Recitation 6: p-n junction

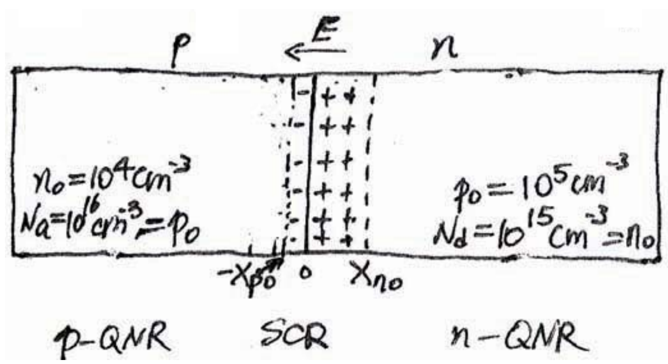
Today's agenda:

1. p-n junction under thermal equilibrium (T.E.):
   - space charge region (depletion layer)
   - quasi-neutral region
2. p-n junction under reverse bias
3. depletion capacitance

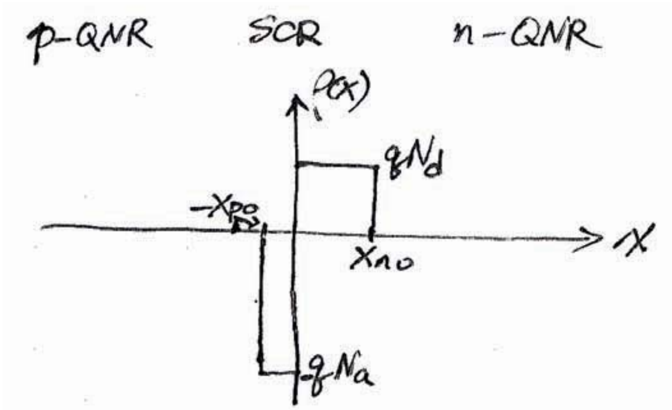
p-n junction under T.E.

Yesterday we talked about p-n junction. We continue here:

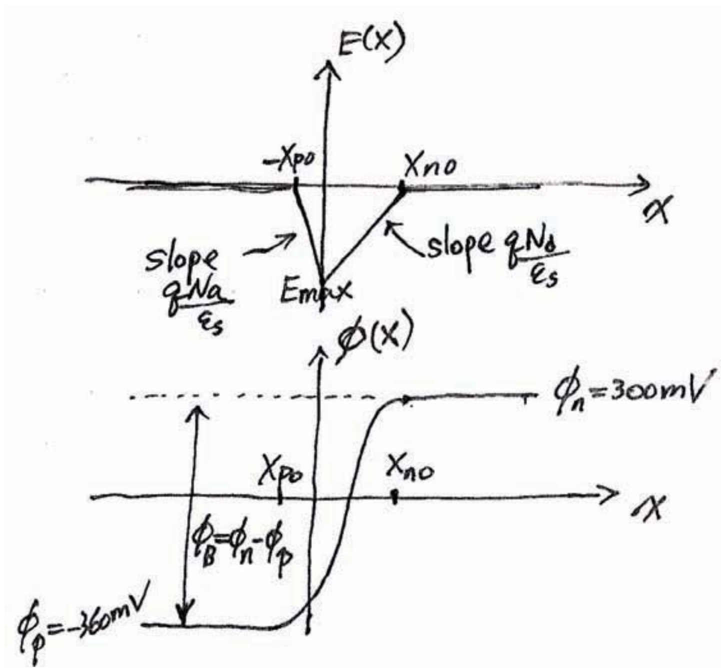
1. When we bring the p and n interface together, what will happen?
   Holes move from p-side to n-side and electrons diffuse from n-side to p-side, leaving behind space charges. The result is an electric field counteracting diffusion.



