MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

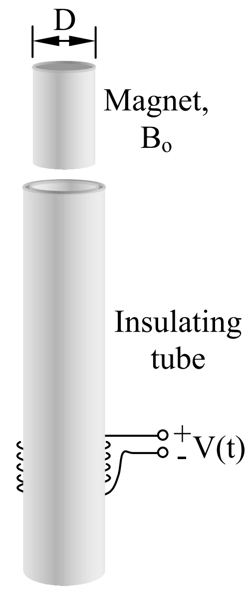
6.013 – Electromagnetics and Applications

Problem Set 4 (six problems)

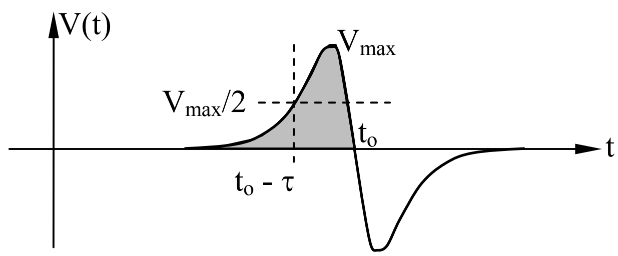
Suggested Reading: Course notes, Sections 5.5.1, 6.1, 6.3, 5.3.1; 5.6, 4.3.1, 4.3.2; 1.3, 4.5.1-4.5.4, 5.2, 4.4.1-4.4.2; 2.3.2-2.3.4

Problem 4.1 (See associated lab instructions)

A pill magnet of diameter D and area $A = \pi D^2/4$ produces a nearly uniform magnetic field B_o across its broad face. We wish to estimate B_o using two different methods: induced voltage and force measurement.



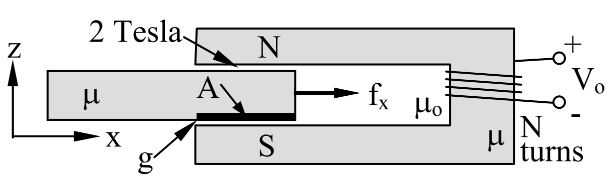
(a) When the magnet is dropped down a tightly fitting but slippery non-conducting cylinder it induces a voltage V(t) across a compact coil of N turns wrapped around the cylinder, as illustrated. Based on this illustrated data, what is the approximate value of B_o? Briefly explain your method and indicate which pole is North (top or bottom). Hint: Before drop, coil flux linkage $\Lambda = 0$. Between t = 0 and t_o: $\Delta\Lambda \cong NB_oA$. $v(t) = d\Lambda/dt$.



(b) What happens when this magnet is dropped down a copper cylinder? Why?

(c) To measure the attraction of the magnet to a thin high-permeability steel plate, the magnet is taped to a digital scale that is preloaded with a large weight. The steel plate is placed on top of the magnet and then pulled away manually using a spring so that the pulling force can be varied in a controlled manner while the "break-away" force F_o [N] for the magnet is being determined. Given F_o, what is the approximate value of B_o? Briefly explain your reasoning.

Problem 4.2



The illustrated horseshoe electromagnet has two thin gaps $g = 10^{-4}$ m separating its pole faces from a sliding high-permeability member, where B in the gaps is 2 Tesla and $\mu >> \mu_o$. The depth of the unit into the page is 3 cm, and the nominal overlap area A of the sliding and stationary members is ~3 cm square (9×10^{-4} m^2) at time t = 0. This is a linear reluctance motor.

(a) What and where is the maximum magnetic energy density [J/m^3] here at t = 0?

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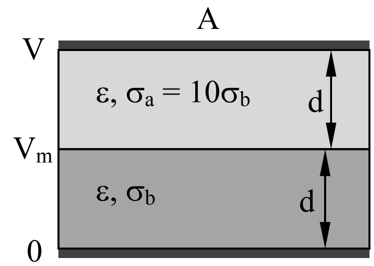
(b) What is the force f_z [N] acting to pull the magnet pole faces together in the z direction?

(c) What is the lateral force f_x [N] pulling the sliding member into the gap (in the x direction)?

(d) What output voltage V_o is induced if the sliding member is withdrawn at velocity v [m/s]? Again assume A = 3 cm square at the time of measurement.

Problem 4.3



A capacitor charged to V = 1000 volts is filled with two equally thick slabs of dielectric having permittivity ε and area A, where the conductivity of the first is $\sigma_a = 10\ \sigma_b$, as illustrated; the total dielectric thickness is 2d.

(a) What is the total capacitance C of this device?

(b) What is the voltage V_m at the midpoint junction between the two dielectrics?

(c) What are the free ρ_{fm} and net polarization ρ_{pm} surface charge densities [C/m^2] at this midpoint junction? The net polarization surface charge ρ_{pm} is the difference between the polarization charges on the two dielectric surfaces at the junction.

(d) If we briefly short circuit this capacitor so that V = 0 and ρ_{fm} remains roughly constant, to what approximate peak value V_p does the open circuit voltage V quickly return, and with what approximate time constant τ does it then decay to zero? Does this suggest that inhomogeneous high-voltage capacitors that are short-circuited only briefly to discharge them could nevertheless be dangerous? Briefly explain your answer.

Problem 4.4

A certain transistor is controlled by its free charge density ρ_f within a zone of width W where $\varepsilon = 4\varepsilon_o$ and $\sigma = 1$ [S/m]. If the voltages on the transistor electrodes bounding that zone abruptly change, forcing ρ_f to take values characterizing the next transistor state, with what time constant τ is the new ρ_f distribution established? Can τ be significantly less than W/c where c is the speed of light within the semiconductor? Discuss briefly.

Problem 4.5

What average pressure p [N/m^2] does a 1-kW/m^2 1-GHz uniform plane wave apply to an absorbing black surface?

Problem 4.6

The separation-of-variables discussion in Section 4.5.2 in the notes deals with two-dimensional potentials $\Phi(x,y)$ [volts]. Consider the three-dimensional case $\Phi(x,y,z)$ and assume the boundary conditions yield $k^2 = 0$.

(a) Derive the general form of $\Phi(x,y,z)$ that satisfies the Laplace equation in this case.

(b) If potential $\Phi = 0$ everywhere along three intersecting edges of a cube (and therefore at four corners), and $\Phi = 10$ volts at the opposite corner, what is the single value of Φ at the remaining three intermediate corners? Explain briefly.