Recitation 4: Electrostatic Potential & Carrier Concentration

Yesterday in lecture, we learned that under T.E. (thermal equilibrium), there is a fundamental relationship between the electrostatic potential $\phi(x)$, at one location (x) within the semiconductor and the carrier concentration at that location.

$$\phi(x) = \frac{kT}{q} \ln \frac{n_o(x)}{n_i} = -\frac{kT}{q} \ln \frac{p_o(x)}{n_i}$$

By the Boltzman relationship (or the 60 mV rule),

$$\phi(x) = (60\,\mathrm{mV}) \ln \frac{\mathrm{n_o(x)}}{\mathrm{n_i}} = -(60\,\mathrm{mV}) \ln \frac{\mathrm{p_o(x)}}{\mathrm{n_i}}$$

How did this relation come about?

Revisit Thermal Equilibrium

1. Under T.E. can we have electrostatic field (or voltage) within the semiconductor? Yes. (we do not have "external" energy source). There can be *static* electrostatic field inside the semiconductor generated by "space" charges.

2. Under T.E. can we have an overall current? No. That will give rise to charge piling up.

Some Fundamentals about Electrostatics

Relationship between electrostatic potential $\phi(x)$, electric field $E(x)$, and (space) charge density $\rho(x)$:

1. First, the charge density we are talking about here is the "*Net*" charge density, we call *space charge density.*

$$\begin{aligned} \text{in n-type material: } \rho(x) &= q(N_d(x) - n(x)) \\ \text{p-type material: } \rho(x) &= q(p(x) - N_a(x)) \end{aligned}$$

   (N_d, N_a are space charges. $\rho(x)$ is the extra which can not be compensated by e^- or hole free charges) $\rightarrow$ *space* charge density.

2. $\dfrac{dE}{dx} = \dfrac{\rho}{\epsilon}$ (ϵ = electric permitivity Farad/cm). In other words,

$$E(x) - E(0) = \frac{1}{\epsilon} \int_0^x e(x')\, dx'$$

Also, $\dfrac{d\phi}{dx} = -E(x)$. In other words:

$$\phi(x) - \phi(0) = -\int_0^x E(x')\,dx'$$

The two equations above can be combined to give the following relation:

$$\frac{d^2\phi(x)}{dx^2} = -\frac{dE(x)}{dx} = -\frac{\rho(x)}{\epsilon}$$

Boltzman Relation

Basically the Boltzman relationship exists due to *thermal equilibrium*. Under T.E., for n-type:

$$\begin{aligned}
J_n &= 0 \\
\text{But } J_{\text{n}} &= q \cdot n_{\text{o}} \cdot \mu_{\text{n}} \cdot E + q \cdot D_{\text{n}} \cdot \frac{dn_{\text{o}}}{dx} = 0 \\
-q \cdot n_{\text{o}} \cdot \mu_{\text{n}} \cdot \frac{d\phi}{dx} &= -q \cdot D_{\text{n}} \cdot \frac{dn_{\text{o}}}{dx} \\
\frac{\mu_{\text{n}}}{D_{\text{n}}} \frac{d\phi}{dx} &= \frac{1}{n_{\text{o}}} \frac{dn_{\text{o}}}{dx} \\
\frac{q}{k \cdot T} \frac{d\phi}{dx} &= \frac{d(\ln(n_{\text{o}}))}{dx} \\
\text{Integrate: } \frac{q}{k \cdot T}(\phi - \phi_{\text{ref}}) &= \ln(n_{\text{o}}) - \ln(n_{\text{o,ref}}) = \ln\left(\frac{n_o}{n_{\text{o,ref}}}\right)
\end{aligned}$$