2. *Space Charge Region (SCR)*: is a depletion region. $x_{\text{no}}, x_{\text{po}}, x_{\text{no}} + x_{\text{po}} = x_{\text{do}}$.
   What is the charge density in SCR? $q\,N_{\text{a}}(-)$ and $q\,N_{\text{d}}(+)$
   And $q\,N_{\text{a}} \cdot x_{\text{po}} = q\,N_{\text{d}} \cdot x_{\text{no}} \implies \dfrac{x_{\text{po}}}{x_{\text{no}}} = \dfrac{N_{\text{d}}}{N_{\text{a}}}$.
   Quasi-Neutral Region (QNR)



3. Summaries of equations:



$$
\begin{aligned}
x_{\mathrm{no}} &= \sqrt{\frac{2\epsilon_{\mathrm{s}}\phi_{\mathrm{B}}N_{\mathrm{a}}}{q(N_{\mathrm{a}}+N_{\mathrm{d}})\cdot N_{\mathrm{d}}}} \\
x_{\mathrm{po}} &= \sqrt{\frac{2\epsilon_{\mathrm{s}}\phi_{\mathrm{B}}N_{\mathrm{d}}}{q(N_{\mathrm{a}}+N_{\mathrm{d}})\cdot N_{\mathrm{a}}}}
\end{aligned}
$$

$x_{\rm no}, x_{\rm po}$ are determined by doping on *both* sides!

$$\begin{aligned}
\text{Built-in potential } \phi_{\rm B} &= \phi_{\rm n} - \phi_{\rm p} = \frac{kT}{q}\ln\frac{N_{\rm a}N_{\rm d}}{n_{\rm i}^2} \\
x_{\rm do} &= x_{\rm no} + x_{\rm po} = \sqrt{\frac{2\epsilon_{\rm s}\phi_{\rm B}}{q}\left(\frac{1}{N_{\rm a}} + \frac{1}{N_{\rm d}}\right)} \\
\text{Maximum field } |E_{\rm max}| &= |E_{\rm o}| = \sqrt{\frac{2q\phi_{\rm B}}{\epsilon_{\rm s}}\frac{N_{\rm a}\cdot N_{\rm d}}{N_{\rm a}+N_{\rm d}}} \\
&= \frac{q\,N_{\rm a}}{\epsilon_{\rm s}}x_{\rm po} = \frac{q\,N_{\rm d}}{\epsilon_{\rm s}}x_{\rm no}
\end{aligned}$$

4. If strongly asymmetric, the lowly doped side controls the electrostatics.

$$p^+n : x_{\rm po} \ll x_{\rm no} \simeq x_{\rm do} = \sqrt{\frac{2\epsilon_{\rm s}\phi_{\rm B}}{q}\frac{1}{N_{\rm d}}} \qquad |E_o| \simeq \sqrt{\frac{2q\phi_{\rm B}}{\epsilon_{\rm s}}N_{\rm d}}$$

Examples

Now let us do some exercises before moving on to the next topic:

$\mathbf{N}_{\rm d}$ [cm^{-3}]	$\mathbf{N}_{\rm a}$ [cm^{-3}]	$\mathbf{x}_{\rm no}$	$\mathbf{x}_{\rm po}$	$\mathbf{E}_{\rm o}$ [V/cm]	$\phi_{\rm B}$
10^{16}	10^{16}	216 nm	216 nm	3.3×10^4 V/cm	720 mV
10^{19}	10^{16}	3.14 $\mathring{A}$	341 nm	5.26×10^4 V/cm	900 mV

$\phi_{\rm B}$ is easier to calculate first.

1. $\phi_{\rm n} = 360\,{\rm mV}, \phi_{\rm p} = -360\,{\rm mV} \quad \phi_{\rm B} = \phi_{\rm n} - \phi_{\rm p} = 720\,{\rm mV}$

$$\begin{aligned}
x_{\rm no} &= \sqrt{\frac{2\epsilon_{\rm s}\phi_{\rm B}}{q(N_{\rm a}+N_{\rm d})}\frac{\cancel{N_{\rm a}}}{\cancel{N_{\rm d}}}} \\
&= \sqrt{\frac{2\times1\times10^{-12}\,({\rm F/cm})\,\times0.72\,({\rm V})}{1.6\times10^{-19}\,({\rm C})\times2\times10^{16}\,({\rm cm}^{-3})}} = \sqrt{4.66}\times10^{-5}\,{\rm cm} = 216\,{\rm nm} \\
x_{\rm po} &= x_{\rm no} \quad \text{(symmetric)} \\
|E_{\rm o}| &= \sqrt{\frac{2q\phi_{\rm B}}{\epsilon_{\rm s}}\frac{N_{\rm a}\cdot N_{\rm d}}{N_{\rm a}+N_{\rm d}}} \\
&= \sqrt{2\times\frac{1.6\times10^{-19}\,({\rm C})\times0.72\,({\rm V})}{1\times10^{-12}\,({\rm F/cm})}\times\frac{10^{16}\times10^{16}\,({\rm cm}^{-3})^2}{2\times10^{16}\,({\rm cm}^{-3})}} = 3.3\times10^4\,{\rm V/cm}
\end{aligned}$$

