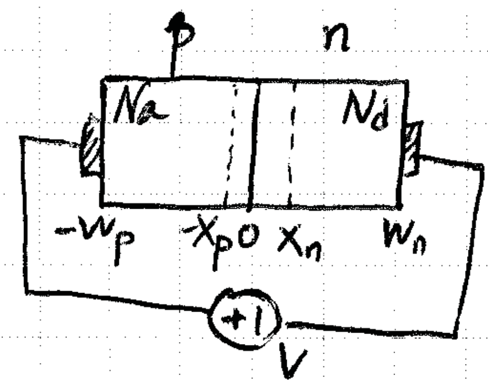
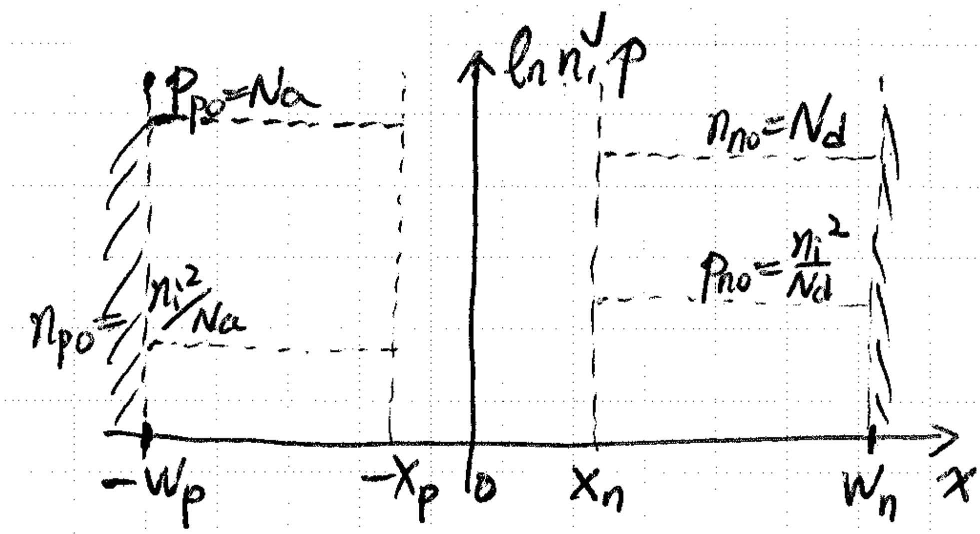
Recitation 15: p-n diode I-V characteristics (II)

Yesterday, we talked about the diode I-V relationship: $I = I_\mathrm{o}(e^{\frac{V}{V_{th}}} - 1)$. Today, we will look more closely how is this relationship and what is I_o?

Brief Review of the "Law of the junction"





When bias voltage V is applied (V can be either positive or negative):
law of the junction $\longrightarrow$ the minority carrier concentration at the SCR edge

$$n_\mathrm{p}(-x_\mathrm{p}) = n_\mathrm{po} \cdot e^{V/V_{th}} = \frac{n_i^2}{N_a} \cdot e^{V/V_{th}}$$

$$p_\mathrm{n}(x_\mathrm{n}) = p_\mathrm{no} \cdot e^{V/V_{th}} = \frac{n_i^2}{N_d} e^{V/V_{th}}$$

