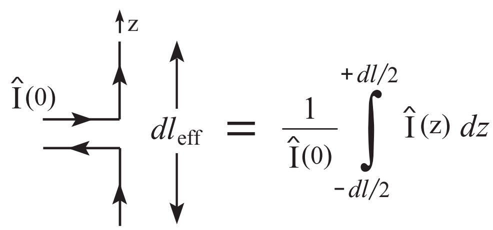
6.013 - Electromagnetics and Applications Fall 2005

Lecture 21 - Receiving Antennas

Prof. Markus Zahn December 6, 2005

I. Review of Transmitting Antennas (Short Dipoles)

A. Far fields ($r \gg \lambda$)



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$$\hat{E}_\theta = \eta \hat{H}_\phi = \frac{\hat{E}_0}{jkr}\sin(\theta)e^{-jkr}, \hat{E}_0 = -\frac{\hat{I}dl_{\text{eff}}k^2\eta}{4\pi}, \eta = \sqrt{\frac{\mu}{\epsilon}}$$

B. Intensity $\langle S_r \rangle$

$$\begin{aligned}
\langle S_r \rangle = \frac{1}{2}\,\text{Re}\left[\hat{\bar{E}} \times \hat{\bar{H}}^*\right] &= \frac{1}{2\eta}|\hat{\bar{E}}|^2 \\
&= \frac{1}{2\eta}\frac{|\hat{E}_0|^2}{k^2r^2}\sin^2(\theta) \\
&= \frac{1}{2\cancel{\eta}}\frac{|\hat{I}dl_{\text{eff}}|^2 k^{\cancel{4}2}\eta^{\cancel{2}}}{16\pi^2\cancel{k^2}r^2}\sin^2(\theta) \\
&= \frac{|\hat{I}dl_{\text{eff}}|^2k^2\eta}{32\pi^2r^2}\sin^2(\theta)
\end{aligned}$$

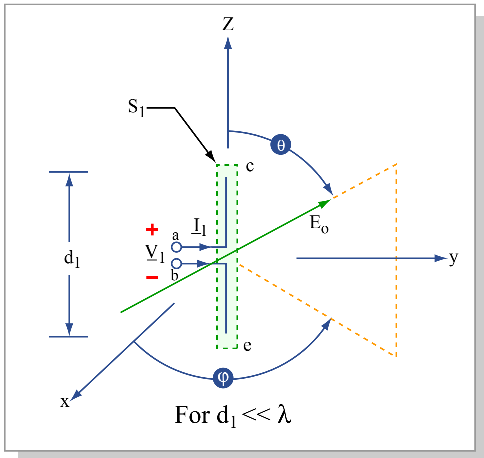
C. Total time average power $\langle P \rangle$

$$\begin{aligned}
\langle P \rangle &= \int_0^\pi d\theta \int_0^{2\pi} d\phi \, \langle S_r \rangle \, r^2 \sin(\theta) \\
&= \frac{|\hat{I}dl_{\text{eff}}|^2\eta k^2}{12\pi} \\
&= \frac{1}{2}|\hat{I}|^2 R \Rightarrow R = \frac{2\pi\eta}{3}\left(\frac{dl_{\text{eff}}}{\lambda}\right)^2 \quad \text{radiation resistance}
\end{aligned}$$

D. Gain

$$\begin{aligned}
G(\theta,\phi) &= \frac{\langle S_r \rangle}{\langle P \rangle / (4\pi r^2)} \\
&= \frac{\cancel{|\hat{I}dl_{\text{eff}}|^2}\cancel{k^2}\cancel{\eta}\sin^2(\theta)\cdot 12\cancel{\pi}(\cancel{4\pi r^2})}{{}_8\cancel{32}\cancel{\pi^2}\cancel{r^2}\cancel{|\hat{I}dl_{\text{eff}}|^2}\cancel{\eta}\cancel{k^2}} \\
&= \frac{3}{2}\sin^2(\theta)
\end{aligned}$$

II. Receiving Antennas



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In absence of receiving antenna: $\bar{E}_{\text{inc}} = \bar{E}_0, \bar{H}_{\text{inc}} = \bar{H}_0$. With $d_1 \ll \lambda$, over size scale of antenna, $\bar{E}_0$ and $\bar{H}_0$ are approximately spatially uniform. In presence of receiving antenna, electric and magnetic fields are perturbed so that tangential $\bar{E}$ and normal $\bar{H}$ are zero along the perfectly conducting length of the antenna.

