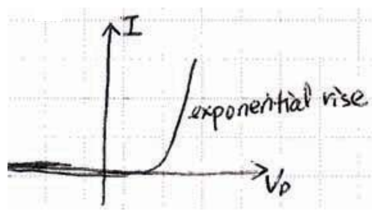
Recitation 16: Small Signal Model of p-n Diode

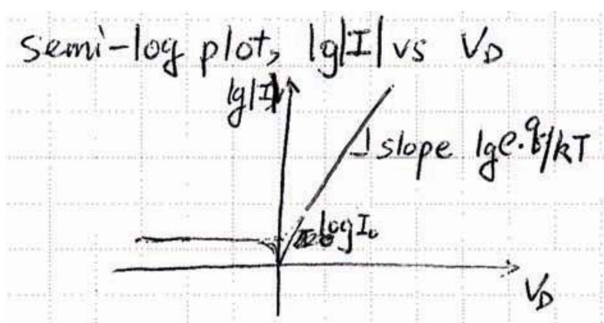
Review

Last week we learned about the IV characteristic of p-n diode:

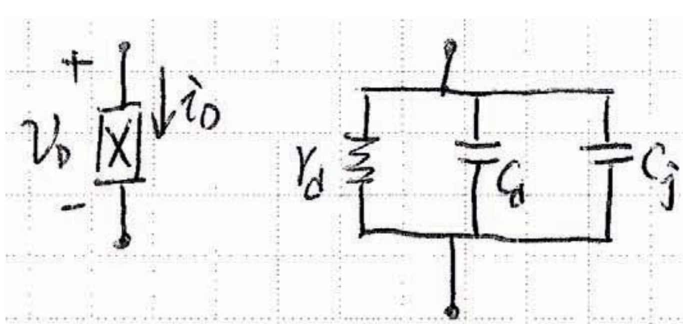
$$I = I_o(e^{qV_o/kT} - 1)$$
$$\text{where } I_o = qAn_i^2\left(\frac{1}{N_a}\frac{D_n}{w_p - x_p} + \frac{1}{N_d}\frac{D_p}{w_n - x_n}\right)$$

If we plot,

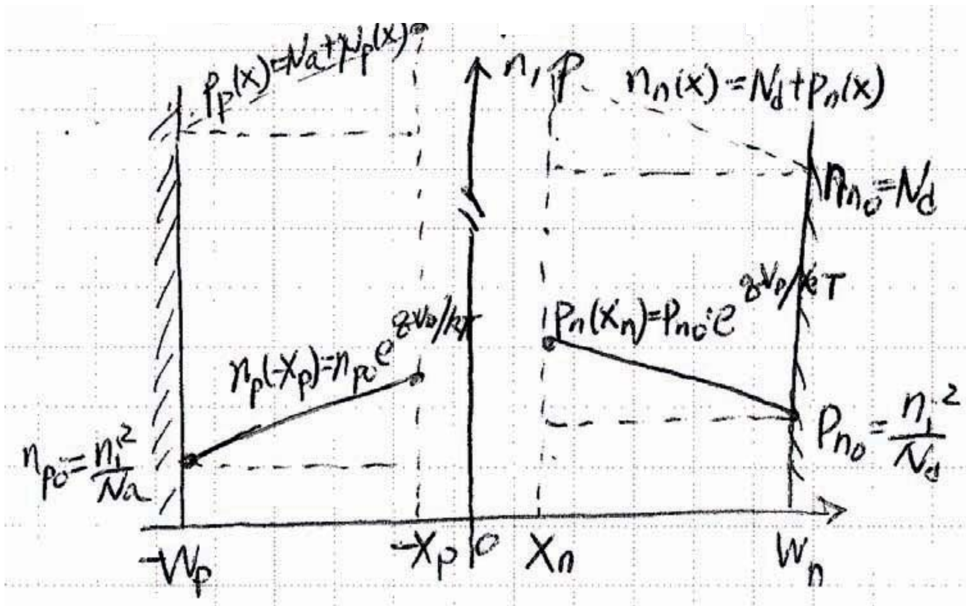




Yesterday, we discussed the small signal model for p-n diode. Under forward bias, the small signal (ss) model of a p-n diode is:

