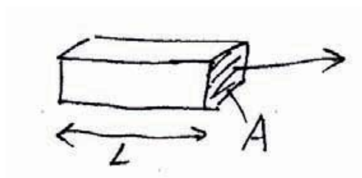
Recitation 3: Carrier Action

Yesterday we talked about the movement of the carriers inside a semiconductor. There is a direct relationship between the velocity of carriers and the electrical current that is generated.

$$\begin{aligned}\text{Current Density} &= |J_n| = |n \cdot q \cdot v_n| \\ &= |J_p| = |p \cdot q \cdot v_p|\end{aligned}$$

This is because:



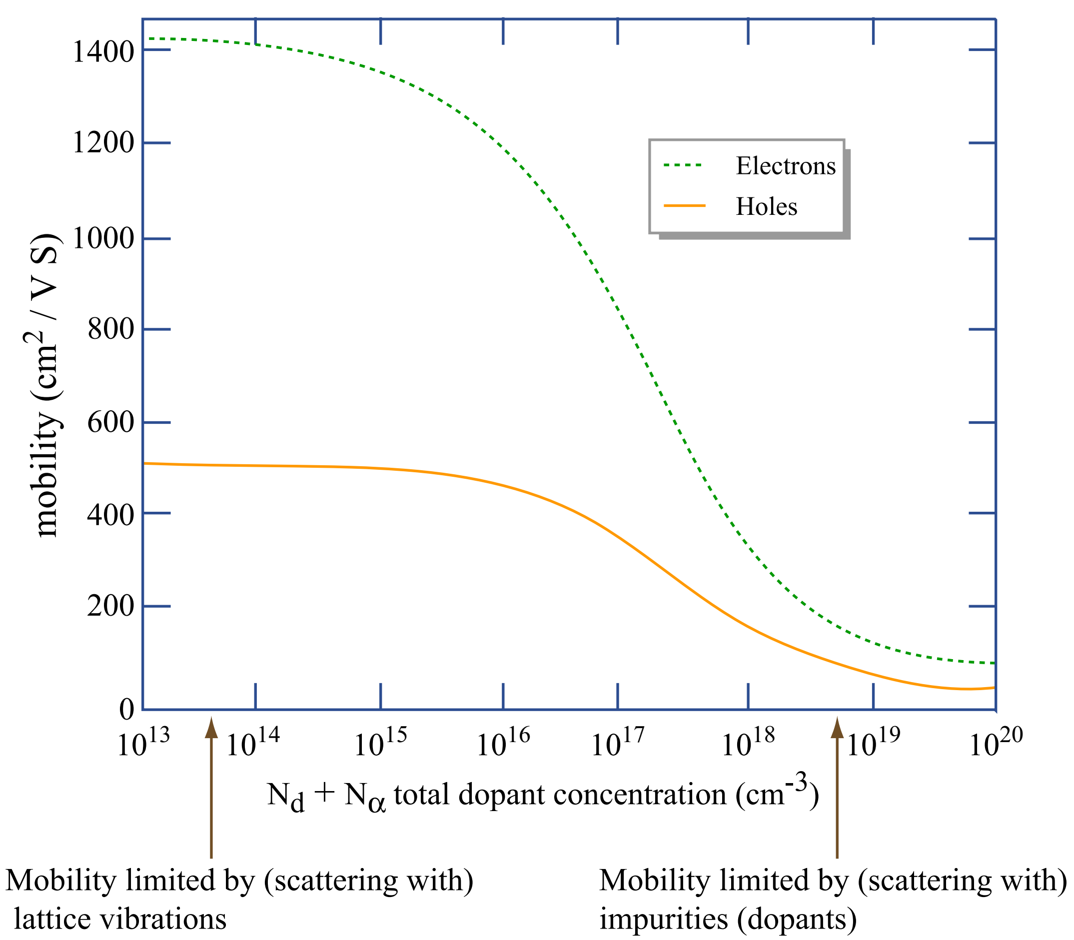
$$\begin{aligned}|I| &= \left|\frac{\text{\# of charges across cross-section area}}{\text{time}}\right| = \left|\frac{Q}{t}\right| = \left|\frac{q \cdot \text{\# of charges across cross-section area}}{t}\right| \\ &= \left|\frac{q \cdot \text{\# density} \cdot \text{volume}}{t}\right| = \left|\frac{q \cdot n \cdot L \cdot A}{t}\right| = |q \cdot n \cdot v_n \cdot A| \because \tfrac{L}{t} = \text{velocity} \\ \left|\frac{I_n}{A}\right| &= |J_n| = |q \cdot n \cdot v_n|\end{aligned}$$