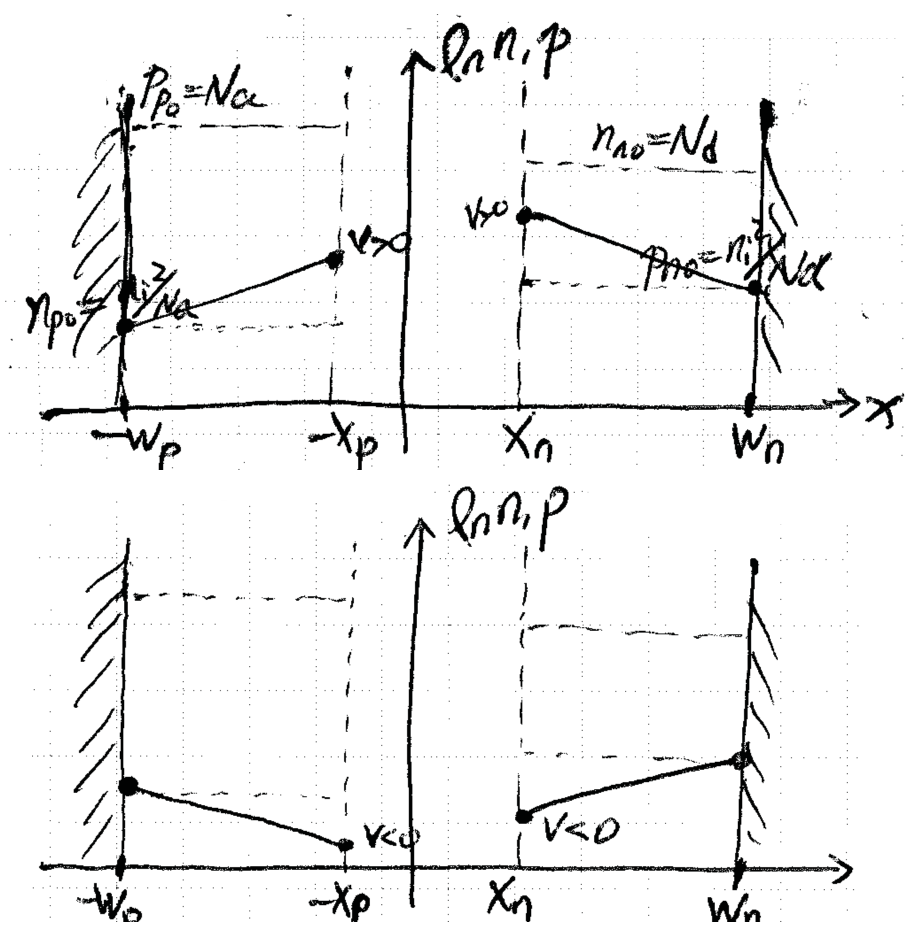
At the contact, recombination/generation occur very fast

$$n_{\rm p}(-w_{\rm p}) = n_{\rm po}$$
$$p_{\rm n}(+w_{\rm n}) = p_{\rm no}$$

We know two data points of the concentration, what is in between the QNRs?

From yesterday's discussion: *Linear profile vs. position.* This is because, approximating there is no recombination/generation inside QNR (and SCR).
$\Longrightarrow$ current need to be constant.
For minority carriers, only diffusion current $\Longrightarrow \dfrac{dp}{dx}$ or $\dfrac{dn}{dx}$ constant.

Note these are log scale, we try to plot linear dependence on log scale;
it should not be linear on log scale



How to Calculate Current

$$
\begin{aligned}
J_\mathrm{p}(x_\mathrm{n}) &= -qD_\mathrm{p}\frac{dp_\mathrm{n}}{dx}\Big|_{x=x_\mathrm{n}} \\
&= -qD_\mathrm{p}\frac{p_\mathrm{n}(x_\mathrm{n}) - \overbrace{p_\mathrm{n}(w_\mathrm{n})}^{p_\mathrm{no}}}{x_\mathrm{n}-w_\mathrm{n}} = qD_\mathrm{p}\frac{p_\mathrm{no}\mathrm{e}^{\mathrm{V/V_{th}}} - \mathrm{p_{no}}}{w_\mathrm{n}-x_\mathrm{n}} \\
J_\mathrm{p}(x_\mathrm{n}) &= qD_\mathrm{p}\frac{p_\mathrm{no}}{w_\mathrm{n}-x_\mathrm{n}}(\mathrm{e}^{\mathrm{V/V_{th}}}-1) = \mathrm{q}\frac{\mathrm{n_i^2}}{\mathrm{N_d}}\cdot\frac{\mathrm{D_p}}{\mathrm{w_n - x_n}}(\mathrm{e}^{\mathrm{V/V_{th}}}-1) \\
J_\mathrm{n}(-x_\mathrm{p}) &= qD_\mathrm{n}\frac{dn_\mathrm{p}}{dx}\Big|_{x=-x_\mathrm{p}} = qD_\mathrm{n}\frac{n_\mathrm{p}(-x_\mathrm{p}) - \overbrace{n_\mathrm{p}(-w_\mathrm{p})}^{=n_\mathrm{po}}}{-x_\mathrm{p}+w_\mathrm{p}} = qD_\mathrm{n}\frac{n_\mathrm{po}(\mathrm{e}^{\mathrm{V/V_{th}}}-1)}{w_\mathrm{p}-x_\mathrm{p}} \\
J_\mathrm{n}(-x_\mathrm{p}) &= q\frac{n_\mathrm{i}^2}{N_\mathrm{a}}\cdot\frac{D_\mathrm{n}}{w_\mathrm{p}-x_\mathrm{p}}(\mathrm{e}^{\mathrm{V/V_{th}}}-1)
\end{aligned}
$$