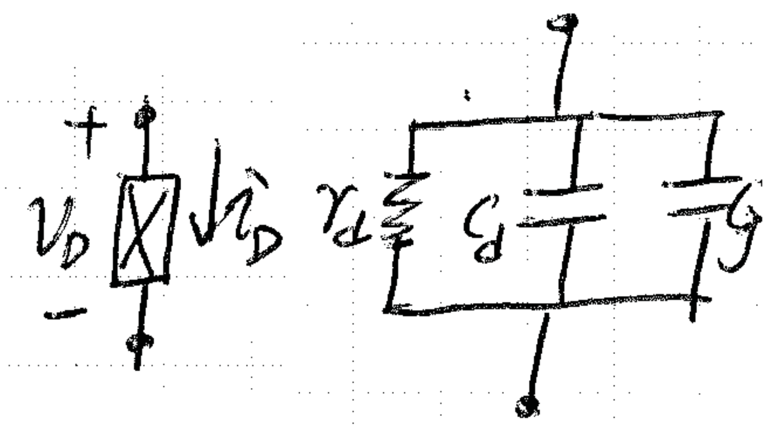


C_j is the junction capacitance as we talked about before, C_d is called "diffusion capacitance", this is new



Small signal circuit model of p-n diode: (two terminal device)

In a small signal model, we have linearized conductance (resistance) and capacitances.



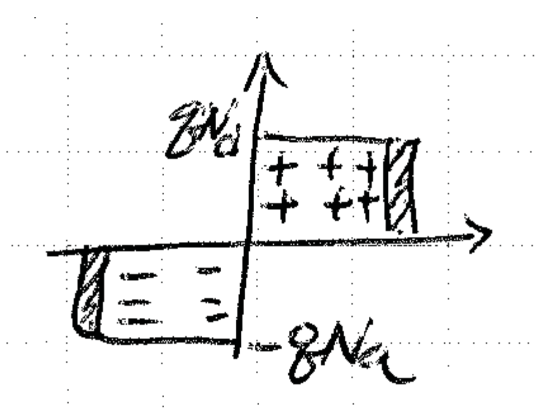
Linearized Conductance (Resistance)

$$\begin{aligned}
\gamma_{\mathrm{d}} &= \frac{1}{g_{\mathrm{d}}}, \quad g_{\mathrm{d}} = \frac{\delta i_{\mathrm{D}}}{\delta V_{\mathrm{D}}}\Big|_{V_{\mathrm{D}}} = \frac{\delta I_{\mathrm{o}} \cdot (\mathrm{e}^{\mathrm{q}\mathrm{V}_{\mathrm{D}}/\mathrm{kT}} - 1)}{\delta V_{\mathrm{D}}}\Big|_{V_{\mathrm{D}}} \\
&= I_{\mathrm{o}} \cdot \frac{q}{kT}\mathrm{e}^{\mathrm{q}\mathrm{V}_{\mathrm{D}}/\mathrm{kT}} \\
&= \frac{q}{kT} \cdot (I_{\mathrm{D}} + I_{\mathrm{o}}) \\
&= \frac{q}{kT} I_{\mathrm{D}}, \quad \frac{kT}{q} = V_{\mathrm{th}} \\
\Longrightarrow \gamma_{\mathrm{d}} &= \frac{V_{\mathrm{th}}}{I_{\mathrm{D}}}
\end{aligned}$$

V_{th} constant, the larger operating current I_{D}, the smaller is γ_{d}

Depletion Capacitance (due to p-n junction)

$$\begin{aligned}
C_{\mathrm{jo}} &= \frac{\epsilon_{\mathrm{s}}}{x_{\mathrm{do}}} = \sqrt{\frac{q\epsilon_{\mathrm{s}} N_{\mathrm{a}} N_{\mathrm{d}}}{2(N_{\mathrm{a}} + N_{\mathrm{d}})\phi_{\mathrm{B}}}} \\
C_{\mathrm{j}}(V_{\mathrm{D}}) &= \frac{C_{\mathrm{jo}}}{\underbrace{\sqrt{1 - \frac{V_{\mathrm{D}}}{\phi_{\mathrm{B}}}}}_{\text{capacitance/unit area!}}} \quad \text{For forward bias limit } V_{\mathrm{D}} \text{ to } \frac{\phi_{\mathrm{B}}}{2}
\end{aligned}$$



Diffusion Capacitance

For this, we need to look at the majority carrier concentration as well. To keep quasi-neutral,

$$\begin{aligned}
n_{\mathrm{n}}(x) &= N_{\mathrm{d}} + p_{\mathrm{n}}(x) \\
p_{\mathrm{p}}(x) &= N_{\mathrm{a}} + n_{\mathrm{p}}(x)
\end{aligned}$$

