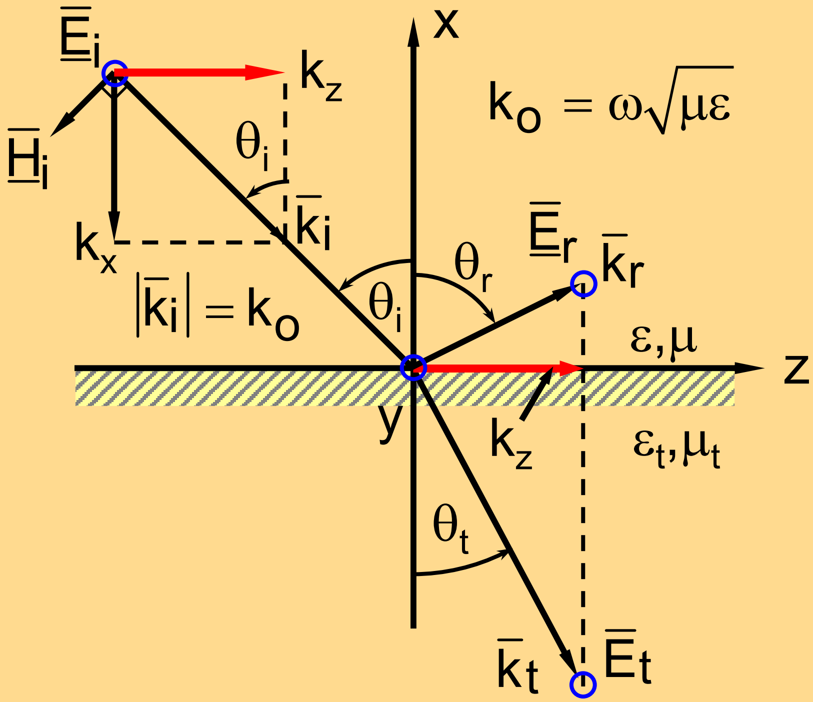
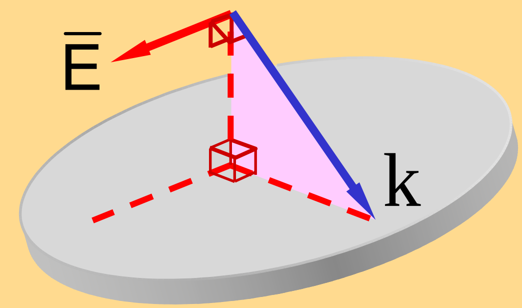
UPW AT A PLANAR BOUNDARY

Case I: TE Wave



"Transverse Electric"

$\triangleq \overline{E} \perp$ Plane of incidence



Trial Solutions:

Incident: $\underline{\overline{E}}_i = \hat{y}E_o e^{+jk_x x - jk_z z} = \hat{y}E_o e^{+j(k_o\cos\theta_i)x\ -\ j(k_o\sin\theta_i)z}$

Reflected: $\underline{\overline{E}}_r = \hat{y}\underline{\Gamma}E_o e^{-jk_x x\ -\ jk_z z}$

Transmitted: $\underline{\overline{E}}_t = \hat{y}\underline{T}E_o e^{+jk_{tx} x\ -\ jk_z z}$