$$\bar{E} = \bar{E}_0 + \bar{E}_1 \tag{1}$$

$$\bar{H} = \bar{H}_0 + \bar{H}_1 \tag{2}$$

Surface S_1 above intimately hugs the antenna so that

$$\oint_{S_1} \bar{E} \times d\bar{a} = \oint_{S_1} da\ \left(\bar{E}_0 + \bar{E}_1\right) \times \bar{n} = 0 \qquad (\text{tangential } \bar{E} = 0) \tag{3}$$

$$\oint_{S_1} da\,\bar{n} \times (\bar{H}_0 + \bar{H}_1) = \oint_{S_1} da\,\bar{K} = \int_{-d_1/2}^{+d_1/2} dz\,\bar{I}_1(z) = I_1 d_{\text{eff}} \bar{i}_z \tag{4}$$

Another useful relationship:

$$\oint_{S_1} da\,(\bar{E}_0 \times \bar{H}_0) \cdot \bar{n} = (\bar{E}_0 \times \bar{H}_0) \cdot \oint_{S_1} da\,\bar{n} = 0 \tag{5}$$

Integral of normal over closed surface is zero:

$$\int_V dV\,\nabla f = \oint_S da\,f\bar{n}, \qquad \text{Take } f = 1, \nabla f = 0 = \oint_S da\,\bar{n} = 0 \tag{6}$$

Scalar Triple Product Identity:

$$(\bar{a} \times \bar{b}) \cdot \bar{c} = \bar{a} \cdot (\bar{b} \times \bar{c}) \tag{7}$$

(Interchange of cross and dot)

Complex power supplied by receiving antenna

$$P = \oint_{S_1} da\,\hat{\bar{S}} \cdot \bar{n} \tag{8}$$

$$\begin{aligned}\hat{\bar{S}} &= \frac{1}{2}(\hat{\bar{E}} \times \hat{\bar{H}}^*) = \frac{1}{2}\left[(\hat{\bar{E}}_0 + \hat{\bar{E}}_1) \times (\hat{\bar{H}}_0^* + \hat{\bar{H}}_1^*)\right] \\ &= \frac{1}{2}\left[\hat{\bar{E}}_0 \times (\hat{\bar{H}}_0^* + \hat{\bar{H}}_1^*) + \hat{\bar{E}}_1 \times (\hat{\bar{H}}_0^* + \hat{\bar{H}}_1^*)\right]\end{aligned} \tag{9}$$

$$P = \oint_{S_1} da\,\hat{\bar{S}} \cdot \bar{n} = \frac{1}{2}\left\{\oint_{S_1} da\,\left[\hat{\bar{E}}_0 \times (\hat{\bar{H}}_0^* + \hat{\bar{H}}_1^*) \cdot \bar{n}\right] + \oint_{S_1} da\,\left[\hat{\bar{E}}_1 \times (\hat{\bar{H}}_0^* + \hat{\bar{H}}_1^*)\right] \cdot \bar{n}\right\} \tag{10}$$

$$\oint_{S_1} da\,\left[\hat{\bar{E}}_0 \times (\hat{\bar{H}}_0^* + \hat{\bar{H}}_1^*)\right]\cdot\bar{n} = \oint_{S_1} da\,\hat{\bar{E}}_0 \cdot \left[(\hat{\bar{H}}_0^* + \hat{\bar{H}}_1^*)\times\bar{n}\right]$$

$$= \hat{\bar{E}}_0 \cdot \oint_{S_1} da\,\left[(\hat{\bar{H}}_0^* + \hat{\bar{H}}_1^*)\times\bar{n}\right]$$

$$= -\hat{\bar{E}}_0 \cdot \hat{I}_1 d_{\text{eff}}\bar{i}_z \qquad \text{(from (4))} \tag{11}$$

$$\oint_{S_1} da\,\left[\hat{\bar{E}}_1 \times (\hat{\bar{H}}_0^* + \hat{\bar{H}}_1^*)\right]\cdot\bar{n} = \oint_{S_1} da\,\left[\hat{\bar{E}}_1 \times \hat{\bar{H}}_0^*\right]\cdot\bar{n} + \underbrace{\oint_{S_1} da\,\left[\hat{\bar{E}}_1 \times \hat{\bar{H}}_1^*\right]\cdot\bar{n}}_{\substack{|\hat{I}_1|^2(R+jX) \\ \text{where } R \text{ is the radiation resistance} \\ \text{and } X \text{ is the antenna reactance}}} \tag{12}$$

