6.013/ESD.013J — Electromagnetics and Applications	Fall 2005

Quiz 2 - Solutions

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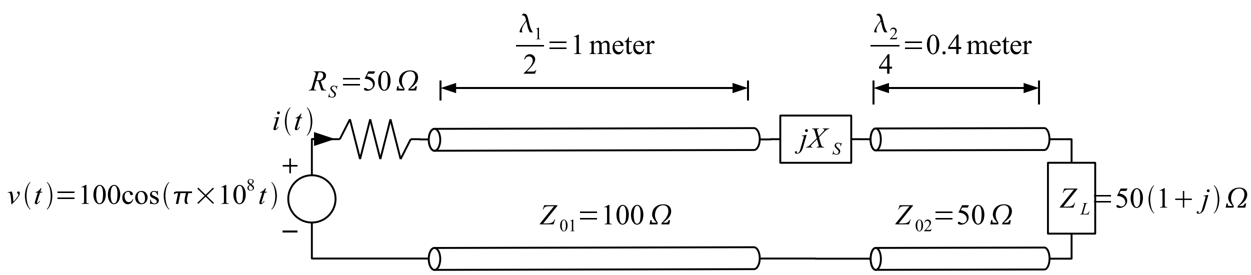
Problem 1



A transmission line system incorporates two transmission lines with characteristic impedances of $Z_{01} = 100\ \Omega$ and $Z_{02} = 50\ \Omega$ as illustrated above. A voltage source is applied at the left end, $v(t) = 100\cos(\pi \times 10^8 t)$. At this frequency, line 1 has length of $\lambda_1/2 = 1$ meter and line 2 has length of $\lambda_2/4 = 0.4$ meter, where λ_1 and λ_2 are the wavelengths along each respective transmission line. The two transmission lines are connected by a series reactance jX_s, and the end of line 2 is loaded by impedance $Z_L = 50(1+j)\ \Omega$. The voltage source is connected to line 1 through a source resistance $R_s = 50\ \Omega$.

A

Question: What are the speeds c_1 and c_2 of electromagnetic waves on each line?

Solution: Using the appropriate values of $f = 50$ MHz, $\lambda_1 = 2$ m, and $\lambda_2 = 1.6$ m together with the expression $f\lambda = c$, we find that

$$c_1 = 5 \times 10^7(2) = 10^8 \text{ m/s, and}$$
$$c_2 = 5 \times 10^7(1.6) = 8 \times 10^7 \text{ m/s.}$$

B

Question: It is desired that X_s be chosen so that the source current $i(t) = I_0\cos(\pi \times 10^8 t)$ is in phase with the voltage source. What is X_s?

Solution: We have that

$$\frac{Z_L}{Z_{02}} = 1 + j,$$
$$Z_n\left(z = -\frac{\lambda_2}{4}\right) = \frac{1}{1+j} = \frac{1-j}{2}, \text{ and}$$
$$Z\left(z = -\frac{\lambda_2}{4}\right) = 25(1-j).$$

However, $jX_s + 25(1-j)$ must be real, hence $X_s = 25\ \Omega$.

C

Question: For the value of X_s in part B, what is the peak amplitude I_0 of the source current $i(t)$?

Solution:

$$Z\left(z = -\frac{\lambda_2}{4}\right) = 25\ \Omega \implies Z\left(z = -\frac{\lambda_2}{4} - \frac{\lambda_1}{2}\right) = 25\ \Omega$$

Hence,

$$i(t) = \frac{v(t)}{75\ \Omega} = \frac{4}{3}\cos(\pi \times 10^8 t) \implies I_0 = \frac{4}{3} \text{ Amperes.}$$

Problem 2

A parallel plate waveguide is to be designed so that only TEM modes can propagate in the frequency range $0 < f < 2$ GHz. The dielectric between the plates has a relative dielectric constant of $\varepsilon_r = 9$ and a magnetic permeability of free space μ_0.

A

Question: What is the maximum allowed spacing $d_{\mathbf{max}}$ between the parallel plate waveguide plates?

Solution:

$$\omega_{co,n} = \frac{n\pi c}{d} = 2\pi f_{co,n} \implies f_{co,n} = \frac{nc}{2d} \text{ with } c = 10^8 \text{ m/s}$$

The lowest cut-off frequency occurs when $n = 1$, therefore

$$f_{co,1} = \frac{c}{2d} > 2 \text{ GHz} \implies d < \frac{c}{4 \times 10^9} = \frac{10^8}{4 \times 10^9} = \frac{1}{40} \text{ m.}$$

Hence $d < 2.5$ cm.

