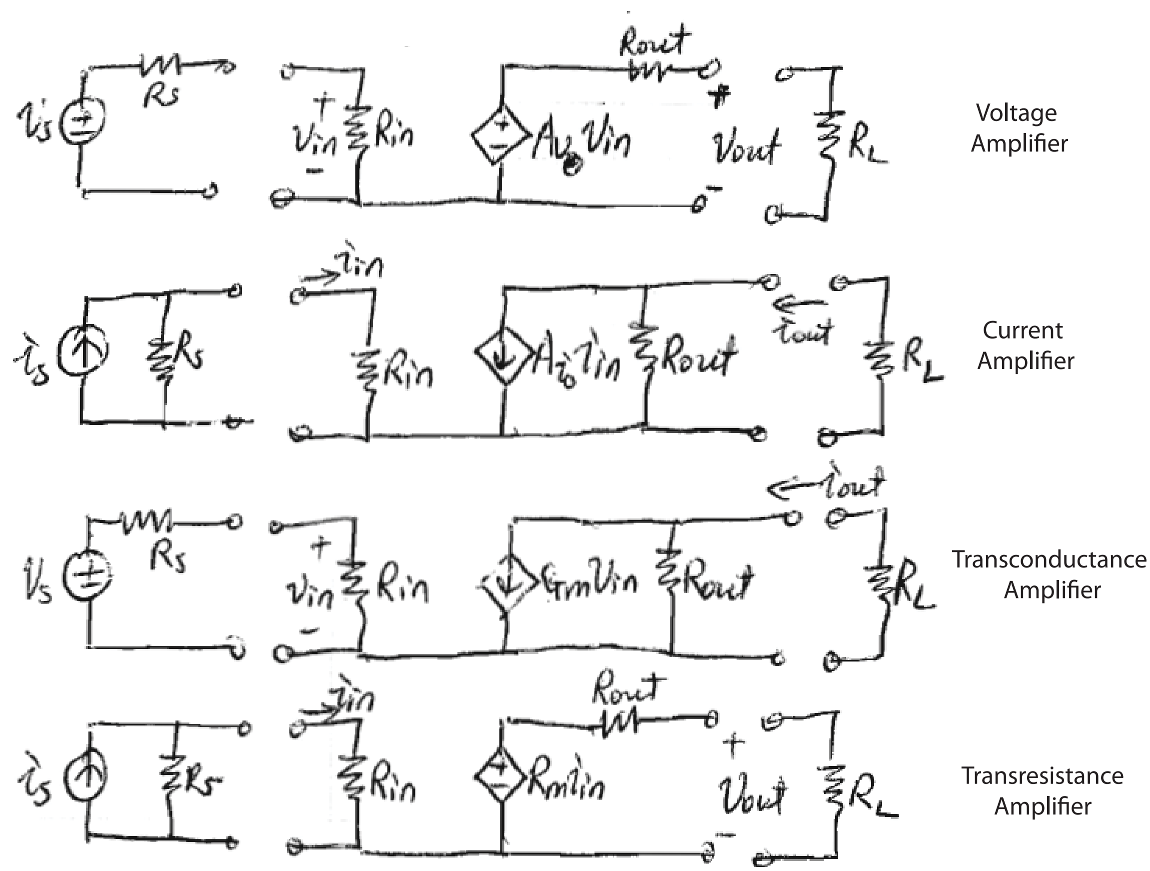
Recitation 20: Amplifiers Review

Yesterday, we introduced two more amplifier circuits: C-drain, C-base.
As we know, there is an analogy between MOS & BJT:

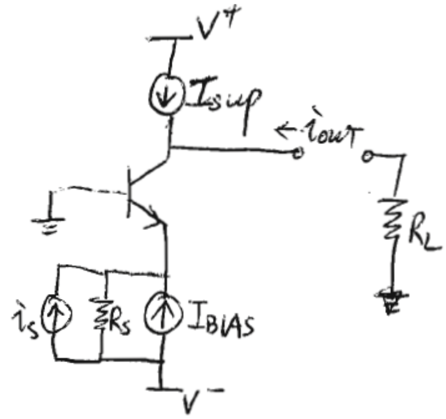
MOS	BJT	Function
Common Source ⟷	Common-Emitter	Voltage or G_m Amp.
Common Drain ⟷	Common-Collector	Voltage Buffer
Common Gate ⟷	Common-Base	Current Buffer

