Chapter 13: Acoustics

13.1 Acoustic waves

13.1.1 Introduction

Wave phenomena are ubiquitous, so the wave concepts presented in this text are widely relevant. Acoustic waves offer an excellent example because of their similarity to electromagnetic waves and because of their important applications. Beside the obvious role of acoustics in microphones and loudspeakers, surface-acoustic-wave (SAW) devices are used as radio-frequency (RF) filters, acoustic-wave modulators diffract optical beams for real-time spectral analysis of RF signals, and mechanical crystal oscillators currently control the timing of most computers and clocks. Because of the great similarity between acoustic and electromagnetic phenomena, this chapter also reviews much of electromagnetics from a different perspective.

Section 13.1.2 begins with a simplified derivation of the two main differential equations that characterize linear acoustics. This pair of equations can be combined to yield the acoustic wave equation. Only longitudinal acoustic waves are considered here, not transverse or "shear" waves. These equations quickly yield the group and phase velocities of sound waves, the acoustic impedance of media, and an acoustic Poynting theorem. Section 13.2.1 then develops the acoustic boundary conditions and the behavior of acoustic waves at planar interfaces, including an acoustic Snell's law, Brewster's angle, the critical angle, and evanescent waves. Section 13.2.2 shows how acoustic plane waves can travel within pipes and be guided and manipulated much as plane waves can be manipulated within TEM transmission lines.

Acoustic waves can be totally reflected at firm boundaries, and Section 13.2.3 explains how they can be trapped and guided in a variety of propagation modes closely resembling those in electromagnetic waveguides, where they exhibit cutoff frequencies of propagation and evanescence below cutoff. Section 13.2.4 then explains how these guides can be terminated at their ends with open or closed orifices, thus forming resonators with Q's that can be controlled as in electromagnetic resonators so as to yield band-stop or band-pass filters. The frequencies of acoustic resonances can be perturbed by distorting the shape of the cavity, as governed by nearly the same equation used for electromagnetic resonators except that the electromagnetic energy densities are replaced by acoustic energy density expressions. Section 13.3 discusses acoustic radiation and antennas, including antenna arrays, and Section 13.4 concludes the chapter with a brief introduction to representative electroacoustic devices.

13.1.2 Acoustic waves and power

Most waves other than electromagnetic waves involve perturbations. For example, acoustic waves involve perturbations in the pressure and velocity fields in gases, liquids, or solids. In gases we may express the total pressure p_T, density ρ_T, and velocity $\overline{u}_T$ fields as the sum of a static component and a dynamic perturbation:

$$p_T(\bar{r},t) = P_o + p(\bar{r},t) \ \left[N/m^2\right] \qquad (13.1.1)$$

$$\rho_T(\bar{r},t) = \rho_o + \rho(\bar{r},t) \ \left[kg/m^3\right] \qquad (13.1.2)$$

$$\bar{u}(\bar{r},t) = \bar{U}_o + \bar{u}(\bar{r},t) \ [m/s] \qquad (13.1.3)$$

Another complexity is that, unlike electromagnetic variables referenced to a particular location, gases move and compress, requiring further linearization.[73] Most important is the approximation that the mean velocity $\bar{U}_o = 0$. After these simplifying steps we are left with two linearized acoustic equations, *Newton's law* (f = ma) and *conservation of mass*:

$$\nabla p \cong -\rho_o \partial\bar{u}/\partial t \ \left[N/m^3\right] \qquad \text{(Newton's law)} \qquad (13.1.4)$$

$$\rho_o \nabla \bullet \bar{u} + \partial\rho/\partial t \cong 0 \ \left[kg/m^3 s\right] \qquad \text{(conservation of mass)} \qquad (13.1.5)$$

Newton's law states that the pressure gradient will induce mass acceleration, while conservation of mass states that velocity divergence $\nabla \bullet \bar{u}$ is proportional to the negative time derivative of mass density.

These two basic equations involve three key variables: p, $\bar{u}$, and ρ; we need the acoustic constitutive relation to reduce this set to two variables. Most acoustic waves involve frequencies sufficiently high that the heating produced by wave compression has no time to escape by conduction or radiation, and thus this heat energy returns to the wave during the subsequent expansion without significant loss. Such *adiabatic processes* involve no heat transfer across populations of particles. The resulting adiabatic *acoustic constitutive relation* states that the fractional change in density equals the fractional change in pressure, divided by a constant γ, called the adiabatic exponent:

$$\partial\rho/\partial p = \rho_o/\gamma P_o \qquad (13.1.6)$$

The reason γ is not unity is that gas heats when compressed, which further increases the pressure, so the gas thereby appears to be slightly "stiffer" or more resistant to compression than otherwise. This effect is diminished for gas particles that have internal rotational or vibrational degrees of freedom so the temperature rises less upon compression. Ideal monatomic molecules without such degrees of freedom exhibit $\gamma = 5/3$, and $1 < \gamma < 2$, in general.

Substituting this constitutive relation into the mass equation (13.1.5) replaces the variable ρ with p, yielding the *acoustic differential equations*:

$$\nabla p \cong -\rho_o \partial\bar{u}/\partial t \ \left[N/m^3\right] \qquad \text{(Newton's law)} \qquad (13.1.7)$$

[73] The Liebnitz identity facilitates taking time derivatives of integrals over volumes deforming in time.

$$\nabla \bullet \bar{u} = -(1/\gamma P_o)\partial p/\partial t \qquad (13.1.8)$$

These two differential equations are roughly analogous to Maxwell's equations (2.1.5) and (2.1.6), and can be combined. To eliminate $\bar{u}$ from Newton's law we operate on it with ($\nabla \bullet$), and then substitute (13.1.8) for $\nabla \bullet \bar{u}$ to form the *acoustic wave equation*, analogous to the Helmholtz wave equation (2.2.7):

$$\nabla^2 p - (\rho_o/\gamma P_o)\partial^2 p/\partial t^2 = 0 \qquad \text{(acoustic wave equation)} \qquad (13.1.9)$$

Wave equations state that the second spatial derivative equals the second time derivative times a constant. If the constant is not frequency dependent, then any arbitrary function of an argument that is the sum or difference of terms linearly proportional to time and space will satisfy this equation; for example:

$$p(\bar{r},t) = p(\omega t - \bar{k} \bullet \bar{r}) \ \left[\mathrm{N/m^2}\right] \qquad (13.1.10)$$

where p(•) is an arbitrary function of its argument (•), and $\bar{k} = k_x\hat{x} + k_y\hat{y} + k_z\hat{z}$; this is analogous to the wave solution (9.2.4) using the notation (9.2.5). Substituting the solution (13.1.10) into the wave equation yields:

$$\left(\partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2\right)p(\omega t - \bar{k} \bullet \bar{r}) - (\rho_o/\gamma P_o)\partial^2 p(\omega t - \bar{k} \bullet \bar{r})/\partial t^2 = 0 \qquad (13.1.11)$$

$$-\left(k_x^2 + k_y^2 + k_z^2\right)p''(\omega t - \bar{k} \bullet \bar{r}) - (\rho_o/\gamma P_o)\omega^2 p''(\omega t - \bar{k} \bullet \bar{r}) = 0 \qquad (13.1.12)$$

$$k_x^2 + k_y^2 + k_z^2 = k^2 = \omega^2 \rho_o/\gamma P_o = \omega^2/v_p^2 \qquad (13.1.13)$$

This is analogous to the electromagnetic dispersion relation (9.2.8).