It is a capacitor without two parallel plates! And the "+" and "-" charges are just mixed with each other! Isn't that amazing!
charge stored on the n-side:

$$\begin{aligned} q_{\mathrm{p_n}} &= -q_{\mathrm{N_n}} = qA\frac{1}{2}(w_\mathrm{n} - x_\mathrm{n}) \cdot \frac{n_\mathrm{i}^2}{N_\mathrm{d}}(\mathrm{e}^{\mathrm{qV_D/kT}} - 1) \\ C_\mathrm{dn} &= \left.\frac{dq_{\mathrm{p_n}}}{dV_\mathrm{D}}\right|_{V_\mathrm{D}} = qA\frac{w_\mathrm{n} - x_\mathrm{n}}{2} \cdot \frac{n_\mathrm{i}^2}{N_\mathrm{d}}\frac{q}{kT}\mathrm{e}^{\mathrm{qV_D/kT}} \end{aligned}$$

1. $C_\mathrm{dn} = \frac{qA}{2V_\mathrm{th}}(w_\mathrm{n} - x_\mathrm{n}) \cdot p_\mathrm{no}\mathrm{e}^{\mathrm{qV_D/kT}} = \frac{\mathrm{qA}}{2\mathrm{V_{th}}}(\mathrm{w_n} - \mathrm{x_n}) \cdot \mathrm{p_n}(\mathrm{x_n})$

2. Write in terms of I_D

$$\begin{aligned} C_\mathrm{dn} &= qA\frac{n_\mathrm{i}^2}{N_\mathrm{d}} \cdot \frac{D_\mathrm{p}}{w_\mathrm{n} - x_\mathrm{n}} \cdot \mathrm{e}^{\mathrm{qV_D/kT}} \cdot \frac{\mathrm{w_n} - \mathrm{x_n}}{2} \cdot \frac{\mathrm{w_n} - \mathrm{x_n}}{\mathrm{D_p}} \cdot \frac{\mathrm{q}}{\mathrm{kT}} \\ C_\mathrm{dn} &= \frac{q}{kT} \cdot \frac{(w_\mathrm{n} - x_\mathrm{n})^2}{2D_\mathrm{p}} \cdot I_{\mathrm{D_p}} \\ \text{Define :} & \quad \text{transit time of holes through n-QNR:} \\ T_{\mathrm{T_p}} &= \frac{(w_\mathrm{n} - x_\mathrm{n})^2}{2D_\mathrm{p}} = \frac{w_\mathrm{n} - x_\mathrm{n}}{2D_\mathrm{p}/(w_\mathrm{n} - x_\mathrm{n})} = \frac{\text{length}}{\text{velocity}} \\ C_\mathrm{dn} &\simeq \frac{q}{kT} \cdot T_{\mathrm{T_p}} \cdot I_{\mathrm{D_p}} \\ C_\mathrm{dn} &= \frac{I_{\mathrm{D_p}} \cdot T_{\mathrm{T_p}}}{V_\mathrm{th}} \\ \text{Similarly, } C_\mathrm{dp} &\simeq \frac{q}{kT} \cdot T_{\mathrm{T_n}} \cdot I_{\mathrm{D_n}} \end{aligned}$$

Charges on both side are added together. The two capacitors are in *parallel*.

$$\begin{aligned} C_\mathrm{d} &= C_\mathrm{dn} + C_\mathrm{dp} = \frac{q}{kT}(T_{\mathrm{T_n}} \cdot I_{\mathrm{D_n}} + T_{\mathrm{T_p}} \cdot I_{\mathrm{D_p}}) \\ C_\mathrm{d} &= C_\mathrm{dn} + C_\mathrm{dp} = \frac{qA}{2V_\mathrm{th}}((w_\mathrm{p} - x_\mathrm{p})n_\mathrm{po} + (w_\mathrm{n} - x_\mathrm{n})p_\mathrm{no})\mathrm{e}^{\mathrm{qV_D/kT}} \end{aligned}$$

Discussions:

1. Where do the extra majority carriers come from?

2. Majority current: -diffusion + drift

3. $C_\mathrm{d} \propto \mathrm{e}^{\mathrm{qV_D/kT}}$: for reverse bias, depletion capacitance dominate. For forward bias, diffusion capacitance dominate

Exercise: p-n Diode

$$
\begin{aligned}
w_{\mathrm{p}} &= 0.5\,\mu\mathrm{m} \quad \mathrm{N_a} = 2.5\times10^{17}\,\mathrm{cm^{-3}} \quad \mathrm{D_n} = 14\,\mathrm{cm^2/s} \\
w_{\mathrm{n}} &= 1.0\,\mu\mathrm{m} \quad \mathrm{N_d} = 4.0\times10^{16}\,\mathrm{cm^{-3}} \quad \mathrm{D_p} = 10\,\mathrm{cm^2/s} \\
V_{\mathrm{d}} &= 720\,\mathrm{mV} \quad \mathrm{I_D} = 50\,\mu\mathrm{A}
\end{aligned}
$$