B

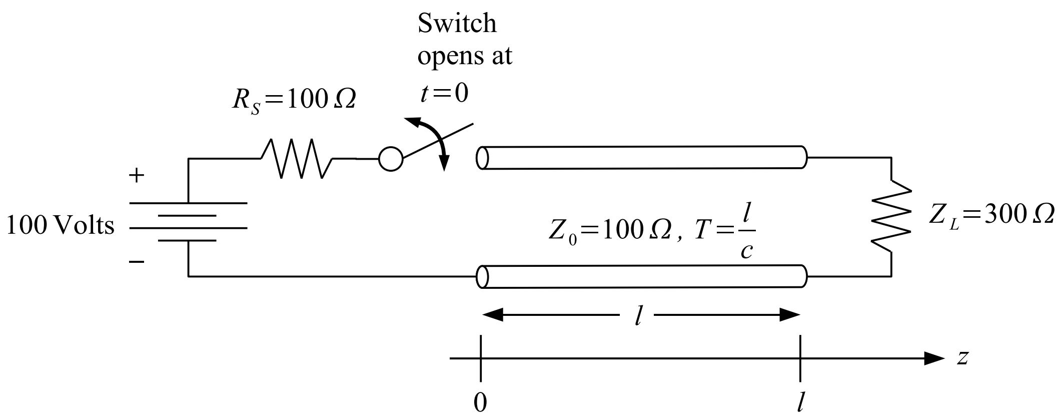
Question: If the plate spacing is 2.1 cm, and $f = 10$ GHz, what $\mathbf{TE}_n$ and $\mathbf{TM}_n$ modes will propagate?

Solution:

$$f_{co,n} = \frac{nc}{2d} < 10 \text{ GHz} \implies n < \frac{2d(10^{10})}{c} = \frac{2(0.021)10^{10}}{10^8} \implies n < 4.2$$

It follows then that we have the propagating modes: TE_1, TE_2, TE_3, TE_4; and TM_1, TM_2, TM_3, TM_4.

Problem 3



A transmission line of length l, characteristic impedance $Z_0 = 100\ \Omega$, and one-way time of flight $T = l/c$ is connected at $z = 0$ to a 100 volt DC battery through a series source resistance $R_s = 100\ \Omega$ and a switch. The $z = l$ end is loaded by a 300 Ω resistor.

A

Question: The switch at the $z = 0$ end has been closed for a very long time so that the system is in the DC steady state. What are the positive and negative traveling wave voltage amplitudes $V_+(z - ct)$ and $V_-(z + ct)$?

Solution:

$$v(z,t) = V_+ + V_- = 75 \text{ Volts}$$
$$Z_0\, i(z,t) = V_+ - V_- = 25 \text{ Volts}$$

Solving for V_+ and V_- yields

$$V_+ = 50 \text{ Volts},$$
$$V_- = 25 \text{ Volts}.$$

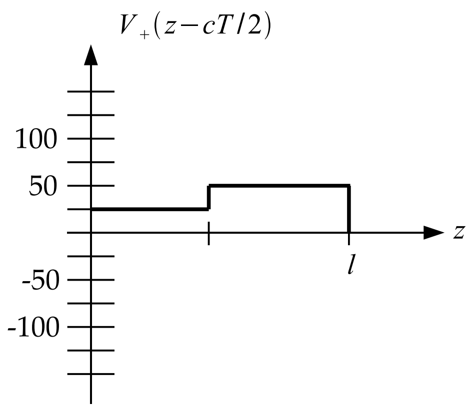
B

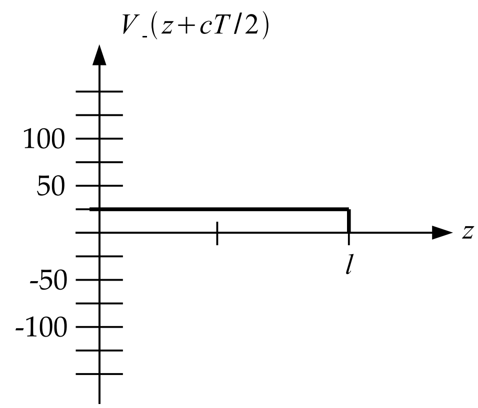
Question: With the system in the DC steady state, the switch is suddenly opened at time $t = 0$.

I. **Plot the positive and negative traveling wave voltage amplitudes,** $V_+(z-ct)$ **and** $V_-(z+ct)$**, as a function of z at time** $t = T/2$.

Solution:

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{300 - 100}{300 + 100} = \frac{200}{400} = \frac{1}{2},\ \Gamma_s = +1.$$





II. **Plot the transmission line voltage** $v(z,t)$ **as a function of** z **at time** $t = T/2$.

Solution: