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MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.013 Electromagnetics and Applications

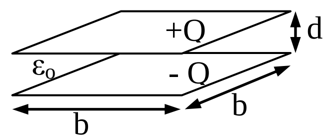
Student Name:

Final Exam | Closed book, no calculators | May 18, 2009

Please note the two pages of formulas provided at the back; the laser and acoustic expressions have been revised slightly. There are 10 problems; some are on the back sides of the sheets. For full credit, please ***simplify all expressions****, present* ***numerical answers to the extent practical*** *without a calculator or tedious computation, and place your* ***final answers within the boxes provided****. You may leave natural constants and trigonometric functions in symbolic form (π, ε_o, μ_o, η_o, h, e, sin(0.9), $\sqrt{2}$, etc.). To receive partial credit, provide all related work on the same sheet of paper and give brief explanations of your answer. Spare sheets are at the back.*

Problem 1. (25/200 points)

Two square capacitor plates in air have separation d, sides of length b, and charge ±Q as illustrated. Fringing fields can be neglected.



a) What is the capacitance C_a of this device?

$C_a = \varepsilon_o A/d = \varepsilon_o b^2/d$

$C_a = \varepsilon_o b^2/d$

b) A perfectly conducting plate is introduced between the capacitor plates, leaving parallel gaps of width d/10 above and below itself. What now is the device capacitance C_b when it is fully inserted?

$C_b = C_a'/2 = \varepsilon_o b^2/2(d/10) = 5\varepsilon_o b^2/d$

$C_b = 5\varepsilon_o b^2/d$

c) What is the magnitude and direction of the force $\overline{f}$ on the new plate of Part (b) as a function of the insertion distance L. Please express your answer as a function of the parameters given in the figure.

$\overline{f} = 2Q^2 d/\varepsilon_o b(b + 4L)^2$

$C = \varepsilon_o[5bL/d + b(b - L)/d]$

$\overline{f} = - dW_T/dL = - dW_T/dC \;\; dC/dL$

$= - (Q^2/2C^2)\, dC/dL = (Q^2/2[\varepsilon_o b/d]^2[b + 4L]^2)4\varepsilon_o b/d = 2Q^2 d/\varepsilon_o b(b + 4L)^2$

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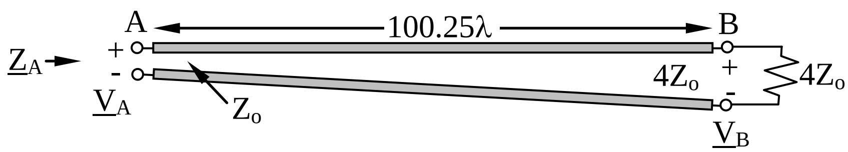
Problem 2. (20/200 points)

The plate separation of a lossless parallel-plate TEM line many wavelengths long (length $D = 100.25\lambda$) very slowly increases from end A to end B, as illustrated. This increases the characteristic impedance of the line from Z_o at the input end A, to $4Z_o$ at the output end B. This transition from A to B is so gradual that it produces no reflections. End B is terminated with a resistor of value $4Z_o$



a) What is the input impedance $\underline{Z}_A$ seen at end A? Explain briefly.

$\underline{Z}_A = Z_o$ [Ω]
Explanation: Line is matched: no reflections.

b) If the sinusoidal (complex) input input voltage is $\underline{V}_A$, what is the output voltage $\underline{V}_B$?