As in the case of electromagnetic waves [see (9.5.19) and (9.5.20)], the *acoustic phase velocity* v_p and *acoustic group velocity* v_g are simply related to k:

$$v_p = \omega/k = (\gamma P_o/\rho_o)^{0.5} = c_s \qquad \text{(acoustic phase velocity)} \qquad (13.1.14)$$

$$v_g = (\partial k/\partial \omega)^{-1} = (\gamma P_o/\rho_o)^{0.5} = c_s \qquad \text{(acoustic group velocity)} \qquad (13.1.15)$$

Adiabatic acoustic waves propagating in 0^oC air near sea level experience $\gamma = 1.4$, $\rho_o = 1.29$ [kg/m^3], and $P_o = 1.01\times10^5$ [N/m^2], yielding $c_s \cong 330$ [m/s].

In solids or liquids the constitutive relation is:

$$\partial\rho/\partial p = \rho/K \qquad \text{(constitutive relation for solids and liquids)} \quad (13.1.16)$$

K [N m^{-2}] is the *bulk modulus* of the medium. The coefficient 1/K then replaces $1/\gamma P_o$ in (13.1.8–10), yielding the *acoustic velocity in solids* and liquids:

$$c_s = (K/\rho_o)^{0.5} \quad [\text{m s}^{-1}] \qquad \text{(acoustic velocity in solids and liquids)} \quad (13.1.17)$$

Typical acoustic velocities are 900 - 2000 m s^{-1} in liquids (~1500 m s^{-1} in water), and 1500–13,000 m s^{-1} in solids (~5900 m s^{-1} in steel).

Analogous to (7.1.25) and (7.1.26), the acoustic differential equations (13.1.8) and (13.1.7) can be simplified for sinusoidal plane waves propagating along the z axis:

$$\nabla \underline{p} \bullet \hat{z} = \frac{d\underline{p}(z)}{dz} = -j\omega\rho_o \underline{u}_z(z) \qquad (13.1.18)$$

$$\nabla \bullet \underline{\bar{u}} = \frac{d\underline{u}_z(z)}{dz} = \frac{-j\omega}{\gamma P_o}\underline{p}(z) \qquad (13.1.19)$$

These can be combined to yield the wave equation for z-axis waves analogous to (7.1.27):

$$\frac{d^2\underline{p}(z)}{dz^2} = -\omega^2 \frac{\rho_o}{\gamma P_o}\underline{p}(z) \qquad (13.1.20)$$

Analogous to (7.1.28) and (7.1.29), the solution is a sum of exponentials of the form:

$$\underline{p}(z) = \underline{p}_+ e^{-jkz} + \underline{p}_- e^{+jkz} \quad [\text{N m}^{-2}] \qquad (13.1.21)$$

$$\underline{u}_z(z) = -\frac{1}{j\omega\rho_o}\frac{d\underline{p}(z)}{dz} = \frac{k}{\omega\rho_o}\left[\underline{p}_+ e^{-jkz} - \underline{p}_- e^{+jkz}\right] \ [\text{m/s}] \qquad (13.1.22)$$

Note that, unlike electromagnetic waves, where the key fields are vectors transverse to the direction of propagation, the velocity vector for acoustic waves is in the direction of propagation and pressure is a scalar.

Analogous to (7.1.31), the characteristic *acoustic impedance* of a gas is:

$$\eta_s = \frac{\underline{p}(z)}{\underline{u}_z(z)} = \frac{\omega\rho_o}{k} = \rho_o c_s = \sqrt{\gamma\rho_o P_o} \quad [\text{N s/m}^3] \qquad (13.1.23)$$

The acoustic impedance of air at room temperature is ~425 [N s m^{-3}]. The acoustic impedance for solids and liquids is $\eta_s = \rho_o c_s = (\rho_o K)^{0.5}$ [N s m^{-3}]. Note that the units are not ohms.

The instantaneous acoustic intensity [W m^{-2}] of this plane wave is $p(t)u_z(t)$, the complex power is $\underline{p}\underline{\bar{u}}^*/2$, and the time average acoustic power is $R_e\{\underline{p}\underline{\bar{u}}^*/2\}$ [W m^{-2}], analogous to (2.7.41).

We can derive an *acoustic power conservation* law similar to the Poynting theorem (2.7.22) by computing the divergence of $\underline{p}\ \underline{\bar{u}}^*$ [W m^{-2}] and substituting in (13.1.18) and (13.1.19):[74]

$$\nabla \bullet \left(\underline{p}\underline{\bar{u}}^*\right) = \underline{\bar{u}}^* \bullet \nabla \underline{p} + \underline{p}\nabla \bullet \underline{\bar{u}}^* = \underline{\bar{u}}^* \bullet \left(-j\omega\rho_o\underline{\bar{u}}\right) + j\omega\underline{p}\underline{p}^*/\gamma P_o \tag{13.1.24}$$

$$= -4j\omega\left(\left[\rho_o|\underline{\bar{u}}|^2/4\right] - \left[|\underline{p}|^2/4\gamma P_o\right]\right) = -4j\omega\left(\langle W_k\rangle - \langle W_p\rangle\right) \tag{13.1.25}$$

The time average *acoustic kinetic energy density* of the wave is W_k [J m^{-3}] $= \rho_o|\underline{\bar{u}}|^2/4$, and the time average *acoustic potential energy density* is $W_p = |\underline{p}|^2/4\gamma P_o$. For liquids or solids $\gamma P_o \rightarrow K$, so $W_p = |\underline{p}|^2/4K$. If there is no divergence of acoustic radiated power $\underline{p}\underline{\bar{u}}^*$, then it follows from (13.1.25) that:

$$\langle W_k\rangle = \langle W_p\rangle \qquad \text{(energy balance in a lossless resonator)} \tag{13.1.26}$$

The *acoustic intensity* I [W m^{-2}] of an acoustic plane wave, analogous to (2.7.41), is:

$$I = R_e\{\underline{p}\underline{u}^*/2\} = |\underline{p}|^2/2\eta_s = \eta_s|\underline{u}|^2/2\ \ [\text{W m}^{-2}] \qquad \text{(acoustic intensity)} \tag{13.1.27}$$

where the acoustic impedance $\eta_s = \rho_o c_s$. The instantaneous acoustic intensity is $p(t)u_z(t)$, as noted above.

Example 13.1A

A loud radio radiates 100 acoustic watts at 1 kHz from a speaker 10-cm square near sea level where ρ_o = 1.29 [kg m^{-3}] and $c_s \cong 330$ m s^{-1}. What are the: 1) wavelength, 2) peak pressure, particle velocity, and displacement, and 3) average energy density of this uniform acoustic plane wave in the speaker aperture?

Solution: $\lambda = c_s/f = 330/1000 = 33$ cm. (13.1.22) yields $|\underline{u}| = (2I/\eta_s)^{0.5}$, and (13.1.18) says $\eta_s = \rho_o c_s$, so $|\underline{u}| = [200/(1.29\times330)]^{0.5} = 0.69$ [m s^{-1}]. $\underline{p} = \eta_s\underline{u} = 425.7\times0.69 = 292$ [N m^{-2}]. Note that this acoustic pressure is much less than the ambient pressure $P_o \cong 10^5$ N m^{-2}, as required for linearization of the acoustic equations. Displacement

[74] Although these two equations apply to waves propagating in the z direction, their right-hand sides also apply to any direction if the subscript z is omitted.

$\underline{d}$ is the integral of velocity $\underline{u}$, so $\underline{d} = \underline{u}/j\omega$ and the peak-to-peak displacement is $2|\underline{u}|/\omega = 2\times 0.69/2\pi 1000 = 0.22$ mm. The average acoustic energy density stored equals $2\langle W_k\rangle = 2\rho_o|\underline{\bar{u}}|^2/4 = 1.29(0.69)^2/2 = 0.31$ [J m^{-3}].

