Recitation 5: Review on Electrostatics

This review is aimed at getting ready for p-n junction. Before the class, have an exercise on the Boltzman relationship:

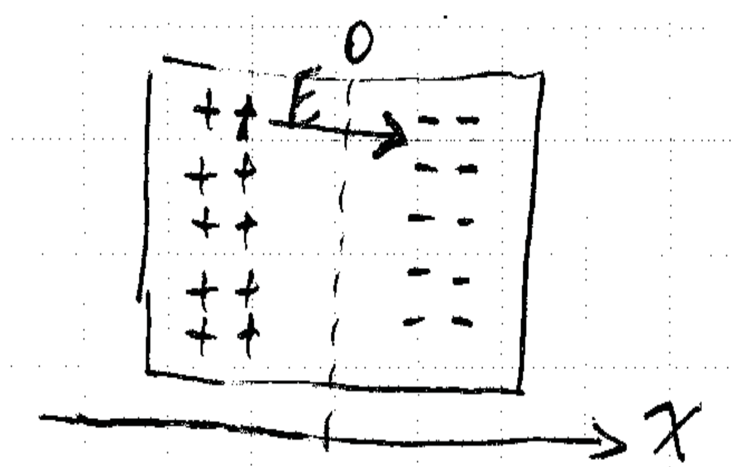
1. If the doping of Si is 10^{16} Boron, what should be the corresponding electrostatic potential be? ($\phi = -6 \times 60 = -360\,\text{mV}$)
2. If $\phi = 480\,\text{mV}$, what is equilibrium electron concentration? ($\frac{480}{60} = 8, n_o = 10^{18}\,\text{cm}^{-3}$)
3. If doping is $N_d = 10^{20}\,\text{cm}^{-3}, \phi =?$ ($\phi \neq 600\,\text{mV}, \phi = \phi_{\text{max}} = 550\,\text{mV}$)

The following is the set of equations which relates $\rho(x), E(x), \phi(x)$ needed for this class (everything is 1D):

1. Relating charge density to the field:

$$\frac{dE}{dx} = \frac{\rho(x)}{\epsilon}$$
$$\text{or } E(x) - E(0) = \frac{1}{\epsilon}\int_0^x \rho(x')\,dx'$$

   $\rho(x)$ = charge density in coloumb/cm^3, ϵ is permittivity or dielectric constant of the material in F/cm, ϵ_o(vacuum) $= 8.85 \times 10^{-14}$ F/cm.



2. Relating electrostatic potential to the field:

$$\frac{d\phi}{dx} = -E(x)$$
$$\text{or } \phi(x) - \phi(0) = -\int_0^x E(x')\,dx'$$

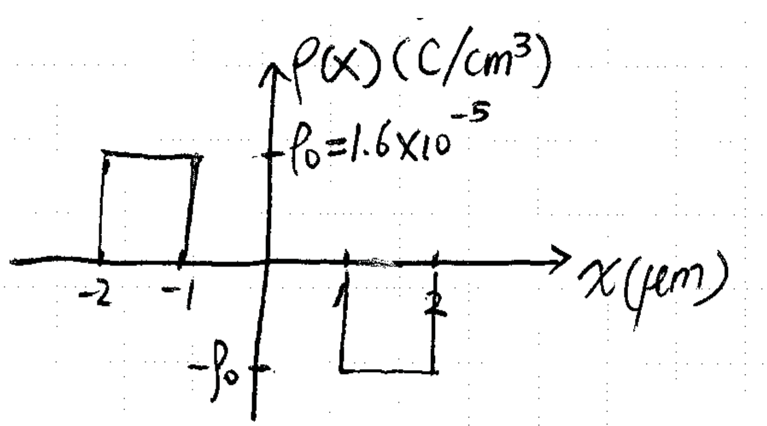
   $E(x)$ has units V/cm and $\phi(x)$ has units of V or mV.