2. $\phi_{\mathrm{B}} = \phi_{\mathrm{n}} - \phi_{\mathrm{p}} = 540\,\mathrm{mV} + 360\,\mathrm{mV} = 900\,\mathrm{mV}$

$$x_{\mathrm{no}} = \sqrt{\frac{2\epsilon_{\mathrm{s}}\phi_{\mathrm{B}}}{q(N_{\mathrm{a}}+N_{\mathrm{d}})}\frac{N_{\mathrm{a}}}{N_{\mathrm{d}}}} = 3.41\mathring{A} \quad \text{(really thin)}$$

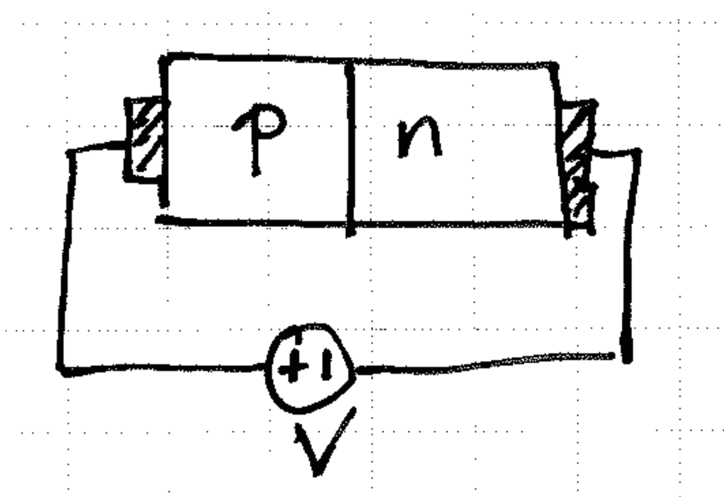
$$x_{\mathrm{po}} = \sqrt{\frac{2\epsilon_{\mathrm{s}}\phi_{\mathrm{B}}}{\cancel{q(N_{\mathrm{a}}+N_{\mathrm{d}})}}\frac{\cancel{N_{\mathrm{d}}}}{N_{\mathrm{a}}}} = \sqrt{\frac{2\epsilon_{\mathrm{s}}\phi_{\mathrm{B}}}{q}\frac{1}{N_{\mathrm{a}}}} = 344\,\mathrm{nm}$$

$$|E_{\mathrm{o}}| = \frac{qN_{\mathrm{a}}}{\epsilon_{\mathrm{s}}x_{\mathrm{po}}} = \frac{1.6\times10^{-19}\,(\mathrm{C})\cdot10^{16}\,(\mathrm{cm}^{-3})}{1\times10^{-12}\,(\mathrm{F/cm})}\times3.41\times10^{-7}\,\mathrm{cm} = 5.26\times10^{4}\,(\mathrm{V/cm})$$

Reverse Bias

Now what happens when we apply a reverse bias?

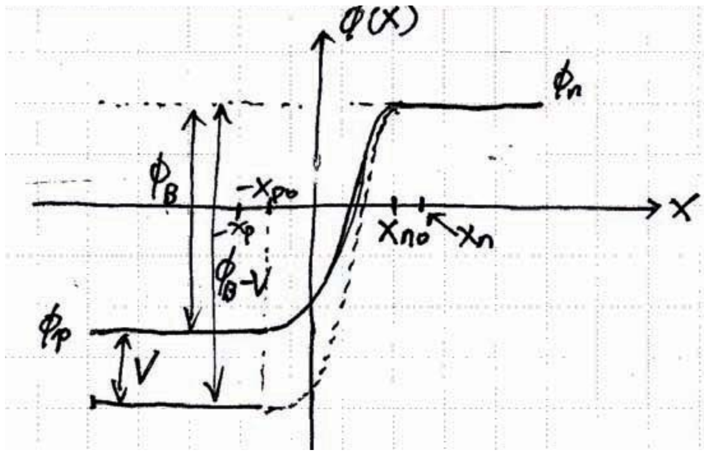
First: Bias convention for pn junction:



