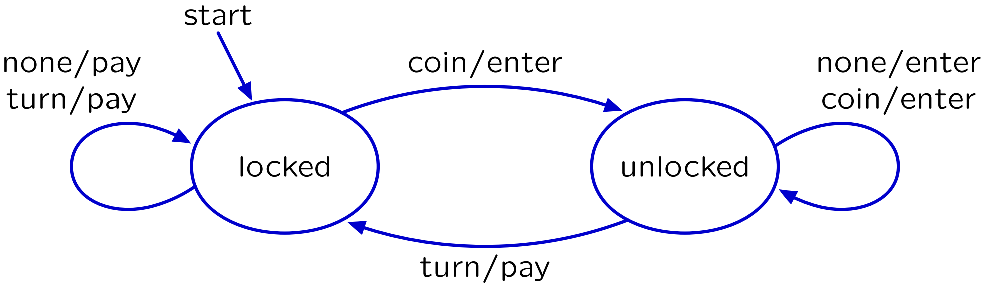
$$\text{output}(s, i) = \begin{cases} \text{enter} & \text{if } \text{nextState}(s, i) = \text{unlocked} \\ \text{pay} & \text{otherwise} \end{cases}$$

$$s_0 = \text{locked}$$

State-transition Diagram

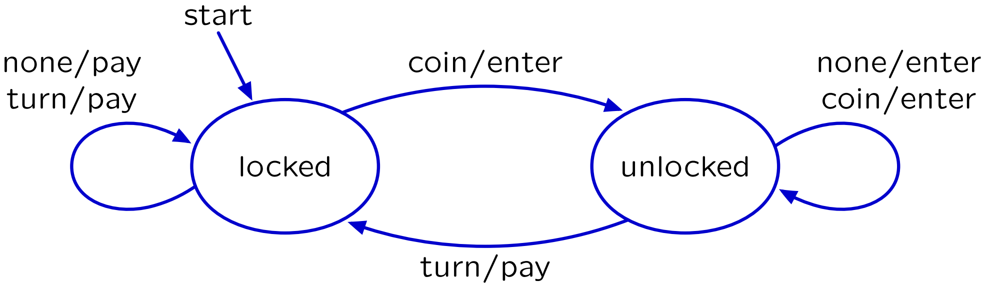
Graphical representation of process.

- Nodes represent states
- Arcs represent transitions: label is input / output



Turn Table

Transition table.



time	0	1	2	3	4	5	6
state	locked	locked	unlocked	unlocked	locked	locked	unlocked
input	none	coin	none	turn	turn	coin	coin
output	pay	enter	enter	pay	pay	enter	enter

State Machines

The state machine representation for controlling processes

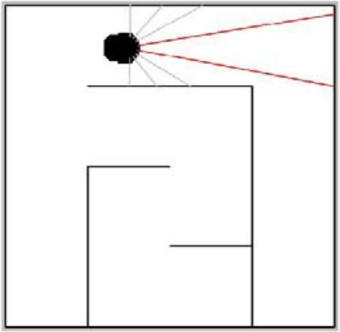
- is simple and concise
- separates system specification from looping structures over time
- is modular

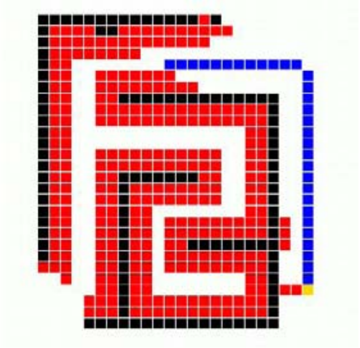
We will use this approach in controlling our robots.

Modular Design with State Machines

Break complicated problems into parts.

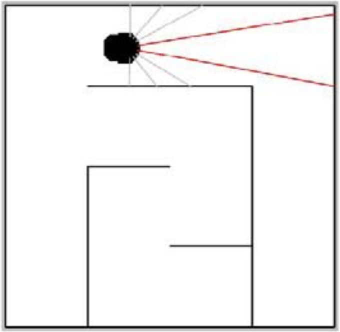
Example: consider exploration with mapping