3. Boundary conditions:

   - Continuation of potential at a boundary (infinite field inside semiconductor not possible)
   $$\phi(x_b^-) = \phi(x_b^+)$$
   - E-field at the boundary can (usually) have a jump, due to:
     - Materials change:
     $$\begin{aligned}\int_{-\Delta}^{\Delta} d(\epsilon E(x)) &= \epsilon_2 E(x=\Delta) - \epsilon_1 E(x=-\Delta) \\ &= \int_{-\Delta}^{\Delta} \rho(x)\,dx = 0 \text{ no charge at interface} \\ E(0^+) &= \frac{\epsilon_1}{\epsilon_2} E(0^-)\end{aligned}$$
     - A sheet of charge at the interface:
     $$\begin{aligned}\int_{-\Delta}^{\Delta} d(\epsilon E(x)) &= \epsilon_2 E(x=\Delta) - \epsilon_1 E(x=-\Delta) \\ &= \int_{-\Delta}^{\Delta} \rho(x)\,dx = Q \text{ coulomb/cm}^2 \\ E(0^+) &= \frac{\epsilon_1}{\epsilon_2} E(0^-) + \frac{Q}{\epsilon_2}\end{aligned}$$

Examples

Like in Figure 1



Then, let us work out $E(x)$, by splitting calculations into 4 regions:

1. $E(x \leq -2\,\mu\text{m}) = \text{E}(\text{x} \geq 2\,\mu\text{m})$ ($\because$ no charge outside these regions)

2. For $-2\,\mu\text{m} < \text{x} < -1\,\mu\text{m}$:

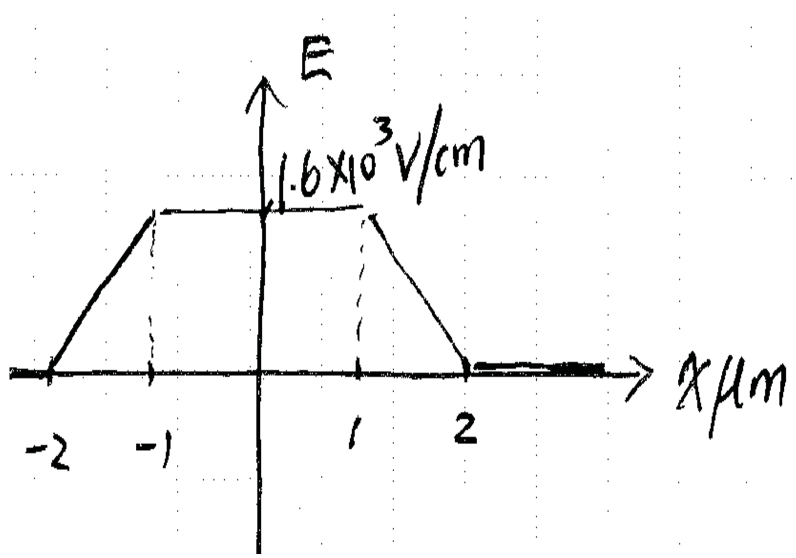
$$\begin{aligned} E(x) - E(-2\,\mu\text{m}) &= \frac{1}{\epsilon_s}\int_{-2}^{x}\rho(x)\,dx = \frac{1}{\epsilon_s}\rho_o\cdot(x+2) \\ \rho_o &= 1.6\times10^{-5}\,\text{C/cm}^3 = 1.6\times10^{-17}\,\text{C/cm}^3 \\ \text{Particularly}, E(-1\,\mu\text{m}) &= \frac{1}{\epsilon_s}\rho_o\times1\,\mu\text{m} \\ \text{Since}\,\epsilon_s &= 1\times10^{-12}\,\text{F/cm}, \\ E(-1\,\mu\text{m}) &= \frac{1.6\times10^{-5}\times10^{-4}\,\text{C/cm}^3\times\,\text{cm}}{1\times10^{-12}\,\text{F/cm}} = 1.6\times10^{3}\,\text{V/cm} \end{aligned}$$