Similarly, for p-type:

$$\begin{aligned}
J_p &= 0 \\
\text{But } J_{\text{p}} &= q \cdot p_{\text{o}} \cdot \mu_{\text{p}} \cdot E - q \cdot D_{\text{p}} \cdot \frac{dp_{\text{o}}}{dx} = 0 \\
-q \cdot p_{\text{o}} \cdot \mu_{\text{p}} \cdot \frac{d\phi}{dx} &= q \cdot D_{\text{p}} \cdot \frac{dp_{\text{o}}}{dx} \\
-\frac{\mu_{\text{p}}}{D_{\text{p}}} \frac{d\phi}{dx} &= \frac{1}{p_{\text{o}}} \frac{dp_{\text{o}}}{dx} \\
-\frac{q}{k \cdot T} \frac{d\phi}{dx} &= \frac{d(\ln(p_{\text{o}}))}{dx} \\
\text{Integrate: } -\frac{q}{k \cdot T}(\phi - \phi_{\text{ref}}) &= \ln(p_{\text{o}}) - \ln(p_{\text{o,ref}}) = \ln\left(\frac{p_o}{p_{\text{o,ref}}}\right)
\end{aligned}$$

Set $\phi_{\text{ref}} = 0$ at $n_{\text{o, ref}} = p_{\text{o, ref}} = n_i$. Then:

$$\begin{aligned}
-\frac{q}{k \cdot T}\phi &= \ln\left(\frac{p_{\text{o}}}{n_{\text{i}}}\right) \\
\phi &= -\frac{k \cdot T}{q} \ln\left(\frac{p_{\text{o}}}{n_{\text{i}}}\right) \\
\text{or } p_{\text{o}} &= n_{\text{i}} e^{-\frac{q\phi}{k \cdot T}}
\end{aligned}$$

Example

Now let us look at a particular example. We have a doping profile $N_d(x) = N_{\mathrm{do}} + \Delta N_{\mathrm{d}}(1 - \mathrm{e}^{-fracxL_{\mathrm{c}}})$. $N_{\mathrm{do}} = 10^{16}\,\mathrm{cm}^{-3}$, $\Delta\mathrm{N_d} = 9 \times 10^{16}\,\mathrm{cm}^{-3}$, $\mathrm{L_c} = 10\,\mu\mathrm{m}$. We would like to know:

1. What is the electrostatic profile $\phi(x)$?
2. How about electric field $E(x)$?
3. Space charge density $\rho(x)$?

First, we have $\phi(x)$ vs. $n_{\mathrm{o}}(x)$, $p_{\mathrm{o}}(x)$ from the Boltzman relationships. If we know $n_{\mathrm{o}}(x)$, or p_{o}, we can find $\phi(x)$.
But does $n_{\mathrm{o}}(x) = N_{\mathrm{d}(x)}$? In reality, it should not, if $n_{\mathrm{o}}(x) = N_{\mathrm{d}(x)}$, we will have a net current due to diffusion Not T.E. anymore.
To obtain an accurate solution, we have:

$$\begin{aligned} J_{\mathrm{n}} &= q \cdot n_{\mathrm{o}} \cdot \mu_{\mathrm{n}} \cdot E + q \cdot D_{\mathrm{n}} \cdot \frac{dn_{\mathrm{o}}}{dx} = 0 \text{ (} N_{\mathrm{d}} \text{ doping, electron majority carrier, only consider } J_{\mathrm{n}} \text{ here.)} \\ \frac{dE}{dx} &= \frac{q}{\epsilon_{\mathrm{Si}}}(N_{\mathrm{d}} - n_{\mathrm{o}}) \end{aligned}$$

With these two, we get:

$$E = -\frac{D_{\mathrm{n}}}{\mu_{\mathrm{n}}}\frac{1}{n_{\mathrm{o}}}\frac{dn_{\mathrm{o}}}{dx} \implies \frac{k \cdot T}{q}\frac{d^2(\ln n_{\mathrm{o}})}{dx^2} = \frac{1}{\epsilon_{\mathrm{Si}}}(n_{\mathrm{o}}(x) - N_{\mathrm{d}})$$