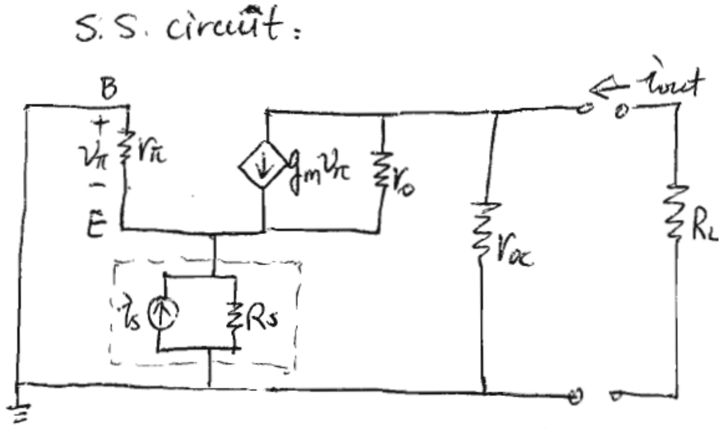
Note: Buffer is an amplifier with gain 1, but input or output impedance changed
We have also learned that there are 4 types of amplifiers, their *two port models* are



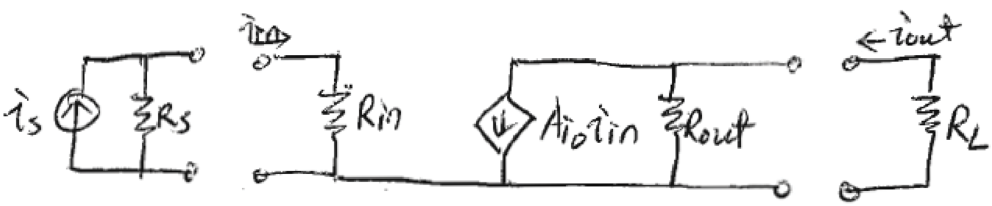
For the above single stage amplifiers (i.e. CS, CD, CG, CE, CC, CB), as we identify their particular function, e.g. current buffer is a type of current amplifier. We can use a *two-port model* for current amplifier to model a CB or CG circuit. Their corresponding $R_{\text{in}}, R_{\text{out}}, A_{\text{io}}$ will depend on the circuit (or device parameter), which we can derive based on the *small signal circuit model* of the circuit.
Yesterday, we looked at the example of CD & CG. Today we will look at CC & CB.

Common-Base Amplifier





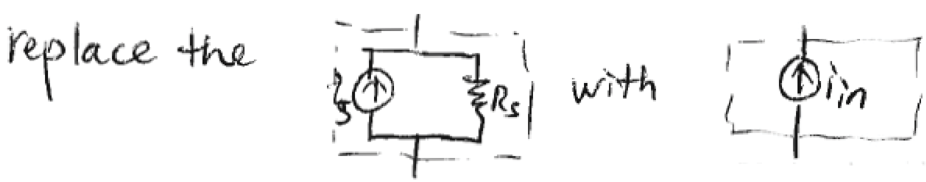
Cast this into two port model



Need to find what is the corresponding $A_{\text{io}}, R_{\text{in}}, R_{\text{out}}$

A_{io}

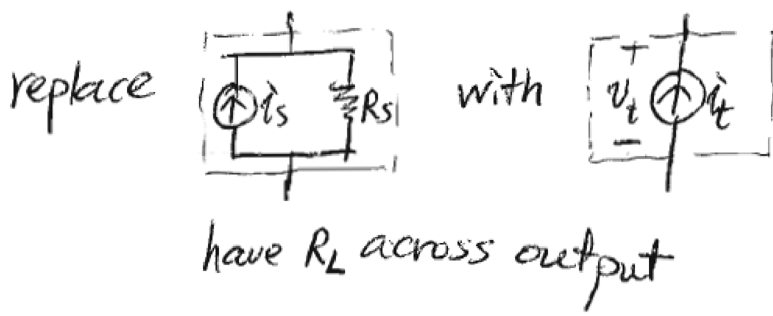
Intrinsic current gain: ignore R_{s}, just consider $i_{\text{in}} = i_{\text{s}}$; R_{L} short.



