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Lecture 23 - Acoustic Resonators

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December 13, 2005

I. Governing Acoustic Equations

$$\begin{aligned}\rho_0 \frac{\partial \bar{v}}{\partial t} &= -\nabla p \\ \frac{\partial p}{\partial t} &= -\rho_0 c_s^2 \nabla \cdot \bar{v} \\ \Rightarrow \nabla^2 p &= \frac{1}{c_s^2} \frac{\partial^2 p}{\partial t^2}\end{aligned}$$

II. Acoustic Resonator - Closed Box

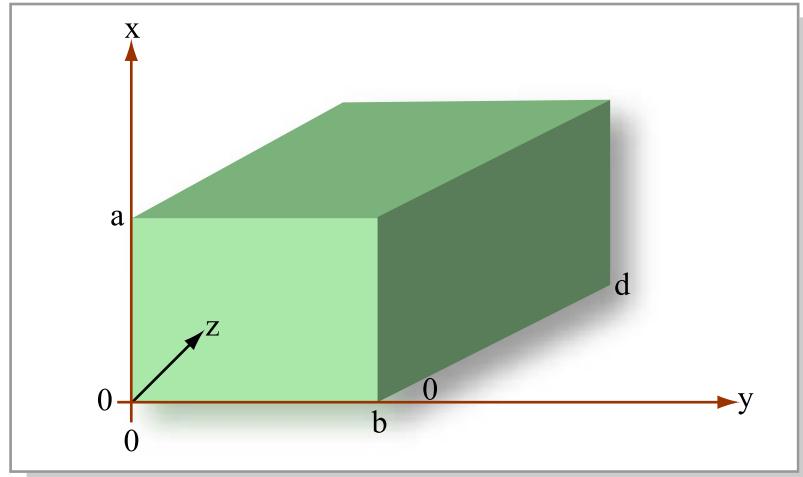


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A. Boundary Conditions

$$\begin{aligned}v_x(x = 0) &= v_x(x = a) = 0 \\ v_y(y = 0) &= v_y(y = b) = 0 \\ v_z(z = 0) &= v_z(z = d) = 0\end{aligned}$$

B. Solutions to Acoustic Equations

$$\begin{aligned}\nabla^2 p &= \frac{1}{c_s^2} \frac{\partial^2 p}{\partial t^2} \\ p(x, y, z, t) &= \operatorname{Re} \left[\hat{p} e^{j(\omega t - \vec{k} \cdot \vec{r})} \right] \\ &= \operatorname{Re} \left[\hat{p} e^{j(\omega t - k_x x - k_y y - k_z z)} \right] \\ \nabla^2 p &= \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} = \frac{1}{c_s^2} \frac{\partial^2 p}{\partial t^2}\end{aligned}$$

$$\Rightarrow -\hat{p} [k_x^2 + k_y^2 + k_z^2] = -\frac{\hat{p}\omega^2}{c_s^2}$$

$$k_x^2 + k_y^2 + k_z^2 = \frac{\omega^2}{c_s^2}$$

$$\begin{aligned}\rho_0 \frac{\partial \bar{v}}{\partial t} &= -\nabla \rho \\ \rho_0 \frac{\partial v_x}{\partial t} &= -\frac{\partial p}{\partial x} \\ \rho_0 \frac{\partial v_y}{\partial t} &= -\frac{\partial p}{\partial y} \\ \rho_0 \frac{\partial v_z}{\partial t} &= -\frac{\partial p}{\partial z}\end{aligned}$$

$$\begin{aligned}v_x &\propto \sin(k_x x), v_x(x=0) = 0 \Rightarrow p \propto \cos(k_x x) \\ v_y &\propto \sin(k_y y), v_y(y=0) = 0 \Rightarrow p \propto \cos(k_y y) \\ v_z &\propto \sin(k_z z), v_z(z=0) = 0 \Rightarrow p \propto \cos(k_z z)\end{aligned}$$

$$\begin{aligned}p(x, y, z, t) &= \operatorname{Re} [\hat{p} \cos(k_x x) \cos(k_y y) \cos(k_z z) e^{j\omega t}] \\ \rho_0 j \omega \hat{v}_x &= \hat{p} k_x \sin(k_x x) \cos(k_y y) \cos(k_z z) \\ \Rightarrow \hat{v}_x &= \frac{\hat{p} k_x}{\rho_0 j \omega} \sin(k_x x) \cos(k_y y) \cos(k_z z) \\ \hat{v}_y &= \frac{\hat{p} k_y}{\rho_0 j \omega} \cos(k_x x) \sin(k_y y) \cos(k_z z) \\ \hat{v}_z &= \frac{\hat{p} k_z}{\rho_0 j \omega} \cos(k_x x) \cos(k_y y) \sin(k_z z)\end{aligned}$$

$$\begin{aligned}\hat{v}_x(x=a) &= 0 \Rightarrow k_x a = m\pi \Rightarrow k_x = \frac{m\pi}{a} \\ \hat{v}_y(y=b) &= 0 \Rightarrow k_y b = n\pi \Rightarrow k_y = \frac{n\pi}{b} \\ \hat{v}_z(z=d) &= 0 \Rightarrow k_z d = p\pi \Rightarrow k_z = \frac{p\pi}{d}\end{aligned}$$

$$\pi \left[\left(\frac{m}{a} \right)^2 + \left(\frac{n}{b} \right)^2 + \left(\frac{p}{d} \right)^2 \right]^{1/2} = \frac{\omega}{c_s}$$

$$\begin{aligned}f_{mnp} &= \frac{c_s}{2} \left[\left(\frac{m}{a} \right)^2 + \left(\frac{n}{b} \right)^2 + \left(\frac{p}{d} \right)^2 \right]^{1/2} \\ c_s &\approx 330 \text{ m/s}, a = .203 \text{ m (8 in)}, b = .203 \text{ m (8 in)}, d = .305 \text{ m (12 in)}\end{aligned}$$