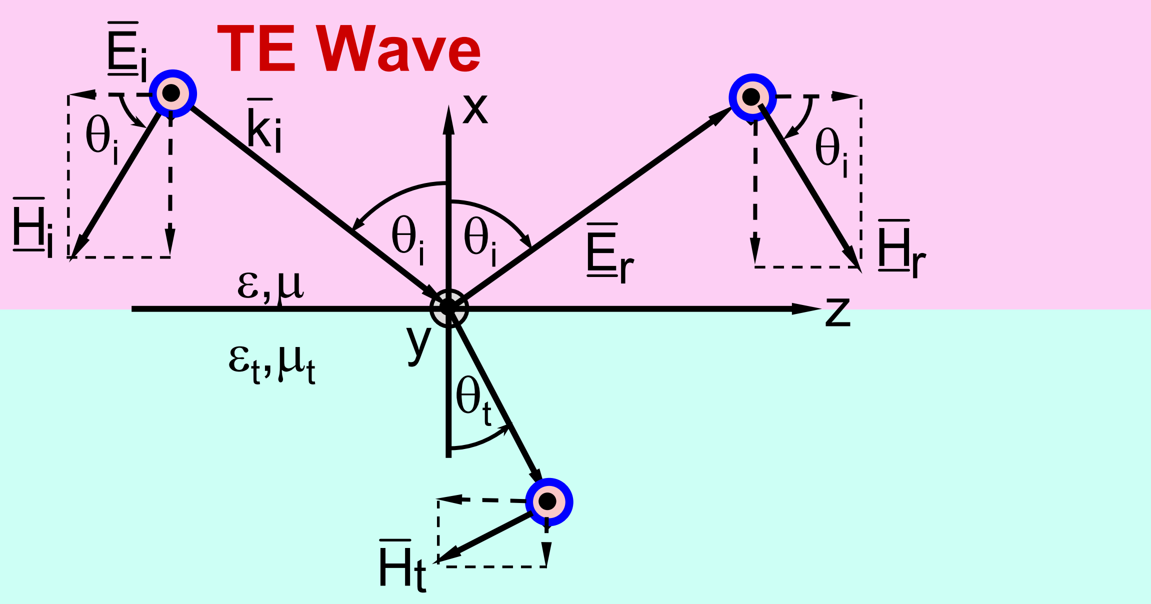
TE WAVE $\overline{H}$ AT A BOUNDARY

Incident: $\underline{\overline{E}}_i = \hat{y}E_o e^{+jk_x x - jk_z z}$

$$\underline{\overline{H}}_i = -\frac{E_o}{\eta_i}(\hat{x}\sin\theta_i + \hat{z}\cos\theta_i)e^{+jk_x x - jk_z z}$$

Reflected: $\underline{\overline{H}}_r = \frac{\underline{\Gamma}E_o}{\eta_i}(-\hat{x}\sin\theta_i + \hat{z}\cos\theta_i)e^{-jk_x x - jk_z z}$

Transmitted: $\underline{\overline{H}}_t = -\frac{\underline{T}E_o}{\eta_t}(\hat{x}\sin\theta_t + \hat{z}\cos\theta_t)\, e^{+jk_{tx}x - jk_{tz}z}$



IMPOSE BOUNDARY CONDITIONS

$\underline{\bar{E}}_{//}$ and $\underline{\bar{H}}_{//}$ **are continuous at x = 0:**

$\underline{\bar{E}}_{i//} + \underline{\bar{E}}_{r//} = \underline{\bar{E}}_{t//}$, and $\underline{\bar{H}}_{i//} + \underline{\bar{H}}_{r//} = \underline{\bar{H}}_{t//}$ for all y, z

Therefore:

$$\hat{y}E_o e^{+jk_x x \,(\to 0)\, - j(k_o \sin\theta_i)z} + \hat{y}\underline{\Gamma}E_o e^{-jk_x x \,(\to 0)\, - j(k_o \sin\theta_r)z} = \hat{y}\underline{T}E_o e^{+jk_{tx} x \,(\to 0)\, - j(k_t \sin\theta_t)z}$$

$$\Rightarrow k_o \sin\theta_i = k_o \sin\theta_r = k_t \sin\theta_t = k_z \Rightarrow \theta_i = \theta_r, \frac{\sin\theta_t}{\sin\theta_i} = \frac{k_o}{k_t} \text{ (Snell's Law)}$$

$$\Rightarrow E_o(1 + \underline{\Gamma}) = E_o\underline{T} \quad \Rightarrow \quad 1 + \underline{\Gamma} = \underline{T}$$

Similarly:

$$-\frac{E_o}{\eta_i}\hat{z}\cos\theta_i\, e^{+jk_x x - jk_z z} + \frac{\underline{\Gamma}E_o}{\eta_i}\hat{z}\cos\theta_i\, e^{-jk_x x - jk_z z} = -\frac{\underline{T}E_o}{\eta_t}\hat{z}\cos\theta_t\, e^{+jk_{tx} x - jk_{tz} z}$$

$$\Rightarrow \frac{E_o}{\eta_i}\cos\theta_i - \frac{\underline{\Gamma}E_o}{\eta_i}\cos\theta_i = \frac{\underline{T}E_o}{\eta_t}\cos\theta_t \quad \Rightarrow \quad 1 - \underline{\Gamma} = \underline{T}\frac{\eta_i \cos\theta_t}{\eta_t \cos\theta_i}$$

SOLVE TE BOUNDARY EQUATIONS

We found:

$$1+\underline{\Gamma} = \underline{T} \quad \text{and} \quad 1-\underline{\Gamma} = \underline{T}\,\frac{\eta_i \cos\theta_t}{\eta_t \cos\theta_i}$$

Solving yields:

$$\Gamma(\theta_i) = \frac{\eta_t \cos\theta_i - \eta_i \cos\theta_t}{\eta_t \cos\theta_i + \eta_i \cos\theta_t} = \frac{\eta_n' - 1}{\eta_n' + 1} \quad \text{where} \quad \eta_n' \triangleq \frac{\eta_t \cos\theta_i}{\eta_i \cos\theta_t}$$

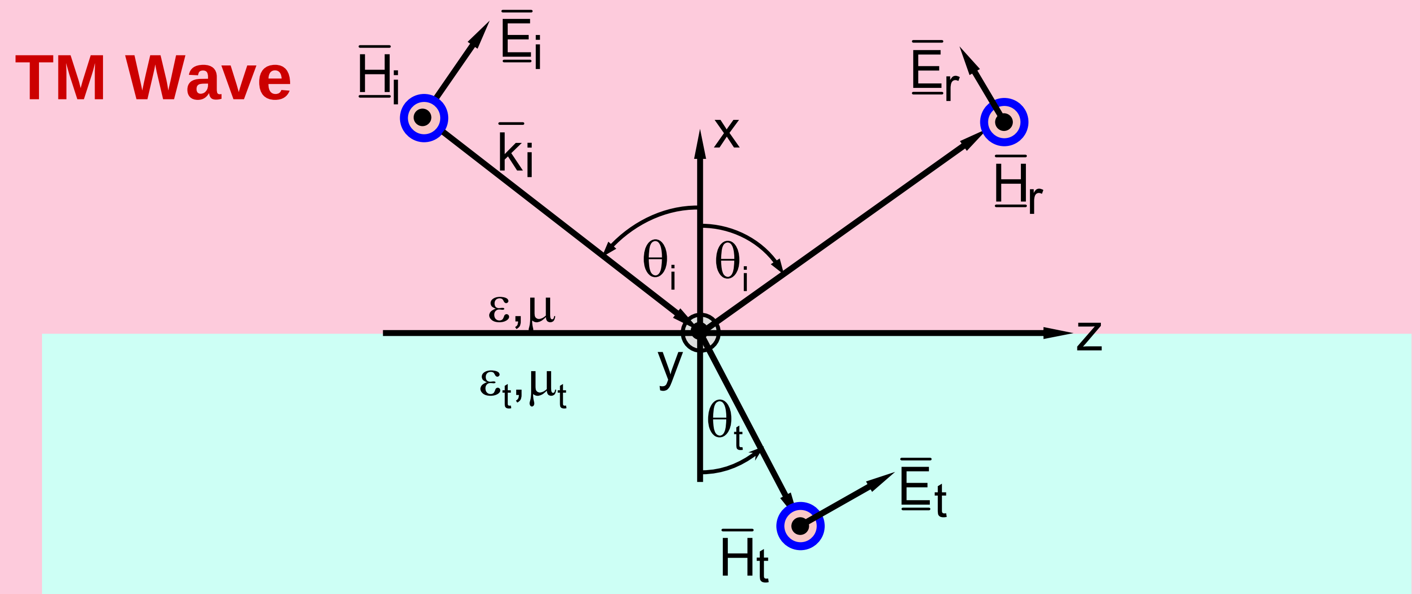
Normal incidence: $\theta = 0$, $\cos\theta = 1$, and $\eta_n' = \eta_t/\eta_i$