But very hard to solve $n_{\mathrm{o}}(x)$. In most cases, an analytical solution is impossible. Can we do something simpler?
We make *approximations*! The first type of scenario is $n_{\mathrm{o}}(x) \simeq N_{\mathrm{d}}$ ("quasi-neutrality"). If we assume $n_{\mathrm{o}}(x) \simeq N_{\mathrm{d}} = N_{\mathrm{do}} + \Delta N_{\mathrm{d}}(1 - \mathrm{e}^{\frac{-x}{L_{\mathrm{c}}}})$:

$$\begin{aligned} \text{Define } a &\triangleq N_{\mathrm{do}} + \Delta N_{\mathrm{d}}(1 - \mathrm{e}^{\frac{-x}{L_{\mathrm{c}}}}) \\ \phi(x) &= \frac{k \cdot T}{q}\ln\frac{n_{\mathrm{o}}(x)}{n_{\mathrm{i}}} \simeq \frac{k \cdot T}{q}\ln\frac{a}{n_{\mathrm{i}}} \\ E(x) &= -\frac{d\phi(x)}{dx} \simeq -\frac{k \cdot T}{q}\frac{1}{a}\Delta N_{\mathrm{d}}\frac{1}{L_{\mathrm{c}}}\mathrm{e}^{-\frac{x}{L_{\mathrm{c}}}} \\ \rho(x) &= \epsilon_{\mathrm{Si}}\frac{dE(x)}{dx} \simeq \epsilon_{\mathrm{Si}}\frac{k \cdot T}{q}\left(\frac{1}{a^2}\Delta N_{\mathrm{d}}^2\frac{1}{L_{\mathrm{c}}^2}\mathrm{e}^{\frac{-2x}{L_{\mathrm{c}}}} + \frac{1}{a}\Delta N_{\mathrm{d}}\frac{1}{L_{\mathrm{c}}^2}\mathrm{e}^{-\frac{x}{L_{\mathrm{c}}}}\right) \\ &\simeq \epsilon_{\mathrm{Si}}\frac{k \cdot T}{q}\frac{1}{a^2}\left(\frac{\Delta N_{\mathrm{d}}(N_{\mathrm{do}} + \Delta N_{\mathrm{d}})}{L_{\mathrm{c}}^2}\right)\mathrm{e}^{-\frac{x}{L_{\mathrm{c}}}} \end{aligned}$$

To satisfy quasi-neutrality, we need:

$$\begin{aligned}
\left|\frac{n_\mathrm{o}(x) - N_\mathrm{d}(x)}{N_\mathrm{d}(x)}\right| &\ll 1, \text{ we know} \,(n_\mathrm{o}(x) - N_\mathrm{d}(x)) = -\frac{\rho(x)}{q} \\
\left|\frac{n_\mathrm{o}(x) - N_\mathrm{d}(x)}{N_\mathrm{d}(x)}\right| &= \left|\frac{\rho(x)}{q \cdot N_\mathrm{d}(x)}\right| \simeq \epsilon_\mathrm{Si}\frac{k \cdot T}{q^2}\frac{\Delta N_\mathrm{d}(N_\mathrm{do} + \Delta N_\mathrm{d})}{a^3}\frac{1}{L_\mathrm{c}^2}\mathrm{e}^{-\frac{x}{L_\mathrm{c}}} \\
\mathrm{e}^{-\frac{x}{L_\mathrm{c}}} &\ll 1 \,(\text{for } x > 0), \text{ and } \frac{\Delta N_\mathrm{d}(N_\mathrm{do} + \Delta N_\mathrm{d})}{a^3} < \frac{\Delta N_\mathrm{d}(N_\mathrm{do} + \Delta N_\mathrm{d})}{(N_\mathrm{do})^3} \\
\left|\frac{n_\mathrm{o}(x) - N_\mathrm{d}(x)}{N_\mathrm{d}(x)}\right| &< \epsilon_\mathrm{Si}\frac{k \cdot T}{q^2}\frac{\Delta N_\mathrm{d}(N_\mathrm{do} + \Delta N_\mathrm{d})}{(N_\mathrm{do})^3}\frac{1}{L_\mathrm{c}^2} = 1.46 \times 10^{-4} \ll 1
\end{aligned}$$

Therefore, our *quasi-neutrality* is valid. This quasi-neutrality satisfies when doping profile is gradual. If we have time, we can verify $J_\mathrm{n}^\mathrm{diff} = q \cdot D_\mathrm{n}\frac{dn_\mathrm{o}(x)}{dx} = J_\mathrm{n}^\mathrm{drift} = q\mu_\mathrm{n} n_\mathrm{o}(x)E(x)$ using the above equations.

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