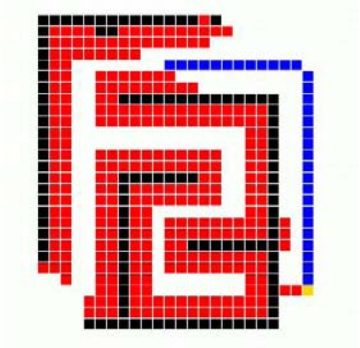




Modular Design with State Machines

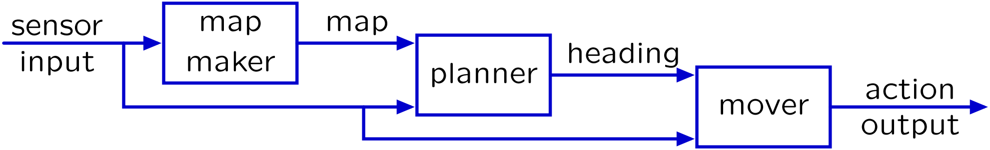
Break complicated problems into parts.





Map: black and red parts.

Plan: blue path, with **heading** determined by first line segment.



State Machines in Python

Represent common features of all state machines in the **SM** class.

Represent kinds of state machines as subclasses of **SM**.

Represent particular state machines as instances.

Example of hierarchical structure

SM Class: All state machines share some methods:

- **`start(self)`** – initialize the instance
- **`step(self, input)`** – receive and process new input
- **`transduce(self, inputs)`** – make repeated calls to **`step`**

Turnstile Class: All turnstiles share some methods and attributes:

- **`startState`** – initial contents of **`state`**
- **`getNextValues(self, state, inp)`** – method to process input

Turnstile Instance: Attributes of this particular turnstile:

- **`state`** – current state of this turnstile

SM Class

The generic methods of the **SM** class use **startState** to initialize the instance variable **state**. Then **getNextValues** is used to process inputs, so that **step** can update **state**.

```
class SM:
    def start(self):
        self.state = self.startState
    def step(self, inp):
        (s, o) = self.getNextValues(self.state, inp)
        self.state = s
        return o
    def transduce(self, inputs):
        self.start()
        return [self.step(inp) for inp in inputs]
```

Note that **getNextValues** should not change **state**.
The **state** is managed by **start** and **step**.

Turnstile Class

All turnstiles share the same **startState** and **getNextValues**.

```
class Turnstile(SM):
    startState = 'locked'

    def getNextValues(self, state, inp):
        if inp == 'coin':
            return ('unlocked', 'enter')
        elif inp == 'turn':
            return ('locked', 'pay')
        elif state == 'locked':
            return ('locked', 'pay')
        else:
            return ('unlocked', 'enter')
```

Turn, Turn, Turn

A particular turnstyle **ts** is represented by an instance.

```
testInput = [None, 'coin', None, 'turn', 'turn', 'coin', 'coin']
ts = Turnstile()
ts.transduce(testInput)
Start state: locked
In: None   Out: pay    Next State: locked
In: coin   Out: enter  Next State: unlocked
In: None   Out: enter  Next State: unlocked
In: turn   Out: pay    Next State: locked
In: turn   Out: pay    Next State: locked
In: coin   Out: enter  Next State: unlocked
In: coin   Out: enter  Next State: unlocked
['pay', 'enter', 'enter', 'pay', 'pay', 'enter', 'enter']
```

Accumulator

```
class Accumulator(SM):
    startState = 0

    def getNextValues(self, state, inp):
        return (state + inp, state + inp)
```

Check Yourself

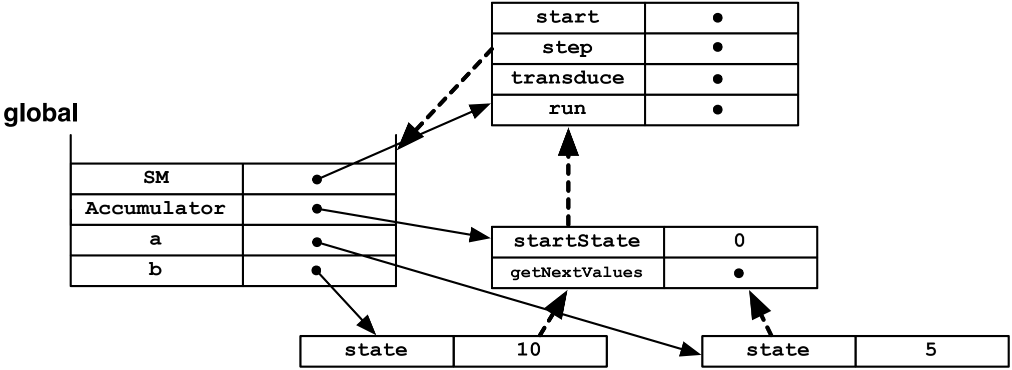
```
>>> a = Accumulator()
>>> a.start()
>>> a.step(7)
>>> b = Accumulator()
>>> b.start()
>>> b.step(10)
>>> a.step(-2)
>>> print a.state,a.getNextValues(8,13),b.getNextValues(8,13)
```

What will be printed?