short R_L at the output

$$
\begin{aligned}
i_{\mathrm{in}} &= -\left(\frac{v_\pi}{\gamma_\pi} + g_{\mathrm{m}} v_\pi + \frac{v_\pi}{\gamma_{\mathrm{o}}}\right), \quad i_{\mathrm{out}} = g_{\mathrm{m}} v_\pi + \frac{v_\pi}{\gamma_{\mathrm{o}}} \\
\Longrightarrow v_\pi &= -\frac{i_{\mathrm{in}}}{\frac{1}{v_\pi} + g_{\mathrm{m}} + \frac{1}{\gamma_{\mathrm{o}}}} = \frac{i_{\mathrm{in}}}{g_\pi + g_{\mathrm{m}} + g_{\mathrm{o}}} \\
\Longrightarrow A_{\mathrm{io}} &= \frac{i_{\mathrm{out}}}{i_{\mathrm{in}}} = -\frac{(g_{\mathrm{m}} + g_{\mathrm{o}}) \cdot \dfrac{i_{\mathrm{in}}}{g_\pi + g_{\mathrm{m}}} + g_{\mathrm{o}}}{i_{\mathrm{in}}} = -\frac{g_{\mathrm{m}} + g_{\mathrm{o}}}{g_\pi + g_{\mathrm{m}} + g_{\mathrm{o}}} \simeq -1 \\
\because \frac{1}{g_{\mathrm{m}}} &\simeq 1\,\mathrm{k}\Omega,\ \gamma_{\mathrm{o}} \approx 100\,\mathrm{k}\Omega \\
g_{\mathrm{m}} \gg g_{\mathrm{o}},\ \gamma_\pi &= \frac{\beta_{\mathrm{F}}}{g_{\mathrm{m}}} \Longrightarrow g_\pi = \frac{g_{\mathrm{m}}}{\beta_{\mathrm{F}}} \ \ g_\pi \ll g_{\mathrm{m}}
\end{aligned}
$$

R_{in}



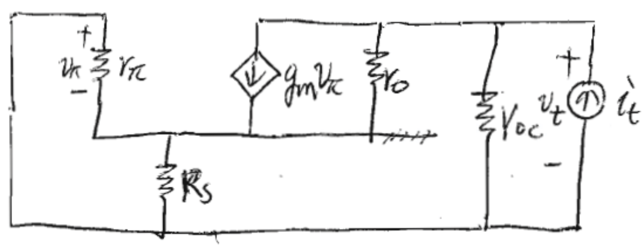
$$
\begin{aligned}
\gamma_\pi, \gamma_{\mathrm{o}} &\gg \frac{1}{g_{\mathrm{m}}} \quad \text{as we just discussed} \\
\therefore & \quad \text{transconductance generator } g_{\mathrm{m}} \text{ dominates currents at the input node} \\
i_{\mathrm{t}} &= -\left(\frac{v_\pi}{\gamma_\pi} + g_{\mathrm{m}} v_\pi + \frac{v_{\mathrm{o}}}{\gamma_{\mathrm{o}}}\right) \simeq -g_{\mathrm{m}} v_\pi = g_{\mathrm{m}} v_{\mathrm{t}} \\
\therefore R_{\mathrm{in}} &= \frac{v_{\mathrm{t}}}{i_{\mathrm{t}}} = \frac{v_{\mathrm{t}}}{g_{\mathrm{m}} v_{\mathrm{t}}} \simeq \frac{1}{g_{\mathrm{m}}} \quad \text{LOW! (good for getting current in)}
\end{aligned}
$$