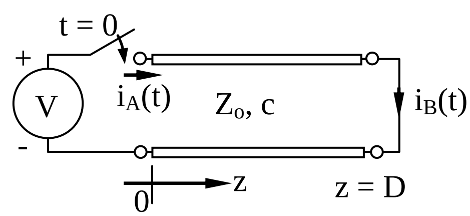
$P_{in} = P_{out}$ so $|\underline{V}_A|^2/2Z_o = |\underline{V}_B|^2/8Z_o \Rightarrow |\underline{V}_B| = 2|\underline{V}_A|$

$e^{-j\pi/2} = -j$

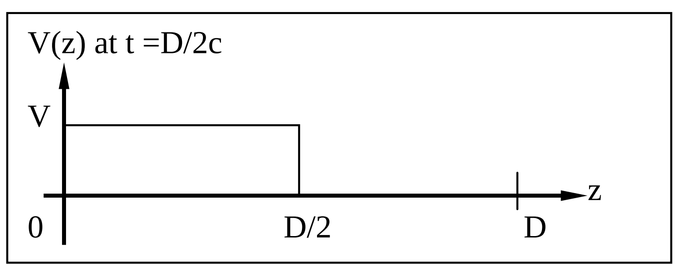
$\underline{V}_B = -2j\underline{V}_A$ [V]

Problem 3. (25/200 points)

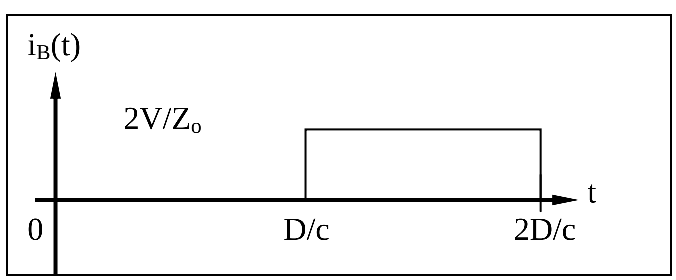
At t = 0 a switch connects a voltage V to a passive air-filled short-circuited TEM line of length D and characteristic impedance Z_o, as illustrated. Please sketch and quantify dimension:



a) The line voltage v(z) at t = D/2c.



b) The current $i_B(t)$ through the short circuit for $0 < t < 2D/c$.



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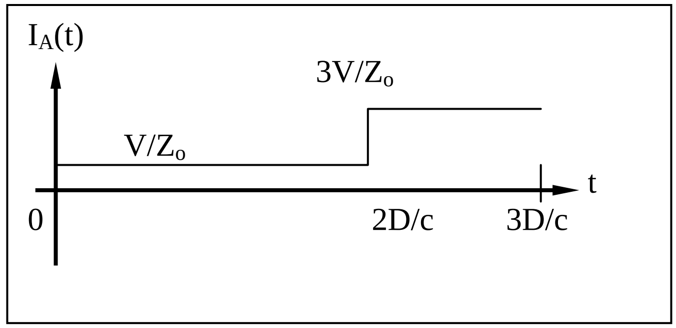
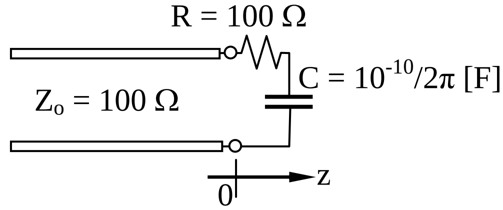
c) The current $i_A(t)$ from the voltage source (z = 0) for 0 < t < 3D/c.



Problem 4. (30/200 points)

A 100-ohm air-filled lossless TEM line is terminated with a 100-ohm resistor and a $10^{-10}/2\pi$ Farad capacitor in series, as illustrated. It is driven at 100 MHz.



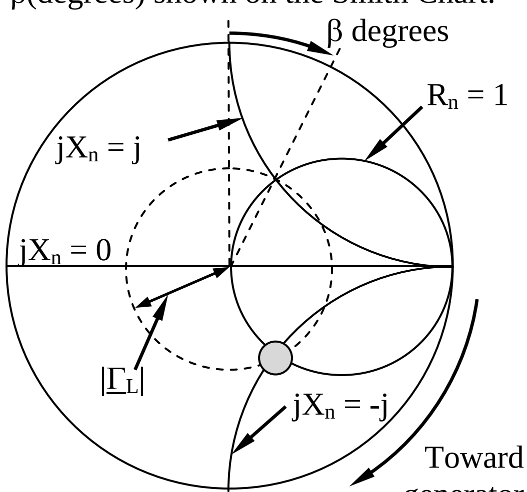
a) What fraction $F = |\underline{\Gamma}_L|^2$ of the incident power is reflected from this load?