1: **5 (18, 18) (23, 23)**
2: **5 (21, 21) (21, 21)**
3: **15 (18, 18) (23, 23)**
4: **15 (21, 21) (21, 21)**
5: none of the above

Classes and Instances for Accumulator

```
a = Accumulator()
a.start()
a.step(7)
b = Accumulator()
b.start()
b.step(10)
a.step(-2)
```



Check Yourself

```
>>> a = Accumulator()
>>> a.start()
>>> a.step(7)
>>> b = Accumulator()
>>> b.start()
>>> b.step(10)
>>> a.step(-2)
>>> print a.state,a.getNextValues(8,13),b.getNextValues(8,13)
```

What will be printed? 2

1: **5 (18, 18) (23, 23)**

2: **5 (21, 21) (21, 21)**

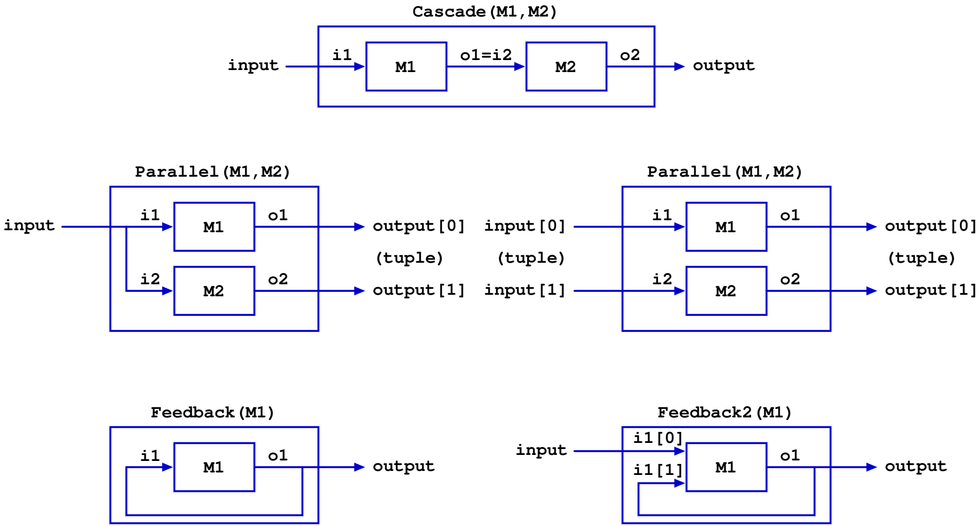
3: **15 (18, 18) (23, 23)**

4: **15 (21, 21) (21, 21)**

5: none of the above

State Machine Combinators

State machines can be **combined** for more complicated tasks.



Check Yourself

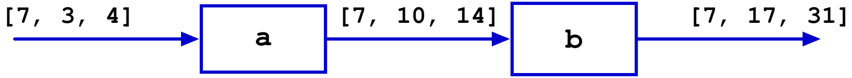
```
>>> a = Accumulator()
>>> b = Accumulator()
>>> c = Cascade(a,b)
>>> print c.transduce([7,3,4])
```

What will be printed?

1: **[7, 3, 4]**
2: **[7, 10, 14]**
3: **[7, 17, 31]**
4: **[0, 7, 17]**
5: none of the above

Check Yourself

```
>>> a = Accumulator()
>>> b = Accumulator()
>>> c = Cascade(a,b)
>>> print c.transduce([7,3,4])
```



Check Yourself

```
>>> a = Accumulator()
>>> b = Accumulator()
>>> c = Cascade(a,b)
>>> print c.transduce([7,3,4])
```

What will be printed? 3

1: **`[7, 3, 4]`**
2: **`[7, 10, 14]`**
3: **`[7, 17, 31]`**
4: **`[0, 7, 17]`**
5: none of the above

This Week

Software lab: Practice with simple state machines

Design lab: Controlling robots with state machines

Homework 1: Symbolic calculator