Solutions to Problem Set 12

12.1 (a) $\theta_c = \sin^{-1}(k_2/k_1) = \sin^{-1}(c_1/c_2) = \sin^{-1}(\varepsilon_2/\varepsilon_1)^{0.5} = 80.9$ degrees
(b) $\alpha_x = (k_z^2 - k_o^2)^{0.5}$ where $k_z/k_o = \sin 85^0$. It follows that $\alpha^{-1} = 0.60\ \lambda_o$
(c) $D = \sim 4\lambda_o$. Note that decay is rapid relative to D, so we might guess $D = \sim 0.6(\lambda_x/2)$ where $\lambda_x = \lambda_z \tan\theta_i$ and $\lambda_z = \sim\lambda_o/2$.

12.2 (a) 3. A_{43} is large, not A_{42} or A_{41}. Also, atoms can't leave 3 because A_{32} and $A_{31} \cong 0$
(b) 33.3 percent, since each atom absorbs E_{41} from the pump, but emits only E_{32}.

12.3 (a) 75 GHz. $f_n = c/\lambda_n$ where $n\lambda_n/2 = 2\times10^{-3}$. $\Delta f_n = c10^3/4 = 7.5\times10^{10}$.
(b) 4+. Note, 0.1 percent of $f = c/10^{-6}$ is $10^{-3}\times3\times10^{14} = 3\times10^{11} = 4\times\Delta f$.
(c) $4\pi\times10^5$. $Q_c = \omega_o W_T/P_d = (2\pi c/10^{-6})(2\times10^{-3}\times4\varepsilon_o E^2/8)/(10^{-2}E^2/2\eta) = f/\Delta f$; $\eta = \eta_o/2$.

12.4 (a) A_{00} has cutoff $f_{00} = 0$ Hz. $f_{01} = f_{10} = c_s/\lambda_{01} = c_s/(2\times5\times10^{-3}) = 34$ kHz. $f_{11} = 2^{0.5}\times34$ kHz.
(b) $d = 0.02 = \lambda_1/4 = 0.75\lambda_2$. $f = c_s/\lambda$. $f_{001} = 340/0.08 = 4.25$ kHz; $f_{002} = 12.75$ kHz.
(c) Only the f_{oo} mode propagates at audible frequencies, reducing confusion. The resonance near 4 kHz could produce problems, except that most speech information lies at lower frequencies, and most people have poorer hearing above ~12+ kHz. Music could be affected, however.
(d) Reinforcement occurs when $n\lambda_n = 6$ cm (no p phase reversal at walls), so $f_n = c_s/\lambda_n = n340/0.06 \cong n\times5.67$ kHz. Nulls occur if $0.06 = (2n+1)\lambda_n/2$, or $f \cong 2.8, 8.4,...$ kHz. Forest footfalls yield white noise, and the frequency at which the ear perceives nulls in such white noise indicates direction, even from behind--very helpful in the wild.
(e) $Q = \omega_o W_T/P_d$. $\omega_o = 2\pi\times4.25$ kHz. $W_T = 2W_p = (2p^2/8\gamma P_o)$area×length. P_d = area × $(1 - |\underline{\Gamma}|^2)p^2/2\eta_s$. $\gamma = 1.4$, $P_o = 1.01\times10^{-5}$. $\eta_s = 425$. $Q = 0.80/(1 - |\underline{\Gamma}|^2)$. Thus Q = 1.6 for $|\underline{\Gamma}|^2 = 0.5$, and for the next resonance Q = 4.8.

12.5 (a) Since $\langle I(t)\rangle = |\underline{p}|^2/2\eta_s \Rightarrow |\underline{p}| = (2\times425\times100)^{0.5} = 206\ N/m^2$.
(b) $\langle I(t)\rangle = \eta_s|\underline{u}|^2/2 \Rightarrow |\underline{u}| = (2\times100/425)^{0.5} = 0.69$ m/s.
(c) $x = \int u\, dt \Rightarrow D = 2|\underline{u}|/\omega = 2\times0.69/(2\pi1000) = 0.22$ mm.
(d) $\theta_c = \sin^{-1}(c_{sc}/c_{sw}) = \sin^{-1}(0.99) = 81.9^o$.
(e) At θ_c there is no decay, so $\alpha = 0$.

12.6 (a) $\eta_o/\eta_d = \rho_o c_o/\rho_d c_d \cong 10^3\times330/(10^6\times1050) = 3.14\times10^{-4}$.
(b) $\underline{\Gamma} = (\underline{Z}_n - 1)/(\underline{Z}_n + 1)$ where $Z_n = 3.14\times10^{-4}$. The door reflects $|\underline{\Gamma}|^2 \cong 0.9987$.
(d) When 5 cm = $n\lambda_d/2$, there is perfect transmission. $\lambda_d = (1050/330)\lambda_o$, so when λ_d = 10 cm (for n = 1), then $f_{pass} = c_s/\lambda_o = 330/(0.1\times330/1050) = 10.5$ kHz.
(d) Maximum mismatch when 5 cm = $\lambda_d/4$ and $\lambda_d = 0.2$; so $f_{stop} \cong 10500/2 = 5.25$ kHz.