13.2 *Acoustic waves at interfaces and in guiding structures and resonators*

13.2.1 Boundary conditions and waves at interfaces

The behavior of acoustic waves at boundaries is determined by the acoustic boundary conditions. At rigid walls the normal component of acoustic velocity must clearly be zero, and fluid pressure is unconstrained there. At boundaries between two fluids or gases in equilibrium, both the acoustic pressure $p(\bar{r},t)$ and the normal component of acoustic velocity $\bar{u}_\perp(\bar{r},t)$ must be continuous. If the pressure were discontinuous, then a finite force normal to the interface would be acting on infinitesimal mass, giving it infinite acceleration, which is not possible. If $\bar{u}_\perp$ were discontinuous, then $\partial p/\partial t$ at the interface would be infinite, which also is not possible; (13.1.8) says $\nabla \bullet \bar{u} = -(1/\gamma P_o)\partial p/\partial t$. These *acoustic boundary conditions* at a boundary between media 1 and 2 can be stated as:

$$p_1 = p_2 \qquad \text{(boundary condition for pressure)} \qquad (13.2.1)$$

$$\bar{u}_{1\perp} = \bar{u}_{2\perp} \qquad \text{(boundary condition for velocity)} \qquad (13.2.2)$$

A uniform acoustic plane wave incident upon a planar boundary between two media having different acoustic properties will generally have a transmitted component and a reflected component, as suggested in Figure 13.2.1. The angles of incidence, reflection, and transmission are θ_i, θ_r, and θ_t, respectively.

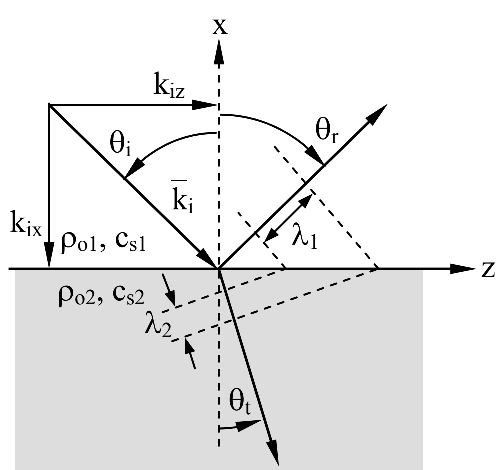

Figure 13.2.1 Acoustic waves at a planar interface with phase matching.

A typical example is the boundary between cold air overlying a lake and warm air above; the warm air is less dense, although the pressures across the boundary must balance. Since $c_s \propto \rho_o^{-0.5}$ and $\eta_s \propto \rho_o^{0.5}$, both the amplitudes and angles of propagation must change at the density discontinuity.

As was done for electromagnetic waves (see Section 9.2.2), we can begin with tentative general expressions for the incident, reflected, and transmitted plane waves:

$$\underline{p}_i(\bar{r}) = \underline{p}_{io} e^{+jk_{ix}x - jk_{iz}z} \quad \text{(incident wave)} \quad (13.2.3)$$

$$\underline{p}_r(\bar{r}) = \underline{p}_{ro} e^{+jk_{rx}x - jk_{rz}z} \quad \text{(reflected wave)} \quad (13.2.4)$$

$$\underline{p}_t(\bar{r}) = \underline{p}_{to} e^{+jk_{tx}x - jk_{tz}z} \quad \text{(transmitted wave)} \quad (13.2.5)$$

At x = 0 the pressure is continuous across the boundary (13.2.1), so $\underline{p}_i(\bar{r}) + \underline{p}_r(\bar{r}) = \underline{p}_t(\bar{r})$, which requires the phases (-jkz) to match:

$$k_{iz} = k_{rz} = k_{tz} \equiv k_z \quad (13.2.6)$$

But k_z is the projection of the $\bar{k}$ on the z axis, so $k_{iz} = k_i \sin\theta_i$, where $k_i = \omega/c_{si}$, and:

$$k_i \sin\theta_i = k_r \sin\theta_r = k_t \sin\theta_t \quad (13.2.7)$$

$$\theta_i = \theta_r \quad (13.2.8)$$

$$\sin\theta_t / \sin\theta_i = c_{si}/c_{st} \quad \text{(\textit{acoustic Snell's law})} \quad (13.2.9)$$

Thus acoustic waves refract at boundaries like electromagnetic waves (9.2.26).

Acoustic waves can also be evanescent for $\theta_i > \theta_c$, where the critical angle θ_c is the angle of incidence (9.2.30) required by Snell's law when $\theta_t = 90°$:

$$\theta_c = \sin^{-1}(c_{si}/c_{st}) \quad \text{(\textit{acoustic critical angle})} \quad (13.2.10)$$

When $\theta_i > \theta_c$, then k_{tx} becomes imaginary, analogous to (9.2.32), the transmitted acoustic wave is evanescent, and there is total reflection of the incident acoustic wave. Thus:

$$k_{tx} = \pm j\left(k_t^2 - k_z^2\right)^{0.5} = \pm j\alpha \quad (13.2.11)$$

$$\underline{p}_t(x,z) = \underline{p}_{to} e^{-\alpha x - jk_z z} \quad (13.2.12)$$

It follows from the complex version of (13.1.7) that:

$$\underline{\bar{u}}_t = -\nabla \underline{p}_t / j\omega\rho_o = (\alpha\hat{x} + jk_z\hat{z})\underline{p}_t / j\omega\rho_o \tag{13.2.13}$$

The complex power flow in this *acoustic evanescent wave* is $\underline{pu}*$, analogous to (9.2.35), so the power flowing in the -x direction is imaginary and the time-average real power flow is:

$$R_e\{\underline{p}\underline{\bar{u}}*\}/2 = \hat{z}(k_z/2\omega\rho_o)|\underline{p}_{to}|^2 e^{-2\alpha z} \ \left[\mathrm{W\ m^{-2}}\right] \tag{13.2.14}$$

The fraction of power reflected from an acoustic boundary can be found by applying the boundary conditions and solving for the unknown reflected amplitude. If we define $\underline{p}_{ro}$ and $\underline{p}_{to}$ as $\underline{\Gamma p}_{io}$ and $\underline{Tp}_{io}$, respectively, then matching boundary conditions at x = z = 0 yields:

$$p_{io} + p_{ro} = p_{to} \Rightarrow 1 + \underline{\Gamma} = \underline{T} \tag{13.2.15}$$

We need an additional boundary condition, and may combine $\underline{\bar{u}} = -\nabla \underline{p}/j\omega\rho_o$ (13.1.7) with the expression for $\underline{p}$ (13.2.3) to yield:

$$\underline{\bar{u}}_i = \left[(-jk_{xi}\hat{x} + jk_z\hat{z})/j\omega\rho_{oi}\right]\underline{p}_{io} e^{+jk_{ix}x - jk_{iz}z} \tag{13.2.16}$$

Similar expressions for $\bar{u}_r$ and $\bar{u}_t$ can be found, and enforcing continuity of $\bar{u}_\perp$ across the boundary at x = z = 0 yields:

$$\frac{k_{xi}}{\omega\rho_{oi}} - \underline{\Gamma}\frac{k_{xi}}{\omega\rho_{oi}} = \frac{k_{xt}}{\omega\rho_{ot}}\underline{T} \tag{13.2.17}$$

$$1 - \underline{\Gamma} = \underline{T}\frac{k_{xt}\rho_{oi}}{k_{xi}\rho_{ot}} = \underline{T}\frac{\eta_i \cos\theta_t}{\eta_t \cos\theta_i} \equiv \frac{\underline{T}}{\eta_n} \tag{13.2.18}$$

where we define the normalized angle-dependent acoustic impedance $\eta_n \equiv (\eta_t\cos\theta_i/\eta_i\cos\theta_t)$ and we recall $k_{xt} = k_t\cos\theta_t$, $k_t = \omega/c_{st}$, and $\eta_t = c_{st}\,\rho_{ot}$. Combining (13.2.15) and (13.2.18) yields:

$$\underline{\Gamma} = \frac{\underline{\eta}_n - 1}{\underline{\eta}_n + 1} \tag{13.2.19}$$

$$\underline{T} = 1 + \underline{\Gamma} = \frac{2\underline{\eta}_n}{\underline{\eta}_n + 1} \tag{13.2.20}$$