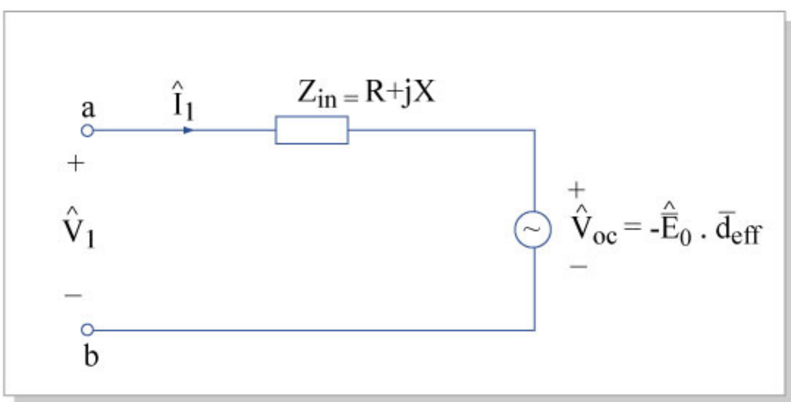
$$\oint_{S_1} da\,\left[\hat{\bar{E}}_1 \times \hat{\bar{H}}_0^*\right]\cdot\bar{n} = -\oint_{S_1} da\,\left[\hat{\bar{H}}_0^* \times \hat{\bar{E}}_1\right]\cdot\bar{n}$$

$$= -\oint_{S_1} da\,\left[\hat{\bar{H}}_0^* \times (\hat{\bar{E}}_0 + \hat{\bar{E}}_1)\right]\cdot\bar{n}$$

$$= -\oint_{S_1} da\,\hat{\bar{H}}_0^* \cdot \left[(\hat{\bar{E}}_0 + \hat{\bar{E}}_1)\times\bar{n}\right] = -\bar{H}_0^* \cdot \oint_{S_1} da\,\left(\hat{\bar{E}}_0 + \hat{\bar{E}}_1\right)\times\bar{n}$$

$$= 0\,\text{(from (3))} \tag{13}$$

$$P = \frac{1}{2}\left[-\hat{\bar{E}}_0 \cdot \hat{I}_1 d_{\text{eff}}\bar{i}_z + |\hat{\bar{I}}_1|^2(R+jX)\right] = \frac{1}{2}\hat{V}_1\hat{I}_1^*$$



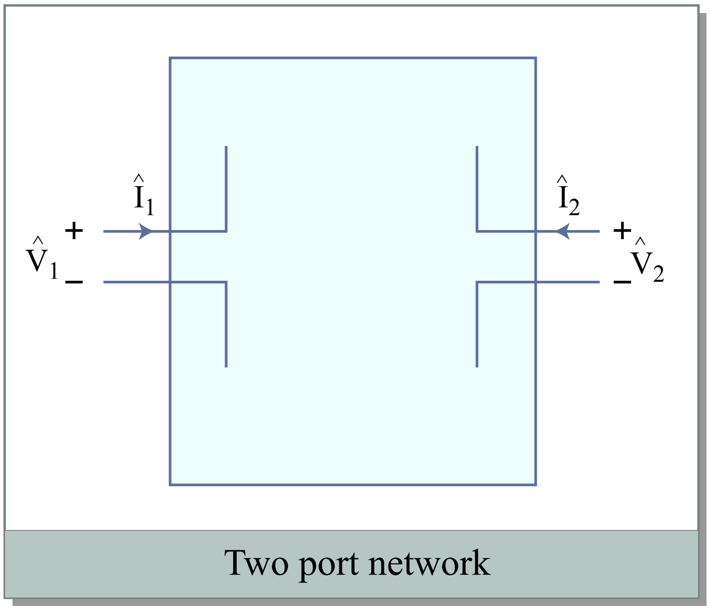
Thevenin Equivalent Circuit

Image by MIT OpenCourseWare.