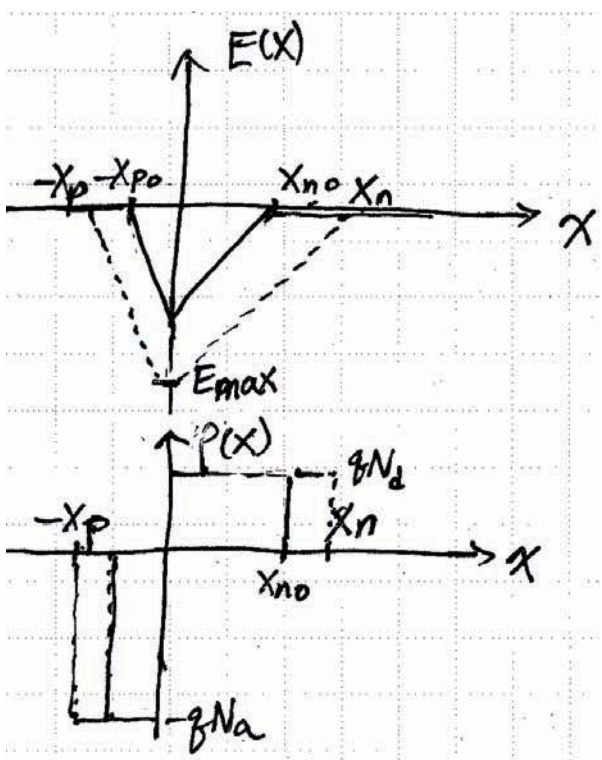
use n-side as reference:
V>0: forward bias
V<0: reverse bias

In your weblab experiment, when you have $V < 0$, what current do you measure? $6\times10^{-15}A$ (very small).
With such a small current, the voltage drop across the QNRs will be really small, safely to ignore.
Then, where is the voltage drop? $\Longrightarrow$ across SCR

Forward bias means $V > 0$, moving "ϕ_p" up and reverse bias means $V < 0$, moving "ϕ_p" down. Reverse bias V: barrier becomes $\phi_\mathrm{B} - V > \phi_\mathrm{B}$ (V is negative, so barrier increases).



Now, what happened to the electric field? Still zero in QNRs. Larger barriers means higher E-field, and $\therefore$ $|E_\mathrm{max}|$ increases. This will mean we need more space charges to support the larger E-field. However, the charge density qN_a or qN_d can not change, $\Longrightarrow$ $x_\mathrm{n}, x_\mathrm{p}$ widens.

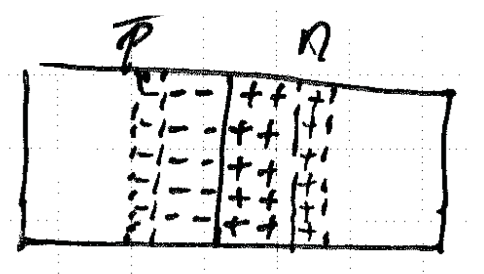


It turns out a straightforward way to do the new calculations is: replace $\phi_{\rm B}$ with $\phi_{\rm B} - V$. So:

$$
\begin{aligned}
x_{\rm n} &= x_{\rm no}\sqrt{\frac{\phi_{rmB} - V}{\phi_{\rm B}}} = x_{\rm no}\sqrt{1 - \frac{V}{\phi_{\rm B}}} \\
&= \sqrt{\frac{2\epsilon_{\rm s}(\phi_{\rm B} - V)}{q(N_{\rm a} + N_{\rm d})}\frac{N_{\rm a}}{N_{\rm d}}} \\
x_{\rm p} &= x_{\rm po}\sqrt{1 - \frac{V}{\phi_{\rm B}}} \\
x_{\rm d} &= x_{\rm do}\sqrt{1 - \frac{V}{\phi_{\rm B}}} \\
E_{\rm max} &= E_{\rm o}\sqrt{1 - \frac{V}{\phi_{\rm B}}}
\end{aligned}
$$

The concept of depletion capacitance

Now let us think about the junction region a bit further.