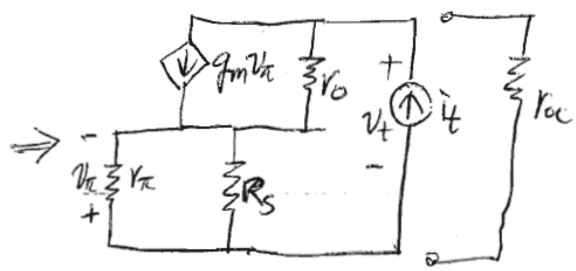
Exact: see pp 150 $R_{\mathrm{in}} = \dfrac{1}{\frac{1}{\gamma_\pi} + g_{\mathrm{m}} + \dfrac{1 - g_{\mathrm{m}}(\gamma_{\mathrm{co}}||R_{\mathrm{L}})}{\gamma_{\mathrm{o}} + (V_{\mathrm{oc}}||R_{\mathrm{L}})}}$

R_{out}

Similarly

1. shut down all independent sources
2. load input with R_{s}
3. put test current source at output
4. $R_{\text{out}} = \dfrac{v_{\text{t}}}{i_{\text{t}}}$

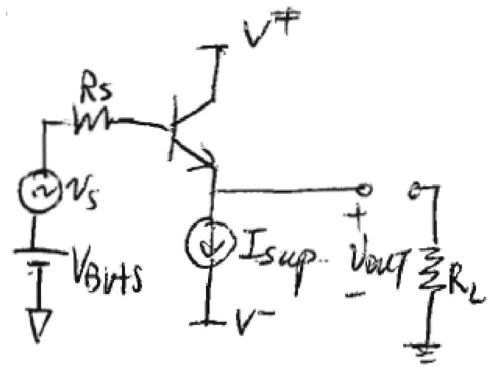


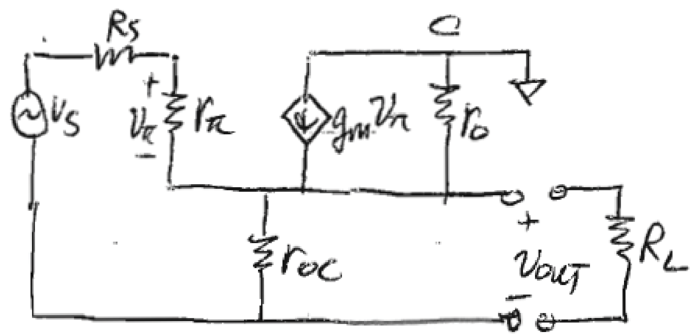


$$
\begin{aligned}
i_{\text{t}} &= g_{\text{m}} v_\pi + \frac{v_{\text{t}} + v_\pi}{\gamma_{\text{o}}} \quad \text{voltage across } \gamma_{\text{o}} \text{ is } v_{\text{t}} + v_\pi && (1)\\
v_\pi &= -i_{\text{t}} \cdot (\gamma_\pi || R_{\text{s}}) && (2)\\
\Longrightarrow \ \text{plug (2) into (1)} & && (3)\\
i_{\text{t}} &= \frac{v_{\text{t}}/\gamma_{\text{o}}}{1 + \dfrac{\gamma_\pi || R_{\text{s}}}{\gamma_{\text{o}}} + g_{\text{m}}(\gamma_\pi || R_{\text{s}})} && (4)\\
\Longrightarrow \frac{v_{\text{t}}}{i_{\text{t}}} &= \gamma_{\text{o}} + (\gamma_\pi || R_{\text{s}}) + g_{\text{m}} \gamma_{\text{o}} (\gamma_\pi || R_{\text{s}}) && (5)\\
\therefore R_{\text{out}} &= \gamma_{\text{oc}} || [\gamma_{\text{o}} + (\gamma_\pi || R_{\text{s}}) + g_{\text{m}} \gamma_{\text{o}} (\gamma_\pi || R_{\text{s}})] \simeq \gamma_{\text{oc}} || \gamma_{\text{o}} [1 + g_{\text{m}} (\gamma_\pi || R_{\text{s}})] && (6)\\
\text{If } R_{\text{s}} \gg \gamma_\pi, \quad R_{\text{out}} &\simeq \gamma_{\text{oc}} || \gamma_{\text{o}} \, [1 + \underbrace{g_{\text{m}} \gamma_\pi}_{\beta_{\text{p}}}] = \underbrace{\gamma_{\text{oc}} || \gamma_{\text{o}} \cdot \beta_{\text{F}}}_{\text{large}} && (7)
\end{aligned}
$$