These expressions for $\underline{\Gamma}$ and $\underline{T}$ are essentially the same as for electromagnetic waves, (7.2.31) and (7.2.32), although the expressions for η_n are different. The fraction of acoustic power reflected is $|\underline{\Gamma}|^2$. Acoustic impedance $\underline{\eta}(z)$ for waves propagating perpendicular to boundaries therefore also are governed by (7.2.24):

$$\underline{\eta}(z) = \eta_o \frac{\underline{\eta}_L - j\eta_o \tan kz}{\eta_o - j\underline{\eta}_L \tan kz} \tag{13.2.21}$$

The Smith chart method of Section 7.3.2 can also be used.

There can even be an *acoustic Brewster's angle* θ_B when $\underline{\Gamma} = 0$, analogous to (9.2.75). Equation (13.2.19) suggests this happens when $\eta_n = 1$ or, from (13.2.18), when $\eta_i\cos\theta_t = \eta_t\cos\theta_B$. After some manipulation it can be shown that Brewster's angle is:

$$\theta_B = \tan^{-1}\sqrt{\frac{(\eta_t/\eta_i)^2 - 1}{1-\left(c_{s_t}/c_{s_i}\right)^2}} \tag{13.2.22}$$

Example 13.2A

A typical door used to block out sounds might be 3 cm thick and have a density of 1000 kg m^{-3}, large compared to 1.29 kg m^{-3} for air. If $c_s = 330$ m s^{-1} in air and 1000 m s^{-1} in the door, what are their respective acoustic impedances, η_a and η_d? What fraction of 500-Hz normally incident acoustic power would be reflected by the door? The fact that the door is not gaseous is irrelevant here if it is free to move and not secured to its door jamb.

Solution: The acoustic impedance $\eta = \rho_o c_s = 425.7$ in air and 10^6 in the door (13.1.23). The impedance at the front surface of the door given by (13.2.21) is $\eta_{fd} = \eta_d(\eta_a - j\eta_d \tan kz)/(\eta_d - j\eta_a \tan kz)$, where $k = 2\pi/\lambda_d$ and $z = 0.03$. $\lambda_d = c_d/f = 1000/500 = 2$, so $kz = \pi z = 0.094$, and $\tan kz = 0.095$. Thus $\eta_{fd} = 425.7 + 3.84$ and $\eta_{fd}/\eta_a = 1.0090$. Using (13.2.19) the reflected power fraction $= |\Gamma|^2 = |(\eta_n - 1)/(\eta_n + 1)|^2$ where $\eta_n = \eta_{fd}/\eta_a = 1.0090$, we find $|\Gamma|^2 \cong 2\times10^{-5}$. Virtually all acoustic power passes through. If this solid door were secure in its frame, shear forces (neglected here) would lead to far better acoustic isolation.

13.2.2 Acoustic plane-wave transmission lines

Acoustic plane waves guided within tubes of constant cross-section satisfy the boundary conditions posed by stiff walls: 1) $u_\perp = 0$, and 2) any $u_{//}$ and p is permitted. If these tubes curve slowly relative to a wavelength then their plane-wave behavior is preserved. The viscosity of gases is sufficiently low that frictional losses at the wall can usually be neglected in small acoustic devices. The resulting waves are governed by the acoustic wave equation (13.1.20), which has the solutions for p, u_z, and η given by (13.1.21), (13.1.22), and (13.1.23), respectively. Wave intensity is governed by (13.1.26), the complex reflection coefficient $\underline{\Gamma}$ is given by

(13.2.19), and impedance transformations are governed by (13.2.21). This set of equations is adequate to solve most acoustic transmission line problems in single tubes once we model their terminations.

Two acoustic terminations for tubes are easily treated: closed ends and open ends. The boundary condition posed at the closed end of an acoustic pipe is simply that u = 0. At an open end the pressure is sufficiently released that $p \cong 0$ there. If we intuitively relate acoustic velocity u(t,z) to current i(z,t) in a TEM line, and p(z,t) to voltage v(t,z), then a closed pipe is analogous to an open circuit, and an open pipe is analogous to a short circuit (the reverse of what we might expect).[75] Standing waves exist in either case, with $\lambda/2$ separations between pressure nulls or between velocity nulls.

13.2.3 Acoustic waveguides

Acoustic waveguides are pipes that convey sound in one or more waveguide modes. Section 13.2.2 considered only the special case where the waves were uniform and the acoustic velocity $\overline{u}$ was confined to the ±z direction. More generally the wave pressure and velocity must satisfy the acoustic wave equation, analogous to (2.3.21):

$$\left(\nabla^2 + \omega^2/c_s^2\right)\begin{Bmatrix}\underline{p}\\ \overline{\underline{u}}\end{Bmatrix} = 0 \tag{13.2.23}$$

Solutions to (13.2.23) in cartesian coordinates are appropriate for rectangular waveguides, as discussed in Section 9.3.2. Assume that two of the walls are at x = 0 and y = 0. Then a wave propagating in the +z direction might have the general form:

$$\underline{p}(x,y,z) = \underline{p}_o \begin{Bmatrix}\sin k_x x\\ \cos k_x x\end{Bmatrix}\begin{Bmatrix}\sin k_y y\\ \cos k_y y\end{Bmatrix} e^{-jk_z z} \tag{13.2.24}$$

The choice between sine and cosine is dictated by boundary conditions on $\overline{\underline{u}}$, which can be found using $\overline{\underline{u}} = -\nabla\underline{p}/j\omega\rho_o$ (13.2.13). Since the velocity $\overline{\underline{u}}$ perpendicular to the waveguide walls at x = 0 and y = 0 must be zero, so must be the gradient ∇p in the same perpendicular x and y directions at the walls. Only the cosine factors in (13.2.24) have this property, so the sine factors must be zero, yielding:

$$\underline{p} = \underline{p}_o \cos k_x x \cos k_y y\, e^{-jk_z z} \tag{13.2.25}$$

$$\begin{aligned}\overline{\underline{H}} = \big[\hat{x}\ k_z\{\sin k_x x \ \textit{or}\ \cos k_x x\} \\ -\hat{y}\left(jk_y/k_o\right)\cos k_x x\ \sin k_y y + \hat{z}\left(k_z/k_o\right)\cos k_x x\ \cos k_y y\big]e^{-jk_z z}\end{aligned} \tag{13.2.26}$$

[75] Although methods directly analogous to TEM transmission lines can also be used to analyze tubes of different cross-sections joined at junctions, the subtleties place this topic outside the scope of this text.

Since $\overline{\underline{u}}_\perp$ (i.e. $\underline{u}_x$ and $\underline{u}_y$) must also be zero at the walls located at x = a and y = b, it follows that $k_x a = m\pi$, and $k_y b = n\pi$, where m and n are integers: 0,1,2,3,... Substitution of any of these solutions (13.2.25) into the wave equation (13.1.9) yields:

$$k_x^2 + k_y^2 + k_z^2 = (m\pi/a)^2 + (n\pi/b)^2 + (2\pi/\lambda_z)^2 = k_s^2 = \omega^2 \rho_o/\gamma P_o = \omega^2/c_s^2 = (2\pi/\lambda_s)^2 \quad (13.2.27)$$

$$k_{z_{mn}} = \sqrt{k_s^2 - k_x^2 - k_y^2} = \sqrt{\left(\frac{\omega}{c_s}\right)^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2} \rightarrow \pm j\alpha \text{ at } \omega_n \quad (13.2.28)$$

Therefore each acoustic mode A_{mn} has its own cutoff frequency ω_{mn} where k_z becomes imaginary. Thus each mode becomes evanescent for frequencies below its cutoff frequency f_{mn}, analogous to (9.3.22), where:

$$f_{mn} = \omega_{mn}/2\pi = \left[(c_s m/2a)^2 + (c_s n/2b)^2\right]^{0.5} \quad [\text{Hz}] \quad \text{(cutoff frequency)} \quad (13.2.29)$$

$$\lambda_{mn} = c_s/f_{mn} = \left[(m/2a)^2 + (n/2b)^2\right]^{-0.5} \quad [\text{m}] \quad \text{(cutoff wavelength)} \quad (13.2.30)$$

Below the cutoff frequency f_{mn} for each acoustic mode the *evanescent acoustic mode* propagates as $e^{-jk_z z} = e^{-\alpha z}$, analogous to (9.3.31), where the wave decay rate is:

$$\alpha = \left[(m\pi/a)^2 + (n\pi/b)^2 - (\omega_{mn}/c_s)^2\right]^{0.5} \quad (13.2.31)$$

The total wave in any acoustic waveguide is that superposition of separate modes which matches the given boundary conditions and sources, where one (A_{00}) or more modes always propagate and an infinite number ($m\rightarrow\infty$, $n\rightarrow\infty$) are evanescent and reactive. The expression for $\underline{p}$ follows from (13.2.25) where $e^{-jk_z z} \rightarrow e^{-\alpha z}$, and the expression for $\overline{\underline{u}}$ follows from $\overline{\underline{u}} = -\nabla\underline{p}/(j\omega\rho_o)$ (13.2.13).

13.2.4 Acoustic resonators

Any closed container trapping acoustic energy exhibits resonances just as do low-loss containers of electromagnetic radiation. We may consider a rectangular room, or perhaps a smaller box, as a rectangular acoustic waveguide terminated at its ends by walls (velocity nulls for u_z). The acoustic waves inside must obey (13.2.27):

$$k_x^2 + k_y^2 + k_z^2 = (m\pi/a)^2 + (n\pi/b)^2 + (q\pi/d)^2 = \omega^2/c_s^2 \quad (13.2.32)$$

where $k_z = 2\pi/\lambda_z$ has been replaced by $k_z = q\pi/d$ using the constraint that if the box is short- or open-circuited at both ends then its length d must be an integral number q of half-wavelengths $\lambda_z/2$; therefore $d = q\lambda_z/2$ and $2\pi/\lambda_z = q\pi/d$. Thus, analogous to (9.4.3), the *acoustic resonant frequencies* of a closed box of dimensions a,b,d are:

$$f_{mnq} = c_s\left[(m/2a)^2 + (n/2b)^2 + (q/2d)^2\right]^{0.5} \ [\text{Hz}] \quad \text{(resonant frequencies)} \quad (13.2.33)$$

A simple geometric construction yields the mode density (modes/Hz) for both acoustic and electromagnetic rectangular acoustic resonators of volume V = abd, as suggested in Figure 13.2.2.

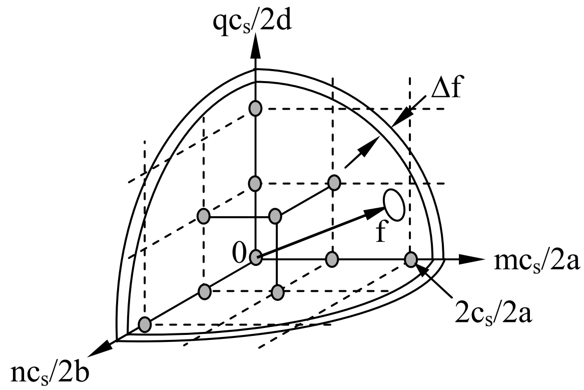

Figure 13.2.2 Resonant modes of a rectangular cavity.

Each resonant mode A_{mnq} corresponds to one set of quantum numbers m,n,q and to one cell in the figure. Referring to (13.2.11) it can be seen that frequency in the figure corresponds to the length of a vector from the origin to the mode A_{mnq} of interest. The total number N_o of acoustic modes with resonances at frequencies less than f_o is approximately the volume of the eighth-sphere shown in the figure, divided by the volume of each cell of dimension $(c_s/2a) \times (c_s/2b) \times (c_s/2d)$, where each cell corresponds to one acoustic mode. The approximation improves as f increases. Thus:

$$N_o \cong \left[4\pi f_o^3/(3\times 8)\right] \Big/ \left(c_s^3/8abd\right) = 4\pi f_o^3 V/3c_s^3 \quad [\text{modes} < f_o] \qquad (13.2.34)$$

In the electromagnetic case each set of quantum numbers m,n,q corresponds to both a TE and a TM resonant mode of a rectangular cavity, so N_o is then doubled:

$$^3\, N_o \cong 8\pi f_o\ V/3C^2 \qquad \text{(electromagnetic modes} < f_o) \quad (13.2.35)$$

The number density n_o of acoustic modes per Hertz is the volume of a thin shell of thickness Δf, again divided by the volume of each cell:

$$n_o \cong \Delta f \times 4\pi f^2 \Big/ \left(8c_s^3 / 8abd\right) = \Delta f\, 4\pi f^2 V / c_s^3 \quad \left[\text{modes Hz}^{-1}\right] \tag{13.2.36}$$

Thus the modes of a resonator overlap more and tend to blend together as the frequency increases. The density of electromagnetic modes in a similar cavity is again twice that for acoustic modes.

Typical examples of acoustic resonators include musical instruments such as horns, woodwinds, organ pipes, and the human vocal tract. Rooms with reflective walls are another example. In each case if we wish to excite a particular mode efficiently the source must not only excite it with the desired frequency, but also from a favorable location.

One way to identify favorable locations for modal excitation is to assume the acoustic source exerts pressure p across a small aperture at the wall or interior of the resonator, and then to compute the incremental acoustic intensity transferred from that source to the resonator using (13.1.27):

$$I = R_e\left\{pu^* / 2\right\} \tag{13.2.37}$$

In this expression we assume $\underline{\bar{u}}$ is dominated by waves already present in the resonator at the resonant frequency of interest and that the vector $\bar{u}$ is normal to the surface across which p is applied. Therefore pressure sources located at velocity nulls for a particular mode transfer no power and no excitation occurs. Conversely, power transfer is maximized if pressure is applied at velocity maxima. Similarly, acoustic velocity sources are best located at a pressure maximum of a desired mode. For example, all acoustic modes have pressure maxima at the corners of rectangular rooms, so velocity loudspeakers located there excite all modes equally.

The converse is also true. If we wish to damp certain acoustic modes we may put absorber at their velocity or pressure maxima, depending on the type of absorber used. A wire mesh that introduces drag damps high velocities, and surfaces that reflect waves weakly (such as holes in pipes) damp pressure maxima. High frequency modes are more strongly damped in humid atmospheres than are low frequency modes, but such bulk absorption mechanisms do not otherwise discriminate among them.