(The value for speed of sound and the determined frequencies are for waves in air.)

m	n	p	f_{mnp} (Hz)	f_{measured} (Hz)
0	0	1	541	550
0	1	0	813	
0	0	2	1082	
1	1	0	1150	1150
1	1	1	1270	
1	1	2	1579	
0	0	3	1623	
0	2	0	1626	
0	1	3	1815	
1	2	2	2115	

C. Comparison of Theory to Experiment

Air -

$$\begin{aligned} T &= 0^\circ \text{ C}, p_0 = 1 \text{ atm} = 1.01 \cdot 10^5 \text{ Nt/m}^2 \quad (1 \text{ Nt/m}^2 = 1 \text{ Pascal}) \\ \rho_0 &= 1.293 \text{ g/L} = 1.293 \text{ kg/m}^3 \\ c_s &= \sqrt{\frac{\gamma p_0}{\rho_0}} = 331 \text{ m/s} \end{aligned}$$

Helium -

$$\begin{aligned} T &= 0^\circ \text{ C}, p_0 = 1 \text{ atm} = 1.01 \cdot 10^5 \text{ Pascals}, \gamma = 1.67 \\ \rho_0 &= 0.178 \text{ g/L} = 0.178 \text{ kg/m}^3 \\ c_s &= \sqrt{\frac{\gamma p_0}{\rho_0}} = 973 \text{ m/s} \end{aligned}$$

Helium/air mixture -

$$\begin{aligned} f_{001} &= \frac{c_s}{2d} = 950 \text{ Hz was measured} \\ c_s &= 2d(950) = 2(.305)950 = 580 \text{ m/s} \\ c_{s,\text{air}} &< c_{s,\text{air/He mixture}} < c_{s,\text{He}} \\ 331 &< 580 < 973 \text{ m/s} \end{aligned}$$

III. Acoustic Resonator - $z = 0$ side open, other 5 sides closed

A. Boundary Conditions

$$\begin{aligned} v_x(x = 0) &= v_x(x = a) = 0 \\ v_y(y = 0) &= v_y(y = b) = 0 \\ v_z(z = d) &= 0, p(z = 0) = 0 \end{aligned}$$

B. Solutions

$$\begin{aligned} p(x, y, z, t) &= \text{Re} [\hat{p} \cos(k_x x) \cos(k_y y) \sin(k_z z) e^{j\omega t}] \\ \hat{v}_x &= \frac{\hat{p} k_x}{\rho_0 j \omega} \sin(k_x x) \cos(k_y y) \sin(k_z z) \\ \hat{v}_y &= \frac{\hat{p} k_y}{\rho_0 j \omega} \cos(k_x x) \sin(k_y y) \sin(k_z z) \\ \hat{v}_z &= -\frac{\hat{p} k_z}{\rho_0 j \omega} \cos(k_x x) \cos(k_y y) \cos(k_z z) e^{j\omega t} \end{aligned}$$

$$\begin{aligned}
\hat{v}_x(x=0) &= 0 \\
\hat{v}_x(x=a) = 0 \Rightarrow k_x a &= m\pi \Rightarrow k_x = \frac{m\pi}{a} \\
\hat{v}_y(y=0) &= 0 \\
\hat{v}_y(y=b) = 0 \Rightarrow k_y b &= n\pi \Rightarrow k_y = \frac{n\pi}{b} \\
\hat{v}_z(z=d) = 0 \Rightarrow k_z d &= (2p+1)\frac{\pi}{2} \Rightarrow k_z = \frac{(2p+1)\pi}{2d} \\
\hat{p}(z=0) &= 0
\end{aligned}$$

$$\begin{aligned}
\frac{\omega^2}{c_s^2} &= \pi^2 \left[\left(\frac{m}{a} \right)^2 + \left(\frac{n}{b} \right)^2 + \left(\frac{2p+1}{2d} \right)^2 \right] \\
f_{mnp} &= \frac{c_s}{2} \left[\left(\frac{m}{a} \right)^2 + \left(\frac{n}{b} \right)^2 + \left(\frac{2p+1}{2d} \right)^2 \right]^{1/2} \\
f_{000} &= \frac{(2p+1)c_s}{4d} \Big|_{p=0}, \quad c_s = 330 \text{m/s (air)}, \quad d = .305 \text{ m} \Rightarrow f_{000} = 270.5 \text{ Hz} \\
f_{100} &= \frac{c_s}{2} \left[\left(\frac{1}{a} \right)^2 + \left(\frac{1}{2d} \right)^2 \right]^{1/2} = 856.6 \text{ Hz}
\end{aligned}$$