

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
 Department of Electrical Engineering and Computer Science

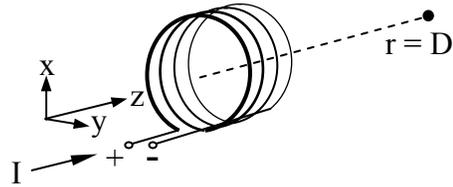
6.013 – Electromagnetics and Applications

Problem Set 10 (four problems)

Suggested Reading: Course notes, Sections 10.1-2; 10.4.1, 10.4.3; 11.2.1-11.2.3

Problem 10.1

An N -turn 10-cm diameter wire coil is excited with $I = 1$ ampere at 100 kHz, as illustrated.

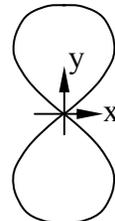


- What *far-field* radiation \bar{H} is observed on the coil axis at distance D ?
- What is the near-field value of \bar{H} at D ? Hint: use the Biot-Savart formula.
- What is the maximum open-circuit voltage V_{\max} induced in a duplicate on-axis coil at D ? (Quasi-static question.)
- The receiving coil, when shunted by a capacitor C , resonates at 100 kHz. For the ideal case of perfectly conducting devices, discuss briefly the physical phenomenon that limits the maximum voltage $V(t)$ that can appear across this resonating coil.
- Assume the LC resonator of part (d) now has a small resistance R in series with the coil, where R has been adjusted to extract and dissipate the maximum possible power P_{\max} from the transmitter. What is P_{\max} in terms of the radiation resistance R_r of the original coil and V_{\max} ?¹ Please sketch your equivalent circuit.
- What technical issues and parameters might determine whether this resonance method of conveying near-field wireless power has commercial significance? Might resonance help far-field receivers using short dipoles with extremely low values of R_r ? Explain briefly.

Problem 10.2

The owner of an FM station at 100 MHz can afford to mount two half-wave dipole antennas on a tall tower to broadcast to the station's isotropically distributed listeners, none of whom reside in space.

- Sketch how the owner might best mount the two antennas if they are to be fed in perfect phase; include quantitative dimensions, orientations, etc.
- Sketch and dimension two different ways in which these two antennas might be mounted and phased in order to achieve approximately the

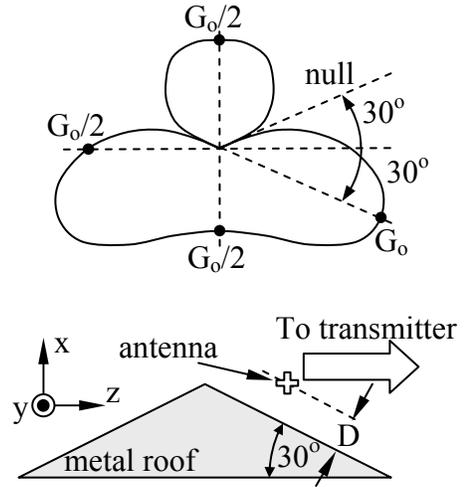


¹ For general interest, an N -turn coil has radiation resistance $R_r \cong 1.9 \times 10^4 N^2 (d/\lambda)^4$, where $d \ll \lambda$ is the coil diameter.

illustrated hour-glass gain pattern in the horizontal x,y plane without transmitting much power directly upward.

(c) How might two vertically polarized half-wave dipoles be positioned and phased to produce the three-lobe antenna gain pattern illustrated to the right? Briefly explain your reasoning.

(d) Many homeowners in this district have flat metal roofs sloped at 30° directly toward the transmitter, as illustrated, and use a single half-wave dipole antenna mounted a distance D from the roof. If the transmitted signal is horizontally polarized and arrives from the horizon, what should D be to optimize reception, and how should the antenna be oriented?

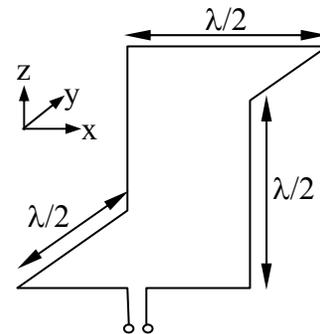


(e) Repeat (d) for a vertically polarized transmitter. Discuss briefly.

(f) What is the increase or decrease in receiver antenna gain G (dB) toward the transmitter relative to the gain if the roof were removed and the dipole antenna was reoriented in an optimum manner? Assume the surface of the earth is absorbing and recall that a power ratio R in dB is $10 \log_{10}R$, and that power is proportional to voltage squared. You may want to be a little creative.

Problem 10.3

The illustrated cubic antenna has 6 arms at right angles, each of which is $\lambda/2$ long. What are the antenna gains G in each of the x , y , and z directions, and in each of vertical and horizontal (x , y , or z) polarization? (Six antenna gains are requested.) Sketch typical instantaneous current distributions and explain your answer briefly.



Problem 10.4

An array of 64 y -oriented short dipole antennas are spaced uniformly $\lambda/2$ apart along the x axis and excited in phase so as to produce maximum gain along the $\pm z$ axis. At what angle from the z axis in the $x-z$ plane is the first null in the antenna pattern? Briefly explain your method.

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