

KENDRA PUGH: Today I'd like to talk to about poles. Last time I ended up talking to you about LTI representations and manipulations. And, in particular, I want to emphasize the relationship between feedforward and feedback systems, in order to segway us into the poles..

Using what we know about that relationship, we can find the base of a geometric sequence. And that geometric sequence actually represents the long-term response of our system to a unit sample response or a delta. That value is also what we refer to the poles.

At this point, I'll go over how to solve for them and very basic properties of them. So that we can use the information that we have about the poles to try to actually predict the future or look at the long-term behavior of our system.

First a quick review. Last time we talked about feedforward systems. And, in particular, I want to emphasize the fact that if you have a transient input to your feedforward system, you're going to end up with a transient response. There's no method by which a feedforward system can retain information over more than the amount of time steps that you feed information into it.

Feedback systems, on the other hand, represent a persistent response to a transient input. Because you're working with a feedback system information that you put in can be reflected in more than one time step and possibly multiple time steps, depending upon how many delays you have working in your feedback system.

Last time I also drew out the relationship between feedforward and feedback systems. You can actually turn a feedback system or talk about a feedback system in terms of a feedforward system that takes infinite samples of the input and feeds them through a summation that takes infinite delays through your system.

You can represent that translation using a geometric sequence. The basis of that geometric sequence is the object that we're going to use in order to predict the

future. And that's what we're talking about when we talk about poles.

You can have multiple geometric sequences involved in actually determining the long-term behaviors of your system. If you only have one, then things are pretty simple. You find your system function. You find the value associated with p_0 in this expression.

It's OK if there's some sort of scalar on the outside of this expression. We're working with linear time and variance systems, so that scalar is going to affect the initial response to your system. But in terms of a long term behavior, it doesn't matter as much. So don't worry about it right now.

Relatedly, if you're solving for these expressions in second or higher order systems, you're going to end up having to solve partial fractions. You can do this. And in part, one of the reasons that you would want to do this is so that you can get out those scalars, if you're going to be talking about the very short-term response to something, like a transient input.

We're not going to be too interested in those in this course. We're mostly going to be talking about long-term response. So we can get around the fact the we're dealing with a higher order of systems and not solving partial fractions by substituting in for an expression called z , which actually represents the inverse power of R and then solving for the roots of that equation.

If you substitute z in for 1 over R in this denominator and then solve for the root associated with that expression, you'll get the same result. You'll actually end up out with p_0 .

All right. So now we know how to find the pole or multiple poles, if we're interested in multiple poles. What do we do now? I still haven't gone over how to figure out the long-term behavior of your system.

The first thing you do is look at the magnitude of all the poles that you've solved for and select the poles with the largest magnitude. If there are multiple poles with the same magnitude, then you'll end up looking at all of them. If you have different

properties than the ones here, you can end up with some, you know, complex behavior. I would not worry about that too much. Or I would ask a professor or TA when that happens.

But in the general sense, if your dominant pole has a magnitude greater than 1, then you're going to see long-term divergence in your system. This makes sense if you think about it. If at every time step your unit sample response is multiplied by a value that is greater than 1, then it's going to increase. And, in fact, the extent to which the magnitude of your dominant pole is greater than 1 is going to determine your rate of increase and also determine how fast your envelope explodes.

Similarly, if the magnitude of your dominant pole is less than 1, then in response to a unit sample input or a delta, your system's going to converge. This also makes sense intuitively. If you are progressively multiplying the values in your system by a scalar that is less than 1, then eventually you're going to end up converging to 0.

To cover the only category we haven't talked about, if your dominant pole is actually equal in magnitude to 1, then you're not going to see convergence or divergence. And this is one of the places in which the magnitude of the scalar that you end up multiplying your system by can become relevant. We're not going to focus on this situation too much. But it's good to know what actually happens when the magnitude of your dominant pole is equal to 1.

The other feature that we're interested in when we're looking at the dominant pole of a system is if we were to represent the dominant pole in this form, what the angle associated with that pole is, if you were to graph that pole on the complex plane using polar coordinates. If your pole stays on the real axis, or if your pole does not have a complex component, then you'll see one of two things.

The first thing that it's possible for you to see is that you'll get absolutely non-alternating behavior. Your system response stays on one side of the x-axis and either converges, diverges, or remains constant as a consequence of input of the unit sample. And you won't see any sort of alternating or oscillating behavior. This only happens when your dominant pole is real and positive.

If your dominant pole is real and negative, this also means that it's still on the real axis, but its value is negative. So if you're looking at polar coordinates, it's going to have an angle π associated with it. This means you get alternating behavior. And what I mean when I say alternating behavior is that your unit sample response is going to jump across the x-axis at every time step. This is also equivalent to having a period of 2.

The other situation you can run into is that this angle is neither 0 nor π . And at that point you're going to be talking about oscillatory behavior or a sinusoidal response that retains its edges at the envelope of your function. In order to find the period, or in order to find the amount of time it takes for your unit sample response to complete one period, you're going to take the angle associated with your dominant pole and divide 2π by it. This is the general equation for a period.

This covers the basics of what you want to do once you already have your poles. Next time I'm actually going to solve a pole problem and show you what the long-term response looks like and also talk about some things about poles that I've pretty much skimmed over. And at that point you should be able to solve and look at poles for yourself.