Recitation 18: BJT-Regions of Operation & Small Signal Model

BJT: Regions of Operation

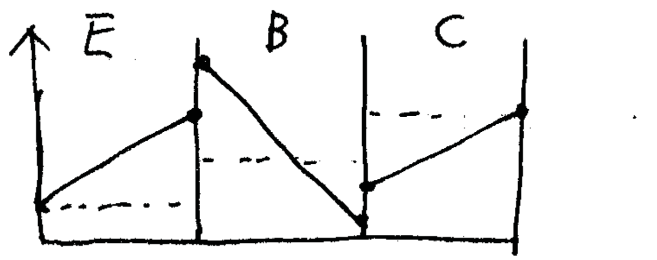
System of equations that describes BJT operation:

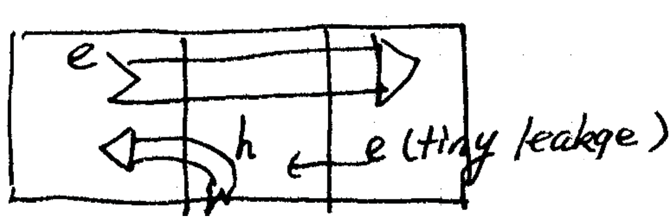
$$\begin{aligned} I_c &= I_s\left(exp(\frac{qV_{BE}}{kT}) - exp(\frac{qV_{BC}}{kT})\right) - \frac{I_s}{\beta_R}(exp(\frac{qV_{BC}}{kT}) - 1) \\ I_B &= \frac{I_s}{\beta_F}(exp(\frac{qV_{BE}}{kT}) - 1) + \frac{I_s}{\beta_R}(exp(\frac{qV_{BC}}{kT}) - 1) \\ I_E &= -I_s(exp(\frac{qV_{BE}}{kT}) - exp(\frac{qV_{BC}}{kT})) - \frac{I_s}{\beta_F}(exp(\frac{qV_{BE}}{kT}) - 1) \end{aligned}$$

$$\begin{aligned} I_s &= \frac{qA_E n_i^2 D_{nB}}{N_{aB} w_B} \\ \beta_F &= \frac{N_{dF}}{N_{aB}}\frac{D_{nB}}{D_{pE}}\frac{w_E}{w_B} \\ \beta_R &= \frac{N_{dC}}{N_{aB}}\frac{D_{nB}}{D_{pC}}\frac{w_C}{w_B} \end{aligned}$$

This set of equations can describe all four regimes of operation for BJT

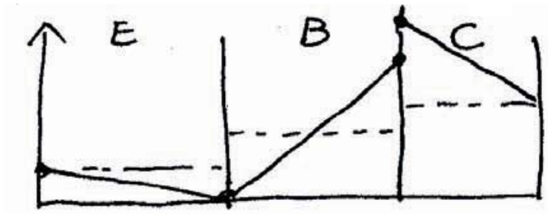
Forward Active: $V_{BE} > 0, V_{BC} < 0$

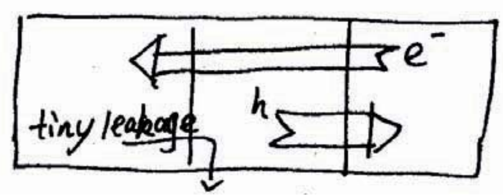




Reverse Active (RAR)