Because the pressure and velocity maxima are located differently for each mode, each mode typically has a different Q, which is the number of radians before the total stored energy w_T decays by a factor of e^{-1}. Therefore the Q of any particular mode m,n,p is (7.4.34):

$$Q = \omega_o w_T / P_d \qquad \text{(acoustic Q)} \tag{13.2.38}$$

The resonant frequencies and stored energies are given by (13.2.33) and (13.1.25), respectively, where it suffices to compute either the maximum stored kinetic or potential energy, for they are equal. The power dissipated P_d can be found by integrating the intensity expression (13.2.37) over the soft walls of the resonator, and adding any dissipation occurring in the interior.

Small changes in resonator shape can perturb acoustic resonant frequencies, much like electromagnetic resonances are perturbed. Whether a gentle indentation increases or lowers a particular resonant frequency depends on whether the time average acoustic pressure for the mode of interest is positive or negative at that indentation. It is useful to note that acoustic energy is quantized, where each *phonon* has energy hf Joules where h is Planck's constant; this is directly analogous to a photon at frequency f. Therefore the total acoustic energy in a resonator at frequency f is:

$$w_T = nhf \ [J] \tag{13.2.39}$$

If the cavity shape changes slowly relative to the frequency, the number n of acoustic phonons remains constant and any change in w_T results in a corresponding change in f. The work Δw_T done on the phonon field when cavity walls move inward Δz is positive if the time average acoustic pressure P_a is outward (positive), and negative if that pressure is inward or negative: $\Delta w_T = P_a \Delta z$. It is well known that gaseous flow parallel to a surface pulls on that surface as a result of the Bernoulli effect, which is the same effect that explains how airplane wings are supported in flight and how aspirators work. Therefore if an acoustic resonator is gently indented at a velocity maximum for a particular resonance, that resonant frequency f will be reduced slightly because the phonon field pulling the wall inward will have done work on the wall. All acoustic velocities at walls must be parallel to them. Conversely, if the indentation occurs near pressure maxima for a set of modes, the net acoustic force is outward and therefore the indentation does work on the phonon field, increasing the energy and frequency of those modes.

The most pervasive example of this phenomenon is human speech, which employs a vocal tract perhaps 16 cm long, typically less in women and all children. One end is excited by brief pulses in air pressure produced as the vocal chords vibrate at the pitch frequency of any vowel being uttered. The resulting train of periodic pressure pulses with period T has a frequency spectrum consisting of impulses spaced at T^{-1} Hz, typically below 500 Hz. The vocal tract then accentuates those impulses falling near any resonance of that tract.

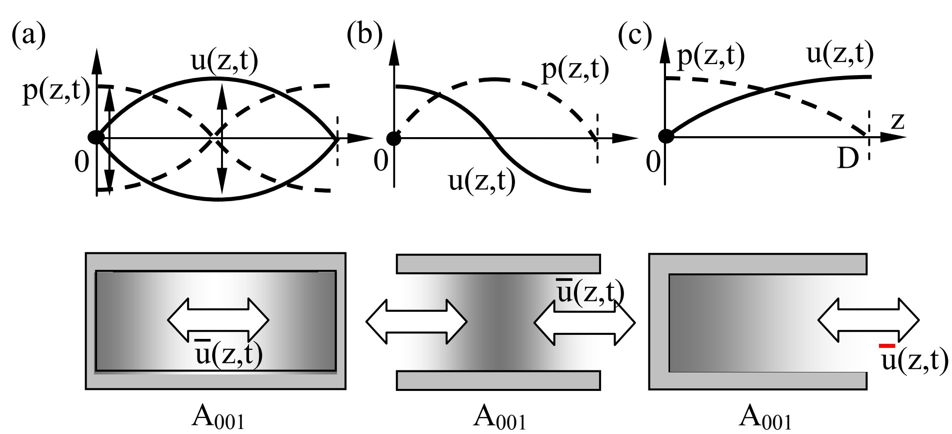

Figure 13.2.3 Acoustic resonances in tubes.

Figure 13.2.3 illustrates the lowest frequency acoustic resonances possible in pipes that are: (a) closed at both ends, (b) open at both ends, and (c) closed at one end and open at the other; each mode is designated A_{001} for its own structure, where 00 corresponds to the fact that the acoustic wave is uniform in the x-y plane, and 1 indicates that it is the lowest non-zero-frequency resonant mode. Resonator (a) is capable of storing energy at zero frequency by pressurization (in the A_{000} mode), and resonator (b) could store energy in the A_{000} mode if there were a steady velocity in one direction through the structure; these A_{000} modes are generally of no interest, and some experts do not consider them modes at all.

A sketch of the human vocal tract appears in Figure 13.2.4(a); at resonance it is generally open at the mouth and closed at the vocal chords, analogous to the resonator pictured in Figure 13.2.3(c). This structure resonates when its length D corresponds to one-quarter wavelength, three-quarters wavelength, or generally (2n-1)/4 wavelengths for the A_{00n} mode, as sketched in Figure 13.2.3(b). For a vocal tract 16 cm long and a velocity of sound $c_s = 340$ m s^{-1}, the lowest resonant frequency $f_{001} = c_s/\lambda_{001} = 340/(4 \times 0.16) = 531$ Hz. The next resonances, f_2 and f_3, fall at 1594 and 2656 Hz, respectively.

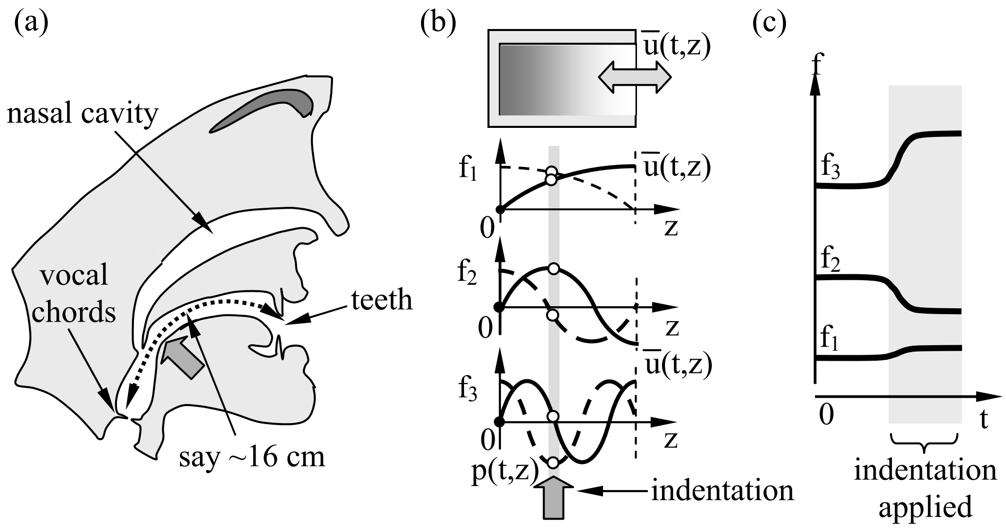

Figure 13.2.4 Human vocal tract.

If the tongue now indents the vocal tract at the arrow indicated in (a) and (b) of Figure 13.2.4, then the three illustrated resonances will all shift as indicated in (c) of the figure. The resonance f_1 shifts only slightly upward because the indentation occurs between the peaks for velocity and pressure, but nearer to the pressure peak. The resonances shift more significantly down for f_2 and up for f_3 because this indentation occurs near velocity and pressure maximum for these two resonances, respectively, while occurring near a null for the complimentary variable. By simply controlling the width of the vocal tract at various positions using the tongue and teeth, these tract resonances can be modulated up and down to produce our full range of vowels.