3. For $-1 < x < 1\,\mu$m:

$$\begin{aligned} \rho(x) &= 0, \text{same dielectric material} \implies E \text{ constant} \\ E(x) &= E(-1\,\mu\text{m}) = 1.6\times10^{3}\,\text{V/cm} \end{aligned}$$

4. For $1 < x < 2\,\mu$m:

$$\begin{aligned} E(x) - E(1\,\mu\text{m}) &= \frac{1}{\epsilon_s}\int_{1}^{x}\rho(x)\,dx = -\frac{1}{\epsilon_s}\rho_o(x-1) \\ \text{particularly}, E(2\,\mu\text{m}) &= E(1\,\mu\text{m}) - \frac{1}{\epsilon_\text{s}}\rho_\text{o}(\text{x}-1)\Big|_{\text{x}=2\,\mu\text{m}} = 0 \end{aligned}$$



Now for the electrostatic potential:
As we do the integration, the results will be relative. Then we can use the doping of Si to find the actual value. For this thought-example, let us make $\phi(0) = 0$.
For $0 < x < 1\,\mu\text{m}, \text{E(x)} = \text{constant} = 1.6 \times 10^3\,\text{V/cm}$.

$$\begin{aligned}\phi(x) - \phi(0) &= -\int_0^x E(x')\,dx' = -1.6 \times 10^3 \cdot x\ (\text{V/cm} \cdot \mu\text{m}) \\ \text{particularly}\,\phi(1\,\mu\text{m}) &= -0.16V\end{aligned}$$

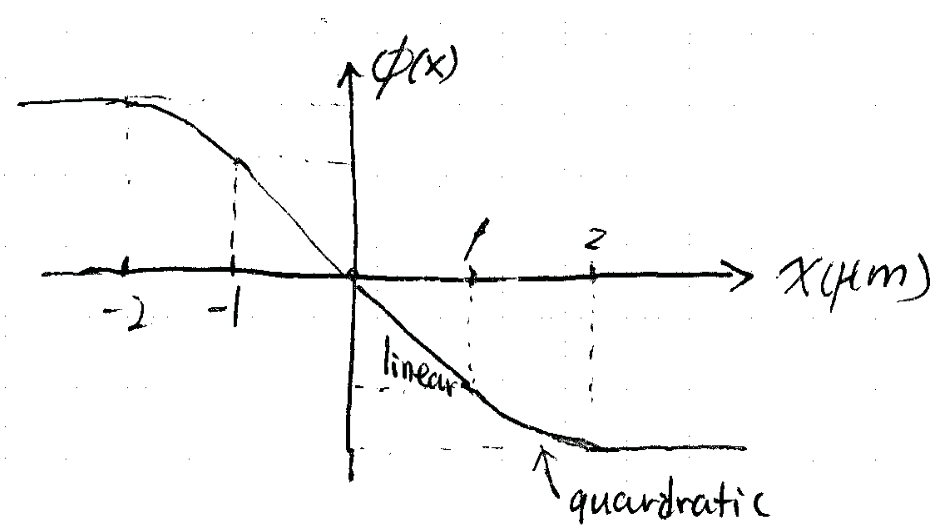
For $1 < x < 2\,\mu\text{m}$,

$$\begin{aligned}\phi(x) - \phi(1\,\mu\text{m}) &= -\int_1^x E(x')\,dx' \\ &= -\int_1^x (1.6 \times 10^3\text{V/cm} - \frac{1}{\epsilon_\text{s}}\rho_\text{o}(\text{x}' - 1))\,\text{dx}' \\ &= -1.6 \times 10^3\,\text{V/cm}\,(\text{x} - 1) + \frac{1}{2\epsilon_\text{s}}\rho_\text{o}(\text{x} - 1)^2\ \text{quadratic} \\ \text{particularly,}\,\phi(2\,\mu\text{m}) &= \phi(1\,\mu\text{m}) - 1.6 \times 10^3\,\text{V/cm} \cdot (1 \times 10^{-4})\,\text{cm} + \frac{1}{2}\frac{1.6 \times 10^{-5}\,\text{C/cm}^3}{1 \times 10^{-12}\,\text{F/cm}} \cdot (1 \times 10^{-8}\,\text{cm}^2) \\ &= -0.16V - 0.08V = -0.24V\end{aligned}$$