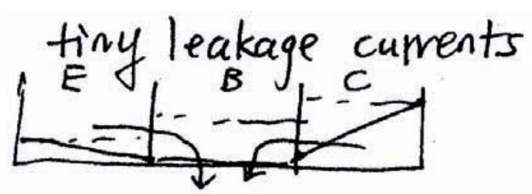
$V_{BE} < 0, V_{BC} > 0$





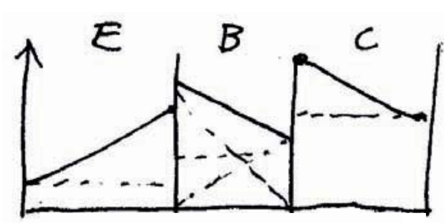
Cut-off

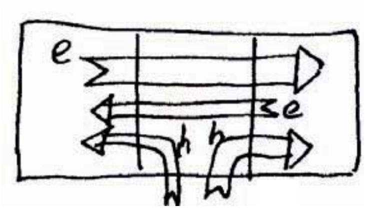
$V_{BE} < 0, V_{BC} < 0$



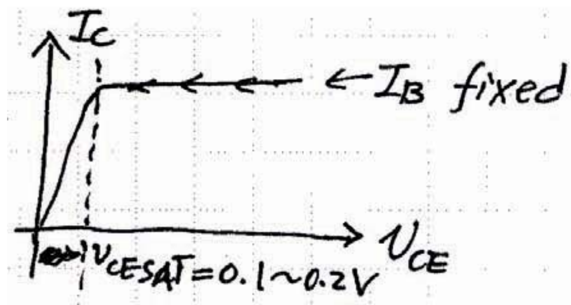
Saturation

$V_{BE} > 0, V_{BC} > 0$





Understanding the I_C vs. V_{CE} curve: I_C drops rapidly below $V_{CE,SAT} \simeq 0.1$ to 0.2 V.

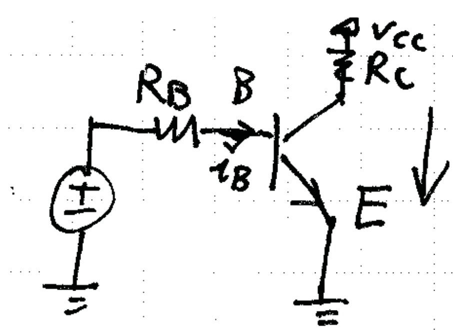


Why?

- Each curve I_B is fixed
- $V_{CE} = V_{BE} - V_{BC}$, $\implies$ $V_{BC} = V_{BE} - V_{CE}$
- When V_{CE} is large, $V_{BC} < 0$, FAR. As we reduce V_{CE}, V_{BC} reduces, at some point, V_{BC} starts to become forward biased. Now, hole flux from B $\rightarrow$ C increases exponentially from Law of Junction; to keep I_B constant, hole flux into emitter must be reduced, $\implies$ V_{BE} drops, $\implies$ I_C drops quickly.

Small Signal Model of BJT

(Next week we will be using BJT & MOSFET for amplifier circuits) Want to know the



small signal circuit model of BJT

1. Transconductance $g_m = \left.\dfrac{\delta i_c}{\delta V_{BE}}\right|_Q$

$$I_c = I_s e^{qV_{BE}/kT} \implies g_m = \frac{q}{kT} I_s e^{qV_{BE}/kT} = \frac{I_c}{V_{th}}$$

Note, different from MOSFET: $g_m \simeq \sqrt{2\frac{w}{L} I_D}$ (depends upon device size), but not for bipolar case.

2. Input resistance:

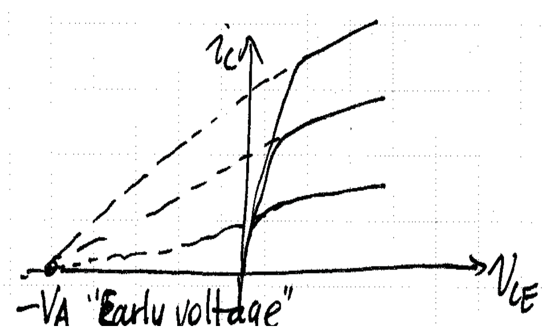
$$
\begin{aligned}
I_{\mathrm{B}} &= \frac{I_{\mathrm{s}}}{\beta_{\mathrm{F}}} e^{\mathrm{qV_{BE}/kT}} \\
g_\pi &= \frac{1}{\gamma_\pi} = \frac{\delta i_{\mathrm{B}}}{\delta V_{\mathrm{BE}}} = \frac{I_{\mathrm{B}}}{V_{\mathrm{th}}} = \frac{g_{\mathrm{m}}}{\beta_{\mathrm{F}}} \\
\text{or } \gamma_\pi &= \frac{\beta_{\mathrm{F}}}{g_{\mathrm{m}}}
\end{aligned}
$$