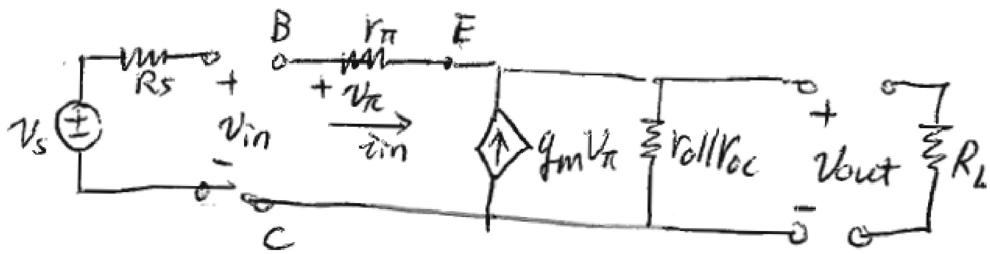
Excellent current buffer: can use current source with source resistance only slightly higher than $R_{\text{in}} \left(\dfrac{1}{g_{\text{m}}} \right)$, and get same current with high R_{out}

Common-Collector Amplifier





Rearrange,



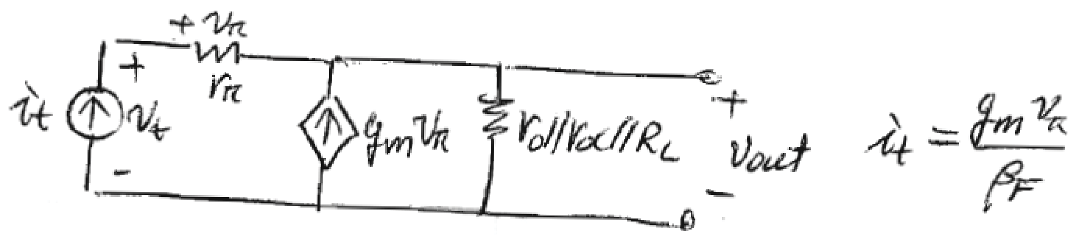
Cast this into two port voltage amplifier model

$A_{\mathrm{vo}}(R_{\mathrm{L}} = \infty, R_{\mathrm{s}} = 0)$

$$\begin{aligned}
V_{\mathrm{out}} &= A_{\mathrm{vo}}V_{\mathrm{in}} = \left(g_{\mathrm{m}}v_\pi + g_{\mathrm{m}}\frac{v_\pi}{\beta_{\mathrm{F}}}\right)\cdot(\gamma_{\mathrm{o}}||\gamma_{\mathrm{oc}}) \\
&= g_{\mathrm{m}}\left(1+\frac{1}{\beta_{\mathrm{F}}}\right)v_\pi(\gamma_{\mathrm{o}}||\gamma_{\mathrm{oc}}) \\
i_{\mathrm{in}} &= \frac{v_\pi}{\gamma_\pi} = v_\pi\frac{g_{\mathrm{m}}}{\beta_{\mathrm{F}}} \\
\text{But } v_\pi &= v_{\mathrm{in}} - v_{\mathrm{out}} \implies v_{\mathrm{out}} = g_{\mathrm{m}}\left(1+\frac{1}{\beta_{\mathrm{F}}}\right)(v_{\mathrm{in}} - v_{\mathrm{out}})(\gamma_{\mathrm{o}}||\gamma_{\mathrm{oc}}) \\
A_{\mathrm{vo}} &= \frac{v_{\mathrm{out}}}{v_{\mathrm{in}}} = \frac{1}{1+\frac{1}{g_{\mathrm{m}}}\left(1+\frac{1}{\beta_{\mathrm{F}}}\right)(\gamma_{\mathrm{o}}||\gamma_{\mathrm{oc}})} \simeq 1
\end{aligned}$$