$$\hat{V}_{TH} = \hat{V}_{oc} = -\hat{\bar{E}}_0 \cdot \bar{d}_{\text{eff}} \qquad (\bar{d}_{\text{eff}} = d_{\text{eff}}\bar{i}_z)$$

III. Transmitting and Receiving Antennas

A. Circuit Description



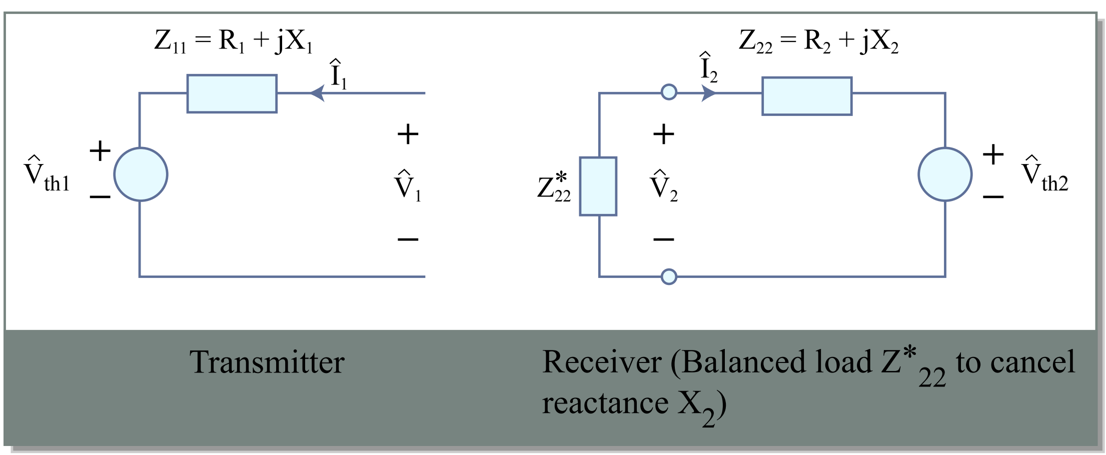
Two port network

Image by MIT OpenCourseWare.

$$\hat{V}_1 = \hat{I}_1 Z_{11} + \hat{I}_2 Z_{12}$$
$$\hat{V}_2 = \hat{I}_1 Z_{21} + \hat{I}_2 Z_{22}$$

$$Z_{12} = \frac{\hat{V}_1}{\hat{I}_2}\Big|_{\hat{I}_1=0} \qquad Z_{21} = \frac{\hat{V}_2}{\hat{I}_1}\Big|_{\hat{I}_2=0}$$

B.



Transmitter

Receiver (Balanced load Z^*_{22} to cancel reactance X_2)

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$$\hat{V}_{th1} = \hat{I}_2 Z_{12} = -\int \hat{\bar{E}}_1 \cdot \overline{dl} = -\hat{\bar{E}}_1 \cdot \overline{dl}_{\text{eff}}$$
$$\hat{V}_{th2} = \hat{I}_1 Z_{21} = -\hat{\bar{E}}_2 \cdot \overline{dl}_{\text{eff}}$$

$$\langle P_2\rangle = \frac{1}{2}\frac{|\hat{V}_{\text{th2}}/2|^2}{R_2} = \frac{1}{8}\frac{|\hat{V}_{\text{th2}}|^2}{R_2} = \frac{1}{8}\frac{|\hat{E}_2 dl_{\text{eff}}\sin(\theta)|^2}{\frac{2\pi\eta}{3}\left(\frac{dl_{\text{eff}}}{\lambda}\right)^2}$$

$$\langle P_2\rangle = A_{\text{rec}}(\theta,\phi)\langle S_r\rangle = A_{\text{rec}}(\theta,\phi)\cdot\frac{1}{2}\frac{|\hat{E}_2|^2}{\eta} = \frac{1}{8}\frac{|\hat{E}_2|^2\sin^2(\theta)\cdot 3\lambda^2}{2\pi\eta}$$

$$A_{\text{rec}}(\theta,\phi) = \frac{3}{2}\sin^2(\theta)\frac{\lambda^2}{4\pi} = G_{\text{rec}}(\theta,\phi)\frac{\lambda^2}{4\pi}$$

C. Representative Parameters

1. Minimum received power $\approx 10^{-20}$ watts
   For total transmitted power of 1 watt, how far away can the receiver be at $f = 1$ GHz?