1. It stores charges
2. By changing the voltage (bias) across the junction, the amount of charges stored there changes $\Longrightarrow$ Capacitor!
3. The charges that it stores:

$$Q_{\rm jo} = |-q\,N_{\rm a}\cdot x_{\rm po}| = |q\,N_{\rm d}\cdot x_{\rm no}| = \sqrt{\frac{2q\epsilon_{\rm s}N_{\rm a}N_{\rm d}}{N_{\rm a}+N_{\rm d}}\cdot\phi_{\rm B}}$$

As voltage changes, $Q_{\rm j}$ becomes:

$$Q_{\rm j}(V) = \sqrt{\frac{2q\epsilon_{\rm s}N_{\rm a}N_{\rm d}}{N_{\rm a}+N_{\rm d}}(\phi_{\rm B} - V)} = Q_{\rm jo}\sqrt{1 - \frac{V}{\phi_{\rm B}}}.$$

Previously, we had $C = \frac{Q}{V}$, but that was for constant capacitance, a more rigorous definition should be:

$$C = \frac{dQ}{dV}$$

Therefore,

$$\begin{aligned}
C_{\mathrm{j}} &= C_{\mathrm{j}}(V_{\mathrm{D}}) = \left|\frac{dQ_{\mathrm{j}}}{dV}\right|\Big|_{V_{\mathrm{D}}} = \left|Q_{\mathrm{jo}}\frac{d(\sqrt{1-\frac{V}{\phi_{\mathrm{B}}}})}{dV}\right|\Big|_{V_{\mathrm{D}}} \\
&= \frac{+qN_{\mathrm{a}}x_{\mathrm{po}}}{2\sqrt{1-\frac{V_{\mathrm{D}}}{\phi_{\mathrm{B}}}}} \cdot \frac{1}{\phi_{\mathrm{B}}} \\
&= \frac{qN_{\mathrm{a}}x_{\mathrm{po}}}{2\phi_{\mathrm{B}}} \cdot \frac{1}{\sqrt{1-\frac{V_{\mathrm{D}}}{\phi_{\mathrm{B}}}}} \\
C_{\mathrm{j}}(V=0) &= C_{\mathrm{jo}} = \frac{qN_{\mathrm{a}}x_{\mathrm{po}}}{2\phi_{\mathrm{B}}} \\
&= \frac{qN_{\mathrm{a}}}{2\phi_{\mathrm{B}}}\sqrt{\frac{2\epsilon_{\mathrm{s}}\phi_{\mathrm{B}}}{q(N_{\mathrm{a}}+N_{\mathrm{d}})} \times \frac{N_{\mathrm{d}}}{N_{\mathrm{a}}}} = \sqrt{\frac{q\epsilon_{\mathrm{s}}}{2\phi_{\mathrm{B}}}\frac{N_{\mathrm{a}}N_{\mathrm{d}}}{N_{\mathrm{a}}+N_{\mathrm{d}}}} \\
\because x_{\mathrm{do}} &= x_{\mathrm{no}} + x_{\mathrm{po}} = \sqrt{\frac{2\epsilon_{\mathrm{s}}\phi_{\mathrm{B}}}{q}\frac{N_{\mathrm{a}}+N_{\mathrm{d}}}{N_{\mathrm{a}}N_{\mathrm{d}}}} \\
\Longrightarrow C_{\mathrm{jo}} &= \frac{\epsilon_{\mathrm{s}}}{x_{\mathrm{do}}}, C_{\mathrm{j}}(V_{\mathrm{D}}) = \frac{C_{\mathrm{jo}}}{\sqrt{1-\frac{V_{\mathrm{D}}}{\phi_{\mathrm{B}}}}} \\
C_{\mathrm{j}}(V_{\mathrm{D}}) &= \frac{\epsilon_{\mathrm{s}}}{x_{\mathrm{do}}\sqrt{1-\frac{V_{\mathrm{D}}}{\phi_{\mathrm{B}}}}} = \frac{\epsilon_{\mathrm{s}}}{x_{\mathrm{d}}}
\end{aligned}$$

Can be considered as "parallel plate capacitor".