$$\Gamma(\theta_i) = \frac{\eta_n' - 1}{\eta_n' + 1} \quad \text{where} \quad \eta_n' \triangleq \frac{\eta_t}{\eta_i} = \frac{\sqrt{\mu_t/\varepsilon_t}}{\sqrt{\mu_i/\varepsilon_i}}$$

Glass has $\varepsilon \cong 4\varepsilon_o$ so $\eta_n' = 0.5 \Rightarrow \underline{\Gamma} = -1/3$ when light hits glass

Reflected power fraction = $|\underline{\Gamma}|^2 = 1/9$ (from both surfaces)

SOLVE TM BOUNDARY EQUATIONS



Option A: Repeat method for TE (write field expressions with unknown $\underline{\Gamma}$ and $\underline{T}$; impose boundary conditions; solve for $\underline{\Gamma}$ and $\underline{T}$)

Option B: Use duality to map TE solution to TM case

DUALITY OF MAXWELL'S EQUATIONS

By making these substitutions to our TE solution:	Our original equations :		Become these:
$\overline{E} \rightarrow \overline{H}$	$\nabla \times \overline{E} = -\mu \dfrac{\partial \overline{H}}{\partial t}$	$\rightarrow$	$\nabla \times \overline{H} = \varepsilon \dfrac{\partial \overline{E}}{\partial t}$
$\overline{H} \rightarrow -\overline{E}$	$\nabla \times \overline{H} = \varepsilon \dfrac{\partial \overline{E}}{\partial t}$	$\rightarrow$	$-\nabla \times \overline{E} = \mu \dfrac{\partial \overline{H}}{\partial t}$
$\varepsilon \leftrightarrow \mu$	$\nabla \bullet \varepsilon \overline{E} = 0$	$\rightarrow$	$\nabla \bullet \mu \overline{H} = 0$
	$\nabla \bullet \mu \overline{H} = 0$	$\rightarrow$	$\nabla \bullet \varepsilon \overline{E} = 0$
			Are they valid?

Under what circumstances does the solution to the resulting set of equations satisfy Maxwell's Equations?

They ARE Maxwell's Equations, just reordered, so the solution to the first set works if $\varepsilon \leftrightarrow \mu$ and ….

DUALITY: TM WAVE SOLUTIONS

For TE waves we found:

$$\underline{\Gamma}_{TE}(\theta_i) = \frac{\eta_n' - 1}{\eta_n' + 1} \quad \text{where} \quad \eta_n' \triangleq \frac{\eta_t \cos\theta_i}{\eta_i \cos\theta_t} \quad \text{and} \quad \eta = \sqrt{\frac{\mu}{\varepsilon}}$$

For TM waves :

Duality swaps $\mu \leftrightarrow \varepsilon$ to yield:

$$\underline{\Gamma}_{TM}(\theta_i) = \frac{\dfrac{\eta_t^{-1}\cos\theta_i}{\eta_i^{-1}\cos\theta_t} - 1}{\dfrac{\eta_t^{-1}\cos\theta_i}{\eta_i^{-1}\cos\theta_t} + 1} = \frac{\eta'_{TMn} - 1}{\eta'_{TMn} - 1} \quad \text{where} \quad \eta'_{TMn} = \frac{\eta_i \cos\theta_i}{\eta_t \cos\theta_t}$$

But this swap requires the boundary conditions to be dual too. They are dual here because both $\underline{\bar{E}}_{//}$ and $\underline{\bar{H}}_{//}$ are continuous across this boundary.

TE AND TM WAVE REFLECTIONS