$$\langle P_{\text{rec}}\rangle = \underbrace{\frac{\langle P_{\text{trans}}\rangle}{4\pi r^2}G_{\text{trans}}}_{\langle S_r\rangle}\underbrace{G_{\text{rec}}\frac{\lambda^2}{4\pi}}_{A_{\text{rec}}(\theta,\phi)}$$

$$f\lambda = c \Rightarrow \lambda = \frac{c}{f} = \frac{3\times 10^8}{10^9} = .3 \text{ m}$$

$$G_{\text{trans}} = G_{\text{rec}} = \frac{3}{2}\sin^2(\theta)\,\text{(for short dipoles)}\,\text{(identical transmitting and receiving antennas)}$$

$$\text{Take } \theta = \frac{\pi}{2} \Rightarrow G_{\text{trans}} = G_{\text{rec}} = \frac{3}{2}$$

$$\begin{aligned} r^2 &= \frac{P_{\text{trans}}}{P_{\text{rec}}}G_{\text{trans}}G_{\text{rec}}\left(\frac{\lambda}{4\pi}\right)^2 \\ &= \frac{1}{10^{-20}}\left(\frac{9}{4}\right)\left(\frac{.3}{4\pi}\right)^2 \\ &= 1.28\times 10^{17}\text{ m}^2 \\ r &= 3.58\times 10^8\text{ m} = 3.58\times 10^5\text{ km} \approx 200{,}000\text{ miles}\end{aligned}$$

2. For data transmission, receivers need $E_b > 4\times 10^{-20}$ Joules/bit
   Power received $= ME_b$ where M is the data rate, bits/s
   10^{-9} watts received power allows $M = \frac{10^{-9}}{E_b} = \frac{10^{-9}}{4\times 10^{-20}} = .25\times 10^{11}$ bits/s
   1 CD $= 700\times 10^6$ bytes $= 5600\times 10^6$ bits (1 byte = 8 bits)
   $M = .25\times 10^{11}$ bits/sec ≈ 4.5 CD/sec
3. Distance is not a barrier to wireless communications
   $r = 1$ lightyear $= 3\times 10^8$ m/s $\cdot 3\times 10^7$ s/yr $= 9\times 10^{15}$ m/yr

$$\begin{aligned} P_{\text{trans}} &= ? \\ f = 3\text{GHz} &\Rightarrow \lambda = \frac{c}{f} = .1\text{ m} \\ M = 1\text{ bit/s}, E_b &= 4\times 10^{-20}\text{ Joules/bit} \\ P_{\text{rec}} &= ME_b = 4\times 10^{-20}\text{ Watts} \\ G_{\text{trans}} &= G_{\text{rec}} = 10^7 \end{aligned}$$

$$
\begin{aligned}
P_{\text{trans}} &= \frac{P_{\text{rec}}\left(\frac{4\pi r}{\lambda}\right)^2}{G_{\text{trans}}G_{\text{rec}}} \\
&= \frac{4\times 10^{-20}\left(\frac{4\pi(9\times 10^{15})}{.1}\right)^2}{10^{14}} \\
&= 512 \text{ Watts}
\end{aligned}
$$

For $M = 2.4$ kb/s $\Rightarrow P_{\text{trans}} \approx 1.2$ MW (with a 1 year delay each way)

4. Optical Communications: $E = hf, h = 6.625 \times 10^{-34}$ Joule-sec (Planck's Constant)
   a. Radio Photons

$$
\begin{aligned}
f = 1 \text{ GHz} &\Rightarrow E = 6.625\times 10^{-25} \text{ Joules/Photon} \\
EN = E_b &\Rightarrow N = \frac{E_b}{E} \text{photons/bit} \\
&= \frac{4\times 10^{-20}}{6.625\times 10^{-25}} \approx 6000 \,\text{photons/bit}
\end{aligned}
$$

   b. Optical Photons

$$
\begin{aligned}
\lambda = 0.5\,\mu\text{m} \Rightarrow f &= \frac{c}{.5\times 10^{-6}} = \frac{3\times 10^{8}}{.5\times 10^{-6}} \approx 6\times 10^{14} \text{ Hz} \\
N = \frac{E_b}{hf} &= \frac{4\times 10^{-20}}{6.625\times 10^{-34}\cdot 6\times 10^{14}} \approx .1 \,\text{photon/bit}
\end{aligned}
$$