Table 1: Drift vs. Diffusion

	Drift	**Diffusion**
Mechanism	Due to electric field E	Due to concentration gradient $\frac{dn}{dx}$ $\frac{dp}{dx}$
Carrier Velocity	$v_n = \mu_n E$ $v_p = \mu_p E$	Flux $F_n = -D_n \frac{dn}{dx}$ $F_p = -D_p \frac{dp}{dx}$ $v_n = \frac{F_n}{n}$ $v_p = \frac{F_p}{p}$
Current Density	$J_n = -q \cdot n \cdot v_n = q \cdot n \cdot \mu_n \cdot E$ $J_p = q \cdot p \cdot v_p = q \cdot p \cdot \mu_p \cdot E$	$J_n = -q \cdot F_n = q \cdot D_n \cdot \frac{dn}{dx}$ $J_p = q \cdot F_p = -q \cdot D_p \cdot \frac{dp}{dx}$
Important Parameter	Mobility μ_n, μ_p $\mu_n = \frac{q \cdot \tau_c}{2 \cdot m_n}$ $\mu_p = \frac{q \cdot \tau_c}{2 \cdot m_p}$	Diffusion Coefficient D_n, D_p $\frac{D_p}{\mu} = \frac{kT}{q}$

Note: ⋆⋆ ***physical intuition rather than remembering the equation***

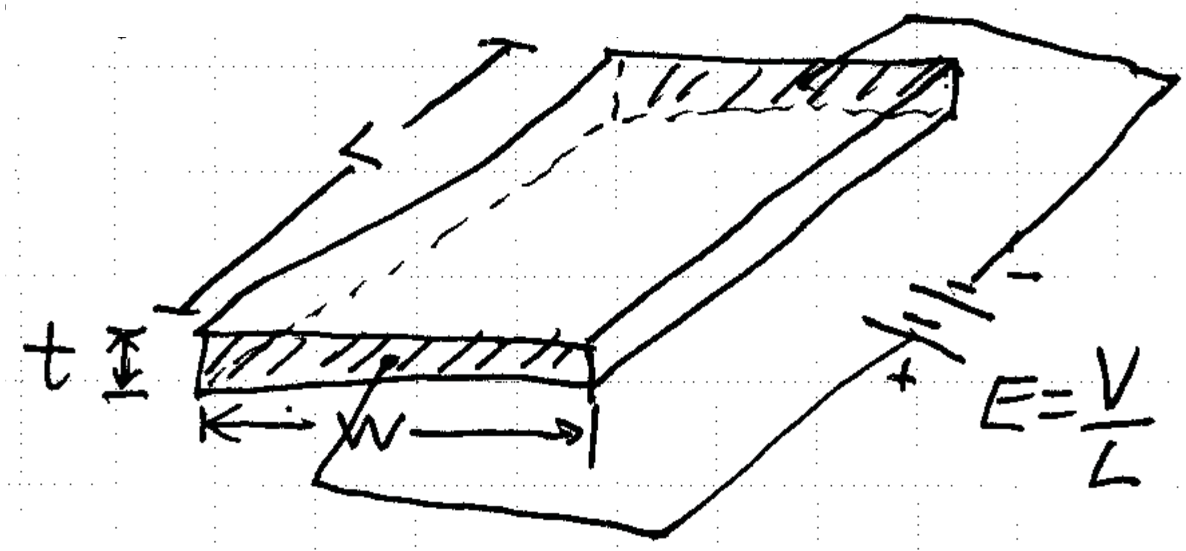
- Current (can usually measure) always related to charge velocity (can back calculate)
- τ_c (collision time) is related to the **imperfection** of the lattice
- Mobility depends on collision time and temperature:
    1. $\mu \propto \tau_c$: doping(impurity) increases $\Longrightarrow$ more collisions $\Longrightarrow$ $\mu \downarrow$
    2. temperature (lattice vibration): higher T $\Longrightarrow$ more collision $\Longrightarrow$ $\mu \downarrow$
- same semiconductor, the difference between μ_n & μ_p are due to m_n & m_p
- high mobility is extremely important for high performance devices
  Si: $\mu_n = 1400\,\text{cm}^2/\text{V}\cdot\text{sec}$ $\mu_\text{p} = 500\,\text{cm}^2/\text{V}\cdot\text{sec}$ for doping $10^{13}\,\text{cm}^{-3}$
  GaAs: $\mu_n = 8000\,\text{cm}^2/\text{V}\cdot\text{sec}$ $\mu_\text{p} = 400\,\text{cm}^2/\text{V}\cdot\text{sec}$