The diode current is carried out by *both* electrons and holes. They need to be summed up

$$
\begin{aligned}
I_\mathrm{total} &= (J_\mathrm{n}+J_\mathrm{p})\cdot A,\ \text{A is cross-sectional area of diode} \\
I_\mathrm{total} &= qA\cdot n_\mathrm{i}^2\left(\frac{1}{N_\mathrm{a}}\cdot\frac{D_\mathrm{n}}{w_\mathrm{p}-x_\mathrm{p}} + \frac{1}{N_\mathrm{d}}\cdot\frac{D_\mathrm{p}}{w_\mathrm{n}-x_\mathrm{n}}\right)\cdot(\mathrm{e}^{\mathrm{V/V_{th}}}-1) \\
I_\mathrm{o} &= qAn_\mathrm{i}^2\left(\frac{1}{N_\mathrm{a}}\cdot\frac{D_\mathrm{n}}{w_\mathrm{p}-x_\mathrm{p}} + \frac{1}{N_\mathrm{d}}\cdot\frac{D_\mathrm{p}}{w_\mathrm{n}-x_\mathrm{n}}\right)
\end{aligned}
$$

Note:

1. I_o is a pretty small value, 10^{-15} A. With a positive voltage, say 0.6 V, $\mathrm{e}^{\mathrm{V/V_{th}}} = 10^{\mathrm{V/60\,mV}} = 10^{10}$, we will get a fairly large current

2. Just by looking at the equation of I_o, can we tell which part is J_n? Which part is J_p?

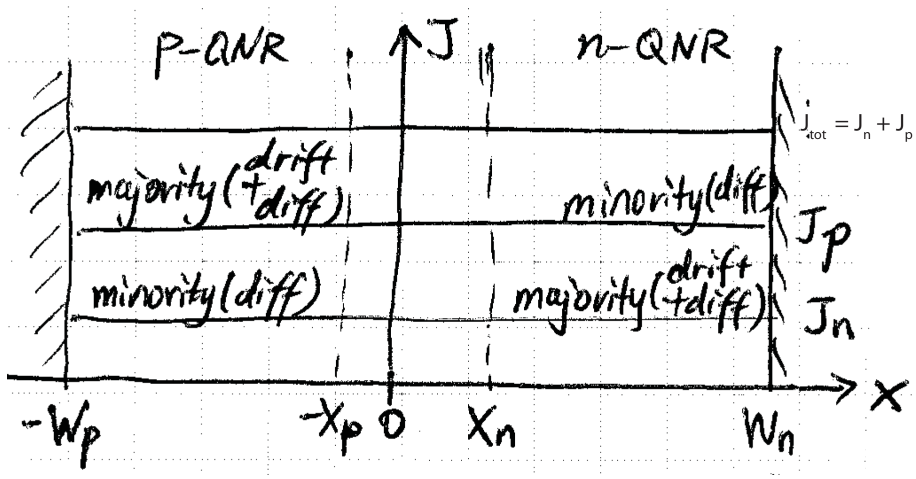
   J_n is *electron diffusion* in p-QNR $\Longrightarrow \dfrac{1}{N_\mathrm{a}}\dfrac{D_\mathrm{n}}{w_\mathrm{p}-x_\mathrm{p}}$

   J_p is *hole diffusion* in n-QNR $\Longrightarrow \dfrac{1}{N_\mathrm{d}}\dfrac{D_\mathrm{p}}{w_\mathrm{n}-x_\mathrm{n}}$

   Be careful where to use $D_\mathrm{n}, D_\mathrm{p}$!

3. For an asymmetrically doped diode, n^+p or p^+n, the J_n and J_p can differ a lot.

Overall Picture of Diode Current



Through the diode, the electron current needs to be constant throughout, and the hole current needs to be constant throughout. This means:

$$\underbrace{\text{the majority hole current on p-QNR side}}_{\text{drift + diffusion}} = \underbrace{\text{minority hole current on n-QNR side}}_{\text{only diffusion}}$$
$$\underbrace{\text{the minority electron current on p-QNR side}}_{\text{diffusion only}} = \underbrace{\text{majority hole current on n-QNR side}}_{\text{drift + diffusion}}$$

For students who are interested in the majority carriers, can discuss a little bit about it.