R_{in}

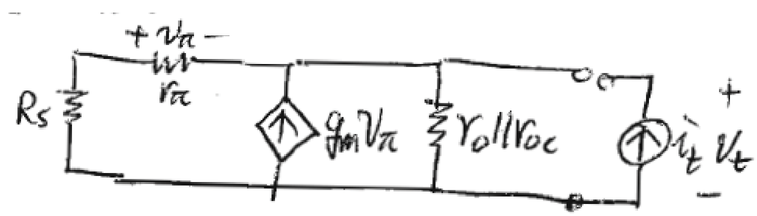
Leave R_{L} in place, replace source with



$$\begin{aligned}
v_{\mathrm{t}} &= i_{\mathrm{t}}\cdot\gamma_\pi + (i_{\mathrm{t}} + g_{\mathrm{m}}v_\pi)\cdot(\gamma_{\mathrm{o}}||\gamma_{\mathrm{oc}}||R_{\mathrm{L}}) \\
&= i_{\mathrm{t}}v_\pi + g_{\mathrm{m}}\left(1+\frac{1}{\beta_{\mathrm{F}}}\right)v_\pi(\gamma_{\mathrm{o}}||\gamma_{\mathrm{oc}}||R_{\mathrm{L}}) \\
R_{\mathrm{in}} &= \frac{v_{\mathrm{t}}}{i_{\mathrm{t}}} = \gamma_\pi + \frac{g_{\mathrm{m}}\left(1+\frac{1}{\beta_{\mathrm{F}}}\right)v_\pi(\gamma_{\mathrm{o}}||\gamma_{\mathrm{oc}}||R_{\mathrm{L}})}{g_{\mathrm{m}}\frac{v_\pi}{\beta_{\mathrm{F}}}} \\
&= \gamma_\pi + (\beta_{\mathrm{F}}+1)(\gamma_{\mathrm{o}}||\gamma_{\mathrm{oc}}||R_{\mathrm{L}}) \quad \text{much larger than } \gamma_\pi
\end{aligned}$$

R_{out}

$v_{\mathrm{s}} = 0$, leave R_{s}, apply $v_{\mathrm{t}}, i_{\mathrm{t}}$ at the output



$$\begin{aligned}
\left(i_{\mathrm{t}} + g_{\mathrm{m}} v_\pi + \overbrace{\frac{v_\pi}{\gamma_\pi}}^{\ll \text{ than } g_{\mathrm{m}} v_\pi}\right) \cdot (\gamma_{\mathrm{o}} || \gamma_{\mathrm{oc}}) &= v_{\mathrm{t}} \\
\text{voltage divider } v_\pi &= -\frac{\gamma_\pi}{\gamma_\pi + R_{\mathrm{s}}} \cdot v_{\mathrm{t}} \\
\Longrightarrow i_{\mathrm{t}} &= -g_{\mathrm{m}} v_\pi + \frac{v_{\mathrm{t}}}{\gamma_{\mathrm{o}} || \gamma_{\mathrm{oc}}} \\
\Longrightarrow i_{\mathrm{t}} &= \frac{g_{\mathrm{m}} \gamma_\pi}{\gamma_\pi + R_{\mathrm{s}}} \cdot v_{\mathrm{t}} + \frac{v_{\mathrm{t}}}{\gamma_{\mathrm{o}} || \gamma_{\mathrm{oc}}} = \left(\frac{\beta_{\mathrm{F}}}{\gamma_\pi + R_{\mathrm{s}}} + \frac{1}{\gamma_{\mathrm{o}} || \gamma_{\mathrm{oc}}}\right) v_{\mathrm{t}} \\
\therefore i_{\mathrm{t}} &\simeq \frac{\beta_{\mathrm{F}}}{\gamma_\pi + R_{\mathrm{s}}} v_{\mathrm{t}} \\
R_{\mathrm{out}} &= \frac{v_{\mathrm{t}}}{i_{\mathrm{t}}} = \frac{\gamma_\pi + R_{\mathrm{s}}}{\beta_{\mathrm{F}}} = \frac{1}{g_{\mathrm{m}}} + \frac{R_{\mathrm{s}}}{\beta_{\mathrm{F}}} \quad \text{LOW!} \because g_{\mathrm{m}}, \beta_{\mathrm{F}} \text{ large}
\end{aligned}$$

In conclusion, see the summary sheet handout

6.012 Microelectronic Devices and Circuits
Spring 2009