These resonances are driven by periodic impulses of air released by the vocal cords at a pitch controlled by the speaker. The pitch is a fraction of the lowest vocal tract resonant frequency, and the impulses are sufficiently brief that their harmonics range up to 5 kHz and more. Speech also includes high-pitched broadband noise caused by turbulent air whistling past the teeth or other obstacles as in the consonants s, h, and f, and impulsive spikes caused by temporary tract closures, as in the consonants b, d, and p. Speech therefore includes both voiced (driven by vocal chord impulses) and unvoiced components. The spectral content of most consonants can similarly be modulated by the vocal tract and mouth.

It is possible to change the composition of the air in the vocal tract, thus altering the velocity of sound c_s and the resonant frequencies of the tract, which are proportional to c_s (13.2.30). Thus when breathing helium all tract resonance frequencies increase by a noticeable fraction, equivalent to shortening the vocal tract. Note that pitch is not significantly altered by helium because the natural pitch of the vocal chords is determined instead primarily by their tension, composition, and length.

13.3 Acoustic radiation and antennas

Any mechanically vibrating surface can radiate acoustic waves. As in the case of electromagnetic waves, it is easiest to understand a point source first, and then to superimpose such radiators in combinations that yield the total desired radiation pattern. Reciprocity applies to linear acoustics, so the receiving and transmitting properties of acoustic antennas are proportional, as they are for electromagnetic waves; i.e. $G(\theta,\phi) \propto A(\theta,\phi)$.

The acoustic wave equation for pressure permits analysis of an *acoustic monopole* radiator:

$$\left[\nabla^2 + (\omega/c_s)^2\right]\underline{p} = 0 \tag{13.3.1}$$

If the acoustic radiator is simply an isolated sphere with a sinusoidally oscillating radius $\underline{a}$, then the source is spherically symmetric and so is the solution; thus $\partial/\partial\theta = \partial/\partial\phi = 0$. If we define $\omega/c_s = k$, then (13.3.1) becomes:

$$\left[r^{-2}d\left(r^2 d/dr\right) + k^2\right]\underline{p} = \left[d^2/dr^2 + 2r^{-1}d/dr + k^2\right]\underline{p} = 0 \tag{13.3.2}$$

This can be rewritten more simply as:

$$d^2(r\underline{p})/dr^2 + k^2(r\underline{p}) = 0 \tag{13.3.3}$$

This equation is satisfied if $r\underline{p}$ is an exponential, so a radial acoustic wave propagating outward would have the form:

$$\underline{p}(r) = \underline{K}r^{-1}e^{-jkr} \quad [\text{N m}^{-2}] \tag{13.3.4}$$

The associated acoustic velocity $\overline{\underline{u}}(r)$ follows from the complex form of Newton's law (13.1.7): $\nabla\underline{p} \cong -j\omega\rho_o\overline{\underline{u}}$ [N m^{-3}]:

$$\underline{\bar{u}}(r) = -\nabla \underline{p}/j\omega r_o = \hat{r}\underline{K}(\eta_s r)^{-1}\left[1+(jkr)^{-1}\right]e^{-jkr} \qquad (13.3.5)$$

The first and second terms in the solution (13.3.5) correspond to the acoustic far field and *acoustic near field*, respectively. When kr >> 1 or, equivalently, r >> λ/2π, then the near field term can be neglected, so that the far-field velocity corresponding to (13.3.4) is:

$$\underline{\bar{u}}_{ff}(r) = \hat{r}\underline{K}(\eta_s r)^{-1}e^{-jkr} \quad \left[\text{m s}^{-1}\right] \qquad \text{(far-field acoustic velocity)} \qquad (13.3.6)$$

The near-field velocity from (13.3.5) is:

$$\underline{\bar{u}}_{nf} = -j\underline{K}\hat{r}\left(k\eta_s r^2\right)^{-1} e^{-jkr} \quad \left[\text{m s}^{-1}\right] \qquad \text{(near-field acoustic velocity)} \qquad (13.3.7)$$

Since $k = \omega/c_s$, the near-field velocity is proportional to ω^{-1}, and becomes very large at low frequencies. Thus a velocity *microphone*, i.e., one that responds to acoustic velocity u rather than to pressure, will respond much more strongly to low frequencies than to high ones when the microphone is held close to one's lips (r <<λ/2π); this effect is usually compensated electronically. The advantage of velocity microphones is that they are largely deaf to ambient noise originating in their far field (r >> λ/2π), although they are sensitive to local wind turbulence.

The acoustic intensity I(r) can be computed using (13.1.22) for a sphere of radius $\underline{a}$ oscillating with a surface velocity $\underline{u}_o$ at r = a. In this case $\underline{\bar{u}}(a) = \hat{r}\underline{u}_o$, and substituting this value for $\underline{\bar{u}}$ into (13.3.7) yields the constant $\underline{K} = j\underline{u}_o\eta_s a^2$; this near-field equation is appropriate only if a << λ/2π. Thus, using (13.3.4) and (13.3.6), the far field intensity is:

$$I = R_e\{\underline{p}\underline{u}^*\}/2 = |\underline{K}|^2/2\eta_s r^2 = \eta_s\left|2\pi\underline{u}_o a^2\right|^2/2 \quad \left[\text{W m}^{-2}\right] \qquad (13.3.8)$$

Integrating I over a sphere of radius r yields the total acoustic power transmitted:

$$P_t = 2\pi\eta_s\left|\omega a^2\underline{u}_o/c_s\right|^2 \quad [\text{W}] \qquad \text{(acoustic power radiated)} \qquad (13.3.9)$$

where $2\pi/\lambda = \omega/c_s$ has been substituted. Thus P_t is proportional to $\eta_s\omega^2 a^4(u_o/c_s)^2$. This suggests the importance of using a high frequency ω and large radius a if substantial power is to be radiated using a velocity source $\underline{u}_o$.

If we imagine a Thevenin equivalent acoustic source providing a "current" of u_o, then, using (13.3.9), the *acoustic radiation resistance* of this acoustic antenna is:

$$R_r = P_t/\left(|\underline{u}_o|^2/2\right) = 4\pi\eta_s\left(ka^2\right)^2 \quad \left[\text{kg s}^{-1}\right] \qquad (13.3.10)$$

Arrays of such acoustic sources can synthesis a wide variety of antenna patterns because superposition applies and thus acoustic pressure and velocities will tend to cancel in some directions and add in others. For example, two such equal sources spaced distance d along the z axis, close compared to a wavelength and driven out of phase, would radiate the far-field pressure:

$$\underline{p}(r) \cong \left(jk\eta_s a^2 \underline{u}_o / r\right)\left(e^{-jkr_1} - e^{-jkr_2}\right) = \left(2k\eta_s a^2 \underline{u}_o / r\right)\sin\left[(kd/2)\cos\theta\right]e^{-jkr} \qquad (13.3.11)$$

where $r_{1,2} \cong r \pm (d/2)\cos\theta$. In the limit where $kd = 2\pi d/\lambda << 1$, (13.3.11) becomes:

$$p(r) \cong \left(k^2 d\eta_s a^2 \underline{u}_o / r\right)\cos\theta\ e^{-jkr} \qquad (13.3.12)$$

The radiated intensity $I(\theta)$ for this acoustic dipole is sketched in Figure 13.3.1(a), and is proportional to p^2 and therefore to k^4 and ω^4.