The input resistance of MOSFET is ∞. In order to have a high input resistance for BJT, need high current gain β_{F}

Example: npn with $\beta_{\mathrm{F}} = 150, I_{\mathrm{c}} =$ mA

$$
\begin{aligned}
g_{\mathrm{m}} &= \frac{I_{\mathrm{c}}}{V_{\mathrm{th}}} = \frac{1 \times 10^{-3}\,\mathrm{A}}{0.025\,\mathrm{V}} = 40\,\mathrm{mS} \\
g_\pi &= \frac{g_{\mathrm{m}}}{\beta_{\mathrm{F}}} = \frac{40\,\mathrm{mS}}{150} = 0.267\,\mathrm{mS} \quad (\gamma_\pi = 3.7\,\mathrm{k\Omega})
\end{aligned}
$$

3. Output resistance: Ebers-Moll model have perfect current source in FAR. Real characteristics show some increase in i_{c} with V_{CE}



$$
\begin{aligned}
g_{\mathrm{o}} &= \frac{\delta i_{\mathrm{c}}}{\delta V_{\mathrm{CE}}} \quad \text{where does } g_{\mathrm{o}} \text{ come from?} \\
\text{In FAR: } I_{\mathrm{c}} &= I_{\mathrm{s}} e^{\mathrm{qV_{BE}/kT}} = \frac{\mathrm{qA_E n_i^2 D_{nB}}}{\mathrm{N_{aB} w_B}} e^{\mathrm{qV_{BE}/kT}}
\end{aligned}
$$

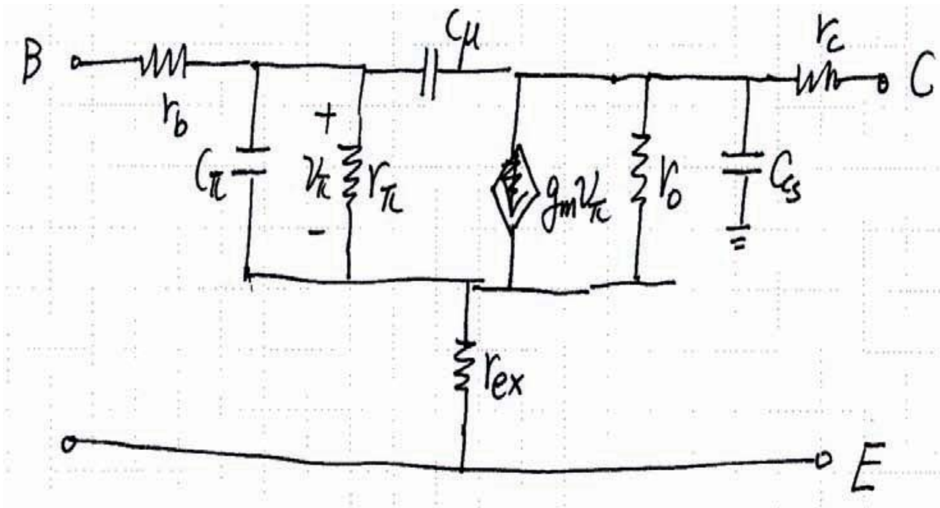
w_{B} shrinks as $|V_{\mathrm{BC}}| \uparrow$, thus $I_{\mathrm{c}} \uparrow$.

$$
\begin{aligned}
\text{Model: } g_{\mathrm{o}} &= \text{slope} = \frac{I_{\mathrm{c}}}{V_{\mathrm{CE}} + V_{\mathrm{A}}} \simeq \frac{I_{\mathrm{c}}}{V_{\mathrm{A}}} \quad (V_{\mathrm{A}} \gg V_{\mathrm{CE}}) \\
g_{\mathrm{o}} &= \frac{1}{\gamma_{\mathrm{o}}} = \frac{I_{\mathrm{c}}}{V_{\mathrm{A}}}
\end{aligned}
$$

Example: $I_{\mathrm{c}} = 100\,\mu\mathrm{A},\ \mathrm{V_A} = 35\,\mathrm{V},\ \Longrightarrow\ \gamma_{\mathrm{o}} = 350\,\mathrm{k\Omega}$

V_{A} *increases with increasing base width and increasing base doping.* This is also why N_{aB} usually $\gg N_{\mathrm{dC}}$

Now what do we have so far? Need to add capacitances...