F = 1/5

$\underline{Z}_n = 0.01(100 + 1/(j2\pi 10^8 10^{-10}/2\pi) = 1 - j$

$\underline{\Gamma} = (\underline{Z}_n - 1)/(\underline{Z}_n + 1) = -j/(2 - j)$, $|\underline{\Gamma}|^2 = F = (1/5^{0.5})^2 = 1/5$

b) What is the minimum distance D(meters) from the load at which the line current $|\underline{I}(z)|$ is maximum? You may express your answer in terms of the angle β(degrees) shown on the Smith Chart.



$D_{min} = 3/8 + \beta/240$ [m]

$D_{min} = l/8 + \beta\lambda/720 = 3(1/8 + \beta/720)$

$\lambda = c/f = 3\times10^8/10^8$

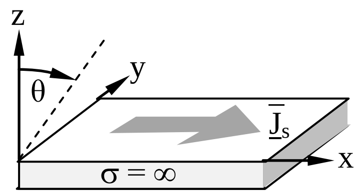
c) Can we match this load by adding another capacitor in series somewhere and, if so, at what distance D and with what value C_m?

Can we match? YES

D = ¾ + β/120 [m]

$C_m = 10^{-10}/2\pi$

Problem 5. (20/200 points)



A flat perfect conductor has a surface current in the xy plane at z = 0 of:

$$\overline{J}_s = \hat{x}\, J_o e^{-jbx} \text{ [A/m].}$$

a) Approximately what is $\overline{H}$ in the xy plane at z = 0+?

$\overline{H}(z = 0+) = - \hat{y}\, J_o e^{-jbx}$

b) How might one easily induce this current sheet at frequency f [Hz] on the surface of a good conductor? Please be reasonably specific and quantitative (not absolute phase).

To induce this current one might: Reflect a TM wave incident from $\phi = \pi$ and $\theta = \sin^{-1}(b\lambda_o/2\pi)$, where $\lambda_o < 2\pi/b$ and $|\underline{H}_y| = J_o/2$.

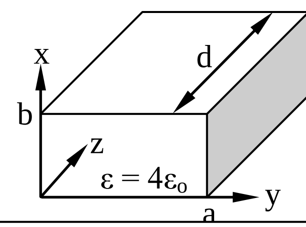
Problem 6. (10/200 points)

A certain evansescent wave at angular frequency ω in a slightly lossy medium has $\overline{E} = \hat{y}\, E_o e^{\alpha(x-0.01z) - jbz}$; assume $\mu = \mu_o$. What is the distance D between phase fronts for this wave?

$b = 2\pi/\lambda_z$

$D = 2\pi/b$

Problem 7. (25/200 points)



A resonator is filled with a dielectric having $\varepsilon = 4\varepsilon_o$ and has dimensions b, a, and d along the x, y, and z directions, respectively, where d > a > b.

a) What is the lowest resonant frequency $f_{m,n,q}$ [Hz] for this resonator?

$f_{m,n,q} = c_\varepsilon[(1/2a)^2 + (1/2b)^2]^{0.5}/2$

$f_{mnp} = (c_\varepsilon/2)[(m/a)^2 + (n/b)^2 + (p/d)^2]^{0.5} = c_\varepsilon[(1/2a)^2 + (1/2b)^2]^{0.5}$
$= c_\varepsilon[(1/2a)^2 + (1/2b)^2]^{0.5}/2$

b) What is the polarization of the electric vector $\overline{E}$ at the center of the resonator for this lowest frequency mode?

Polarization of $\overline{E}$ is: $\hat{x}$ (linear)

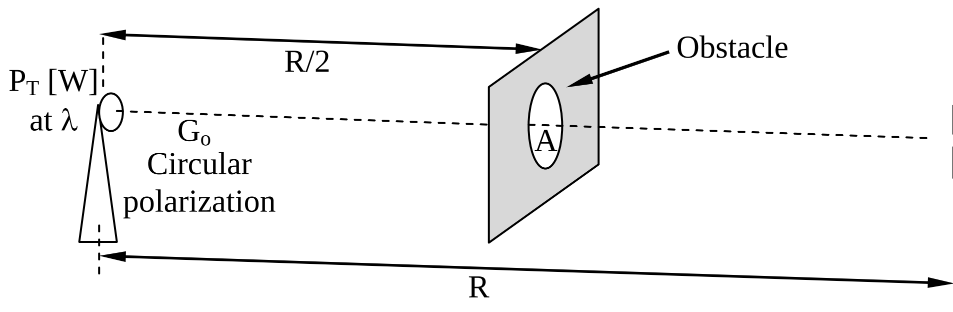
c) What is the Q of this resonance if the dielectric has a slight conductivity σ? Hint: a ratio of integrals may suffice, so the integrals might not need to be computed.

$Q = (\pi/\sigma\eta_o)(a^{-2} + d^{-2})$

$Q = w_o W_T/P_d = 2\pi f_{101}\, [2\int_V (\varepsilon_o|\underline{E}|^2/4)dV]/[\int_V (\sigma|\underline{E}|^2/2)dV$
$= 2\pi\varepsilon_o f_{101}/\sigma = (\pi/\sigma\eta_o)(a^{-2} + d^{-2})$

Problem 8. (20/200 points)

A certain transmitter transmits P_T watts of circularly polarized radiation with antenna gain G_o (in circular polarization) toward an optimally oriented matched short-dipole receiving antenna (gain = 1.5) located a distance R away. The wavelength is λ.



a) In the absence of any obstacles or reflections, what power P_R is received?

$P_R = 0.75\ P_T G_o(\lambda/4\pi R)^2$

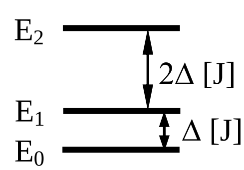
$P_R = (P_T G_o/4\pi R^2)\ A_e$, $A_e = 0.5\ 1.5\lambda^2/4\pi$

b) A large metal fence is then erected half way between the transmitter and receiver, and perpendicular to the line of sight. Fortunately it has a round hole of area A centered on that line of sight. Assume the hole is sufficiently small that the electrical phase of the incident wave is constant over its entirety. What power is received now?

$P_R = 0.75 P_T G_o(A/\pi R^2)^2$ [W]

Problem 9. (15/200 points)

An ideal lossless three-level laser has the illustrated energy level structure. Level 1 is Δ Joules above the ground state, and Level 2 is 3Δ Joules above the ground state. All rates of spontaneous emission A_{ij} have the same finite value except for A_{21}, which is infinite.



a) What should be the laser frequency f_L [Hz]?
$\Delta E = hf$

f_L [Hz] $= \Delta/h$

b) What is this laser's maximum possible efficiency η = (laser power)/(pump power)?

$\eta = 1/3$

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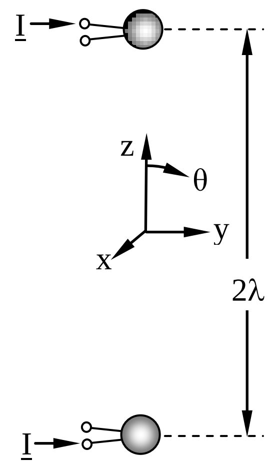
Problem 10. (10/200 points)

Two monopole (isotropic) acoustic antennas lying on the z axis are aligned in the z direction and separated by 2λ, as illustrated. <u>They are fed 180° out of phase</u>. In what directions θ does this acoustic array have maximum gain $G(\theta)$? Simple expressions suffice. If more than one direction has the same maximum gain, please describe all such directions.



$\theta = \pm\cos^{-1}(1/4)$ and $\pm\cos^{-1}(3/4)$

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