Find $V_{\mathrm{d}}, C_{\mathrm{j}}$, and C_{d}. For this calculation, ignore $x_{\mathrm{n}}, x_{\mathrm{p}}$.

$$
\begin{aligned}
J_{\mathrm{o}} &= q n_{\mathrm{i}}^2\left(\frac{D_{\mathrm{n}}}{N_{\mathrm{a}} w_{\mathrm{p}}} + \frac{D_{\mathrm{p}}}{N_{\mathrm{d}} w_{\mathrm{n}}}\right) = 5.79\times10^{-11}\,\mathrm{A/cm^2} \\
I_{\mathrm{D}} &= I_{\mathrm{o}}(\mathrm{e}^{\mathrm{qV_o/kT}} - 1) \simeq \mathrm{I_o}\cdot\mathrm{e}^{\mathrm{qV_o/kT}} = \mathrm{I_o}\cdot 10^{720/60} = 50\,\mu\mathrm{A} \\
\Longrightarrow I_{\mathrm{o}} &= 5\times10^{-17}\,\mathrm{A} \Longrightarrow \mathrm{J_o} = \mathrm{I_o/A} \Longrightarrow \mathrm{A} = 8.64\times10^{-7}\,\mathrm{cm^2} \\
\gamma_{\mathrm{d}} &= \frac{V_{\mathrm{th}}}{I_{\mathrm{D}}} = \frac{0.025\,\mathrm{V}}{50\,\mu\mathrm{A}} = 500\,\omega \quad \phi_{\mathrm{B}} = \frac{\mathrm{kT}}{\mathrm{q}}\ln\frac{\mathrm{N_a N_d}}{\mathrm{n_i^2}} = 840\,\mathrm{mV} \\
C_{\mathrm{jo}} &= A\cdot\sqrt{\frac{q\epsilon_{\mathrm{s}} N_{\mathrm{d}} N_{\mathrm{a}}}{2(N_{\mathrm{a}}+N_{\mathrm{d}})\phi_{\mathrm{B}}}} \\
&= A\sqrt{\frac{1.6\times10^{-19}\,\mathrm{C}\times 8.85\times10^{-14}\,\mathrm{F/cm}\cdot 11.9\times 4\times10^{16}\times 2.5\times10^{17}\,\mathrm{cm^{-6}}}{2(4\times10^{16}+2.5\times10^{17})0.84\,\mathrm{V}\cdot\mathrm{cm^{-3}}}} \\
&= 8.64\times10^{-7}\,\mathrm{cm^2}\times 5.88\times10^{-8}\,\mathrm{F/cm^2} \\
&= 50.8\,\mathrm{fF} = 50.8\times10^{-15}\,\mathrm{F} \\
C_{\mathrm{j}} &= \frac{C_{\mathrm{jo}}}{\sqrt{1-\frac{V_{\mathrm{D}}}{\phi_{\mathrm{B}}}}} = \frac{50.8\,\mathrm{fF}}{\frac{1}{\sqrt{2}}} = 71.8\,\mathrm{fF} \\
C_{\mathrm{d}} &= \frac{qA}{2V_{\mathrm{th}}}\cdot(w_{\mathrm{p}}\cdot n_{\mathrm{p}}(-x_{\mathrm{p}}) + w_{\mathrm{n}}\cdot p_{\mathrm{n}}(x_{\mathrm{n}})) \\
&= \frac{1.6\times10^{-19}\,\mathrm{C}\times 8.64\times10^{-7}\,\mathrm{cm^2}}{2\times0.025}\left(0.5\times10^{-4}\times\frac{10^{20}}{2.5\times10^{17}} + 1\times10^{-4}\times\frac{10^{20}}{4\times10^{16}}\right)\underbrace{\mathrm{e}^{\mathrm{qV_o/kT}}}_{10^{12}} \\
&= \frac{1.6\times10^{-19}\,\mathrm{C}\times 8.64\times10^{-7}\,\mathrm{cm^2}}{2\times0.025\,\mathrm{V}}(\underbrace{0.5\times10^{-4}}_{\mathrm{cm}}\times\underbrace{4\times10^{14}}_{\mathrm{cm^{-3}}}+\underbrace{1\times10^{-4}}_{\mathrm{cm}}\times\underbrace{2.5\times10^{15}}_{\mathrm{cm^{-3}}}) \\
&= \frac{1.6\times10^{-19}\,\mathrm{C}\times 8.64\times10^{-7}}{2\times0.025\,\mathrm{V}}\times 2.7\times10^{11} = 746\,\mathrm{fF}
\end{aligned}
$$