Junction Capacitance (depletion capacitance)

$$\text{(B-E):}\ C_{\text{jE}} = \sqrt{\frac{q\epsilon_{\text{s}}N_{\text{aB}}N_{\text{dE}}}{2(N_{\text{aB}}+N_{\text{dE}})(\phi_{\text{BE}}-V_{\text{BE}})}}\ \ (\because N_{\text{dE}} \gg N_{\text{aB}})$$

$$\text{(B-E):}\ C_{\text{jC}} = \sqrt{\frac{q\epsilon_{\text{s}}N_{\text{aB}}N_{\text{dC}}}{2(N_{\text{aB}}+N_{\text{dC}})(\phi_{\text{BC}}-V_{\text{BC}})}} \simeq \sqrt{\frac{q\epsilon_{\text{s}}}{2}\frac{N_{\text{dC}}}{(\phi_{\text{BC}}-V_{\text{BE}})}}$$

- Both are functions of bias
- Since $N_{\text{aB}} \gg N_{\text{dC}}, C_{\text{jE}} \gg C_{\text{jC}}$. C_{jC} is often called C_μ.

Diffusion Capacitance

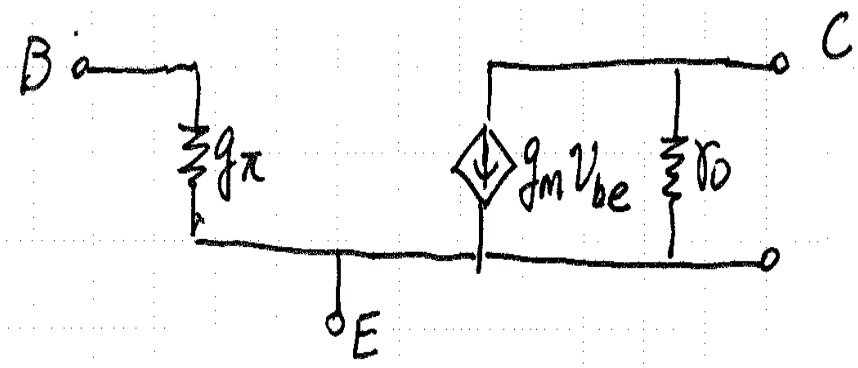
$$C_{\text{b}} = \frac{\delta}{\delta V_{\text{BE}}}|Q_{\text{nB}}|$$

$$|Q_{\text{nB}}| = \frac{1}{2}qA_{\text{E}}w_{\text{B}}n_{\text{pBO}}\text{e}^{\text{qV}_{\text{BE}}/\text{kT}}$$

$$= \frac{1}{2}w_{\text{B}}\left(\frac{w_{\text{B}}}{D_{\text{nB}}}\right)\left(\frac{qA_{\text{E}}D_{\text{nB}}}{w_{\text{B}}}\right)n_{\text{pBO}}\text{e}^{\text{qV}_{\text{BE}}/\text{kT}} = \left(\frac{\text{w}_{\text{B}}^2}{2\text{D}_{\text{nB}}}\right)\text{I}_{\text{c}}$$

$$C_{\text{b}} = \frac{\delta}{\delta V_{\text{BE}}}\left(\left(\frac{w_{\text{B}}^2}{2D_{\text{nB}}}\right)i_{\text{c}}\right) = \left(\frac{w_{\text{B}}^2}{2D_{\text{n}}}\right)g_{\text{m}}$$

$$\frac{w_{\text{B}}^2}{2D_{\text{n}}} = \tau_{\text{F}}\ \ \text{base diffusion transit time}$$



C_b is in between base and emitter:

$$C_b + C_{jE} = C_\pi$$

Add the following

- depletion capacitance: collector to bulk C_{CS}
- parasitic resistances: γ_b of base, γ_{ex} of emitter, γ_c of collector

Complete small signal model