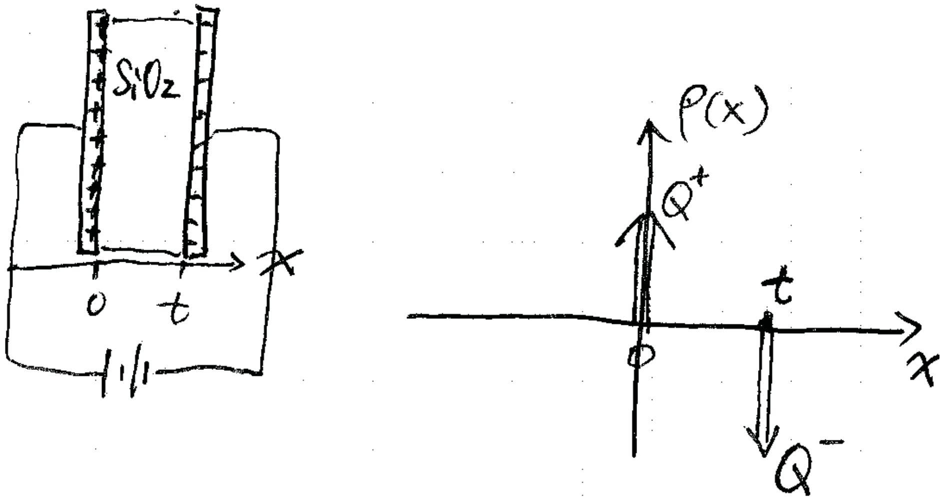
For $x > 2\,\mu\text{m}$:

$$\because E(x) = 0 \text{ electrostatic potential will be constant}$$

Similarly, we can work out the other side:



Parallel Plate Capacitor

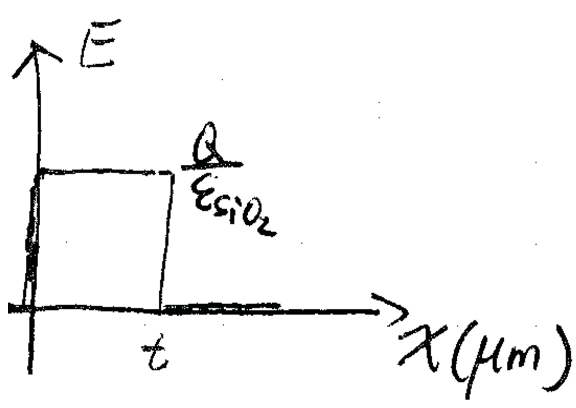


Two sheets of charge at the interface of the capacitor Q+/Q-
In 1D, this can be modeled as δ function

Now let us consider the electric field:
Inside the metal, no charge, no field, $\implies E(x<0) = E(x>t_d) = 0$:

$$\begin{aligned}
\epsilon_{\mathrm{SiO_2}} - 0 &= \int_{0^-}^{0^+} \rho(x')\,dx' = \int_{0^-}^{0^+} \delta(x)\,dx = Q \\
\therefore E(0^+) &= \frac{Q}{\epsilon_{\mathrm{Si0_2}}} \\
\epsilon_{\mathrm{Si0_2}} &= 3.45 \times 10^{-13}\,\mathrm{F/cm}
\end{aligned}$$

As there is no charge inside $\mathrm{SiO_2}$ region, E is constant $E(x) = \frac{Q}{\epsilon_{\mathrm{SiO_2}}}$ for $0 < x < t$



If we use $\phi(0) = 0$ again,