Exercise

$N_\mathrm{a} = 5\times 10^6\,\mathrm{cm}^{-3}, \mathrm{N_d} = 10^{17}\,\mathrm{cm}^{-3}, \mathrm{w_p} = 0.3\,\mu\mathrm{m}, \mathrm{w_n} = 0.3\,\mu\mathrm{m}$. If $V = 0.75\phi_\mathrm{B}$, find $J_\mathrm{n}, J_\mathrm{p}$. First D_n and D_p:

- D_n is electron diffusion coefficient in p-region: doping

$$5\times 10^6\,\mathrm{cm}^{-3},\ \mu_\mathrm{n},\ \mu_\mathrm{n} = 900\,\mathrm{cm}^2/\mathrm{Vs},\ \frac{\mathrm{D}}{\mu} = \frac{\mathrm{kT}}{\mathrm{q}} \implies \mathrm{D_n} \simeq 22.5\,\mathrm{cm}^2/\mathrm{s}$$

- D_p is hole diffusion coefficient in n-region:

$$N_\mathrm{d} = 10^{17}, \mu_\mathrm{n} = 350\,\mathrm{cm}^2/\mathrm{Vs},\ \mathrm{D_p} = 8.75\,\mathrm{cm}^2/\mathrm{s}$$

$$
\begin{aligned}
\phi_\mathrm{B} &= \frac{kT}{q}\ln\frac{N_\mathrm{a}N_\mathrm{d}}{n_\mathrm{i}^2} = 0.025\ln\left(\frac{5\times10^6\times10^{17}}{10^{20}}\right) = 0.789\,\mathrm{V}\\
V &= 0.75\phi_\mathrm{B} = 0.591\,\mathrm{V}\\
x_\mathrm{po} &= \sqrt{\frac{2\epsilon_\mathrm{s}\phi_\mathrm{B}N_\mathrm{d}}{qN_\mathrm{a}(N_\mathrm{a}+N_\mathrm{d})}} = \sqrt{\frac{2\times1\times10^{-12}\,\mathrm{F/cm}\times0.79\,\mathrm{V}\times10^{17}\,\mathrm{cm^{-3}}}{1.6\times10^{-19}\,\mathrm{C}\times5\times10^{16}\times(5\times10^{16}+10^{17})\,\mathrm{cm^{-6}}}} = 0.118\,\mu\mathrm{m}\\
N_\mathrm{a}\cdot x_\mathrm{po} &= N_\mathrm{d}\cdot x_\mathrm{no},\implies x_\mathrm{no} = \frac{1}{2}x_\mathrm{po} = 0.059\,\mu\mathrm{m}\\
x_\mathrm{n} &= x_\mathrm{no}\sqrt{1-V/\phi_\mathrm{B}} = 0.059\,\mu\mathrm{m}\times\frac{1}{2} = 0.0295\,\mu\mathrm{m}\\
x_\mathrm{p} &= x_\mathrm{po}\sqrt{1-V/\phi_\mathrm{B}} = 0.059\,\mu\mathrm{m}\\
J_\mathrm{n} &= qn_\mathrm{i}^2\left(\frac{D_\mathrm{n}}{N_\mathrm{a}(w_\mathrm{p}-x_\mathrm{p})}\right)(\mathrm{e}^{\mathrm{V/V_{th}}}\cancel{-1})\quad\text{forward bias } V\gg V_\mathrm{th}\\
&= \left(\frac{1.6\times10^{-19}\times10^{20}\times22.5\cdot\ \mathrm{C}\times\mathrm{cm^{-6}}\times\mathrm{cm^2/s}}{5\times10^{16}\,\mathrm{cm^3}(0.3-0.059)\times10^{-4}\,\mathrm{cm}}\right)\cdot\underbrace{\mathrm{e}^{\mathrm{V/V_{th}}}}_{10^{9.85}}\\
&= 2.98\times10^{-10}\,\mathrm{A/cm^2}\cdot10^{9.85} = 2.98\times10^{-10}\times7.07\times10^9\,\mathrm{A/cm^2} = 2.10\,\mathrm{A/cm^2}\\
J_\mathrm{p} &= qn_\mathrm{i}^2\frac{D_\mathrm{p}}{N_\mathrm{d}(w_\mathrm{n}-x_\mathrm{n})}(\mathrm{e}^{\mathrm{V/V_{th}}}-1)\\
&= \frac{1.6\times10^{-19}\times10^{20}\times8.75}{10^{17}\times(0.3-0.029)\times10^{-4}\,\mathrm{cm}}\cdot\mathrm{e}^{\mathrm{V/V_{th}}} = 5.17\times10^{-11}\,\mathrm{A/cm^2}\times10^{9.85} = 0.36\,\mathrm{A/cm^2}
\end{aligned}
$$