Carrier Mobility vs. Doping Concentration in Si

Example 1: Integrated Resistor

Our first IC device:



Area: w x t
n-type slab of Si
Fabricated by ion implantation from top
As long as doping is uniform along L, no diffusion along L
Therefore, only drift current along L
Note: doping need not be uniform along t (as you will see)

$$
\begin{aligned}
J &= J_n + J_p = q(n\mu_n + p\mu_p)E \\
E &= \frac{V}{L},\ A = w \cdot t \\
I &= J \cdot A = \left(q(n\mu_n + p\mu_p)\frac{V}{L} \right) \cdot (w \cdot t) \\
I &= \left[q(n\mu_n + p\mu_p)\frac{w \cdot t}{L} \right] \cdot V \\
\text{But } I &= \frac{V}{R} \quad \text{Ohm's Law} \\
\therefore R &= \frac{1}{\left[q(n\mu_n + p\mu_p)\frac{w \cdot t}{L}\right]} = \frac{1}{q(n\mu_n + p\mu_p)} \cdot \frac{L}{w \cdot t} = \rho \cdot \frac{L}{w \cdot t}
\end{aligned}
$$

$$\text{Resistivity } = \rho = \frac{1}{q(n\mu_n + p\mu_p)} \text{ or } \sigma = q(n\mu_n + p\mu_p)$$

Usually majority dominates resistivity (n-type majority $\Longrightarrow \rho \approx \frac{1}{q \cdot n \cdot \mu_n}$, and vice versa). Since ρ (or σ) can be measured easily, it can be used to derive doping of a semiconductor (n or p). If we take a Si wafer, it will be hard to know the doping *a priori* unless someone specifies the doping level, but we can use resistivity to find out.

Example 2: Resistivity of Si

What is the resistivity of (1) intrinsic Si, (2) Si with $N_d = 10^{13}$ and (3) Si with $N_a = 10^{20}$?

1. $n_o = p_o = 10^{10}\,\text{cm}^{-3}$. Therefore, ρ is:

$$= \frac{1}{1.6 \times 10^{-19}\,\text{C}(1450\,\text{cm}^2/\text{V}\cdot\text{sec} \times 10^{10}\,\text{cm}^{-3} + 500\,\text{cm}^2/\text{V}\cdot\text{sec} \times 10^{10}\,\text{cm}^{-3})}$$

$$= \frac{1}{1.6 \times 10^{-19} \times 1.95 \times 10^{13}} = 3.2 \times 10^5\,\Omega\cdot\text{cm} \text{ (make sure the units are correct)}$$

   Poor conductivity, quite insulating

2. $N_d = 10^{13}\,\text{cm}^{-3} \gg \text{n}_\text{i} = 10^{10} \Longrightarrow \text{n}_\text{o} \approx \text{N}_\text{d} = 10^{13}$, $\text{p}_\text{o} = \frac{\text{n}_\text{i}^2}{\text{n}_\text{o}} = 10^7$