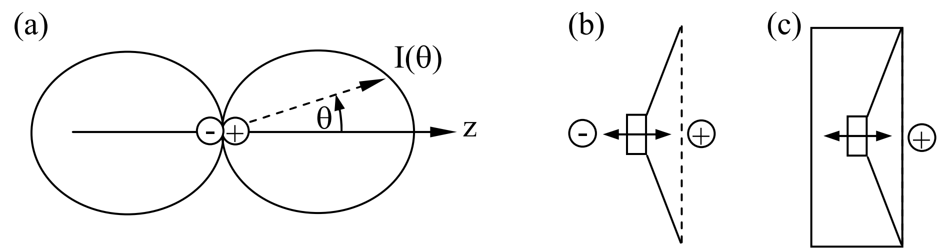

Figure 13.3.1 Acoustic radiators: (a) dipole, (b) loudspeaker, (c) baffled loudspeaker.

Thus it radiates poorly at low frequencies. Its *acoustic antenna gain* $G(\theta)$ is $3\cos^2\theta$, which can be computed by comparing the acoustic intensity I to the total acoustic power radiated P_t, just as is done for electromagnetic antennas. That is, the acoustic gain over an isotropic radiator is:

$$G(\theta,\phi) = I(\theta,\phi,r) \Big/ \left[P_t / 4\pi r^2\right] \qquad \text{(acoustic antenna gain)} \quad (13.3.13)$$

$$P_t = \int_0^{2\pi}\int_0^{\pi} I(\theta,\phi,r) r^2 \sin\theta\, d\theta\, d\phi \ \ [W] \qquad (13.3.14)$$

A common way to produce this dipole acoustic pattern is illustrated in Figure 13.3.1(b) for the case of a *loudspeaker* with no baffling to block radiation from the back side of its vibrating speaker cone; the back side is clearly 180^o out of phase with the velocity of the front side. The radiation from an unbaffled loudspeaker can unfortunately reflect from the walls of the room and interfere with the sound from the front side, reinforcing those frequencies for which the two rays add in phase, and diminishing those frequencies for which they are out of phase. As a result, most good loudspeakers are baffled so the reverse wave is trapped and cannot interfere with the primary wave radiated forward. This alters the acoustic impedance of the loudspeaker, but it can be electrically compensated. The result is an acoustic monopole that radiates total power in proportion to p^2, k^2, and therefore ω^2, rather than ω^4 as for the dipole.

A linear array of monopole acoustic sources of total length L has a diffraction pattern similar to that for an array of Hertzian dipoles. If the sources are all in phase, then they radiate maximum power broadside ($\theta \equiv 0$) where all rays remain in phase. They exhibit their first null at $\theta \cong \pm\lambda/L$. See Section 10.4 for more discussion of arrays of radiators. *Acoustic array microphones* have similar directional patterns, and microphones feeding parabolic reflectors of large dimension L have even higher gains, where the gain of an acoustic antenna is proportional to its effective area. The effective area of a parabolic reflector large compared to a wavelength is approximately its physical cross-section if it is uniformly illuminated without spillover, as shown in (11.1.25) for electromagnetic waves.

13.4 Electrodynamic-acoustic devices

13.4.1 Magneto-acoustic devices

One of the most common electro-acoustic devices is the loudspeaker, where larger units typically employ a magnetic solenoid (see Section 6.4.1) to drive a large lightweight cone that pushes air with the driven waveform. The frequency limits are within the mechanical resonances of the system, which are the natural frequencies of oscillation of the cone. The low frequency mechanical limit is typically set by the resonance of the rigid cone oscillating within its support structure. An upper mechanical limit is set by the natural resonance modes of the cone itself, which are lower for larger cones because the driven waves typically propagate outward from the driven center, and can reflect from the outer edge of the cone, setting up standing waves. The amplitude limit is typically set by the strength of the system and its linearity. As shown in Section 6.1.2, mechanical motion can generate electric voltages in the same systems, so they also function as microphones.

Another magneto-acoustic device uses magnetostriction, which is the shrinkage of some magnetic materials when exposed to large magnetic fields. They are used when small powerful linear motions are desired, typically on the order of microns. To obtain larger motions the drive head can be connected to a mechanically tapered acoustic transmission line resembling a small solid version of a trumpet horn that smoothly matches the high mechanical impedance of the driver over a large area to the low mechanical impedance of the small tip. The small tip moves much greater distances because acoustic power is conserved if the taper is slow compared to a quarter-wavelength, much like a series of quarter-wave transformers being used for impedance transformation; small tips moving large distances convey the same power as large areas moving small distances. Such acoustic-transmission-line transformers can be used in either direction, depending on whether high displacements or high forces are desired.

13.4.2 Electro-acoustic devices

The simplest electro-acoustic device is perhaps a capacitor with one plate that is free to move and push air in response to time-varying electric forces on it, as discussed in Section 6.2.2.

These can be implemented macroscopically or within micro-electromechanical systems (MEMS).

Some materials such as quartz are piezo-electric and shrink or distort when high voltages are place across them. Because this warping yields little heat, periodic excitation of quartz crystals can cause them to resonate with a very high Q, making them useful for time-keeping purposes in watches, computers, and other electronic devices. These mechanical resonances for common crystals are in the MHz range and have stabilities that are $\sim 10^{-4}$–10^{-6}, depending mostly on temperature stability; larger crystals resonate at lower frequencies. They can also be designed to drive tiny resonant loudspeakers at high acoustic frequencies and efficiencies for watch alarms, etc.

By reciprocity, good piezo-electric actuators are also good sensors and can be used as microphones. Mechanical distortion of such materials generates small measurable voltages. The same is true when the plate separation of capacitors is varied, as shown in Section 6.6.1. Mechanically tapered solid acoustic waveguides can also be used for impedance transformations between low-force/high-motion terminals and high-force/low motion terminals, as noted in Section 13.4.1. Levers can also be used for the same purpose.

13.4.3 Opto-acoustic-wave transducers

When transparent materials are compressed their permittivity generally increases, slowing lightwaves passing through. This phenomenon has been used to compute Fourier transforms of broadband signals that are converted to acoustic waves propagating down the length of a transparent rectangular rod. A uniform plane wave from a laser then passes through the rod at right angles to it and to the acoustic beam, and thereby experiences local phase lags along those portions of the rod where the acoustic wave has temporarily compressed it. If the acoustic wave is at 100 MHz and the velocity of sound in the bar is 1000 m/s, then the acoustic wavelength is 10 microns. If the laser has wavelength 1 micron, then the laser light will pass straight through the bar and will also diffract at angles $\pm\lambda_{laser}/\lambda_{acoustic} = 10^{-6}/10^{-5} = 0.1$ radian. Several other beams will emerge too, at $\sim\pm 0.2$, 0.3, etc. [radians]. There will therefore be a diffracted laser beam at an angle unique to each Fourier component of the acoustic signal, the strength of which depends on the magnitude of the associated optical phase delays along the rod. Lenses can then focus these various plane waves to make the power density spectrum more visible.

If several exit ports are provided for the emerging light beams, one per angle, the laser beam can effectively be switched at acoustic speeds among those ports. If 100 exit ports are provided, then the rod length L should be at least 100 wavelengths, or 1mm for the case cited above. At an acoustic velocity c_s of 1000 m/s a new wave can enter the device after $L/c_s = 10^{-3}/1000 = 10^{-6}$ seconds.

13.4.4 Surface-wave devices

Only compressive acoustic waves have been discussed so far, but acoustic shear waves can also be generated in solids, and exhibit most of the same wave phenomena as compressive waves, such as guidance and resonance. The dominant velocity in a shear wave is transverse to the direction of wave propagation. By generating shear waves on the surface of quartz devices, and by periodically loading those surfaces mechanically with slots or metal, multiple reflections are induced that, depending on their spacing relative to a wavelength, permit band-pass and band-stop filters to be constructed, as well as transformers, resonators, and directional couplers. Because quartz has such high mechanical Q, it is often used to construct high-Q resonators at MHz frequencies.