   $\rho = \frac{1}{q(n \cdot \mu_n + p \cdot \mu_p)} = \frac{1}{1.6 \times 10^{-19} \times (1450 \times 10^{13} + 500 \times 10^7)} = 430\,\Omega\cdot\text{cm}$
   (check on the curve)

3. $N_a = 10^{20} \gg n_i = 10^{10}$, $\Longrightarrow p_o \approx N_a = 10^{20}$, $n_o = \frac{n_i^2}{p_o} = 1$

   $\rho \approx \frac{1}{q \cdot p \cdot \mu_p} = \frac{1}{1.6 \times 10^{-19} \times 50 \times 10^{20}} = 1.25 \times 10^{-3}\,\Omega\cdot\text{cm}$ like metal

From this example, we can see that Si resistivity can be tuned several orders of magnitude by doping, from insulator-like to metal-like.

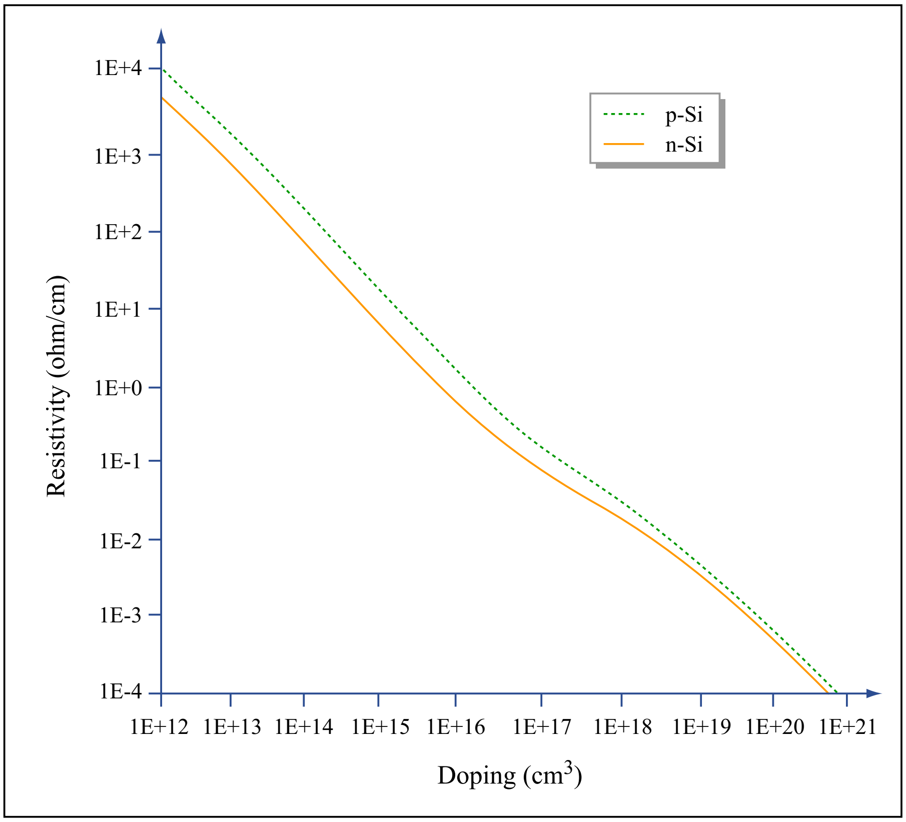


Figure by MIT OpenCourseWare.

Sheet Resistance

$$R = \left(\frac{\rho}{t}\right)\left(\frac{L}{w}\right)$$

The unit of ρ is $\Omega \cdot$ cm, the unit of t is cm meaning that the unit of $\left(\frac{\rho}{t}\right)$ is Ω - that of resistance. We call $\left(\frac{\rho}{t}\right)$ the sheet resistance R_s. This is a convenient metric for IC design as:

- ρ, t: process and material parameters
- $\frac{L}{w}$: # of squares with dimensions w - layout design parameter

Sheet resistance is also a very useful parameter to characterize (thin) film resistivity.

Fabricating an IC Resistor

How to fabricate an IC resistor?
Make an n-type region in a p-type substrate. We will see why this isolation can work soon.