Lecture 16
The pn Junction Diode (III)

Outline

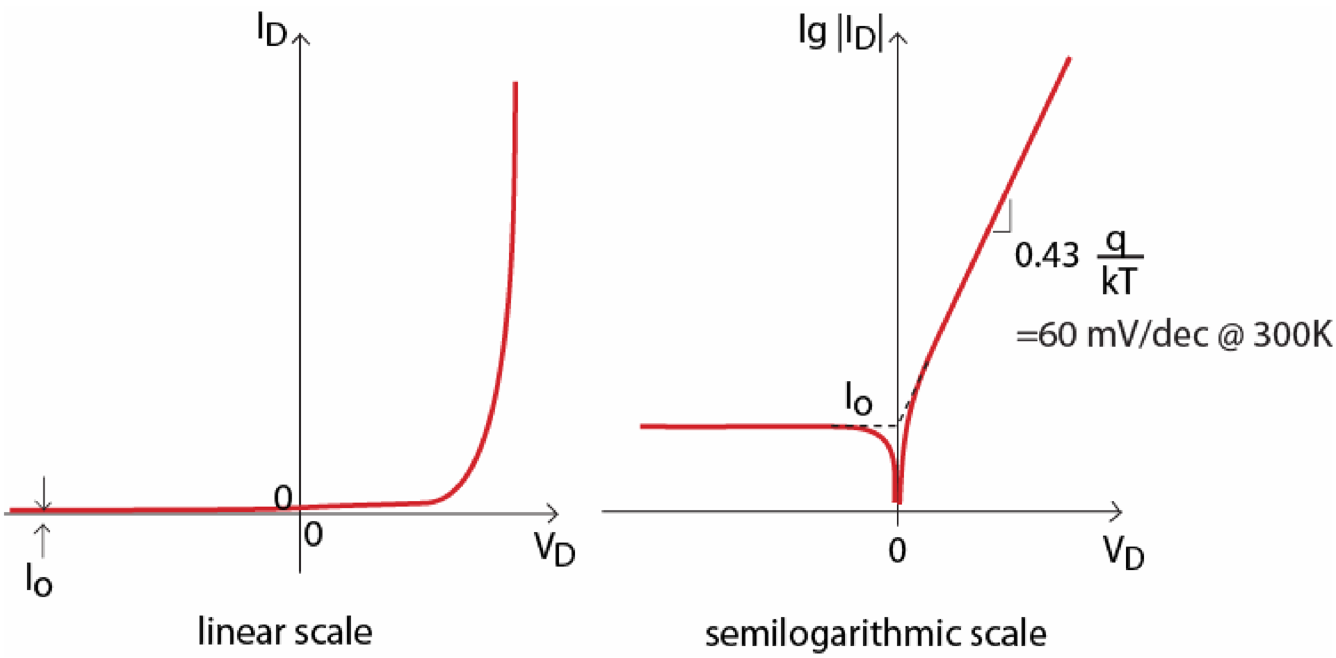
- I-V Characteristics (Review)
- Small-signal equivalent circuit model
- Carrier charge storage
  - –Diffusion capacitance

Reading Assignment:
Howe and Sodini; Chapter 6, Sections 6.4 - 6.5

1. I-V Characteristics (Review)

Diode Current Equation:

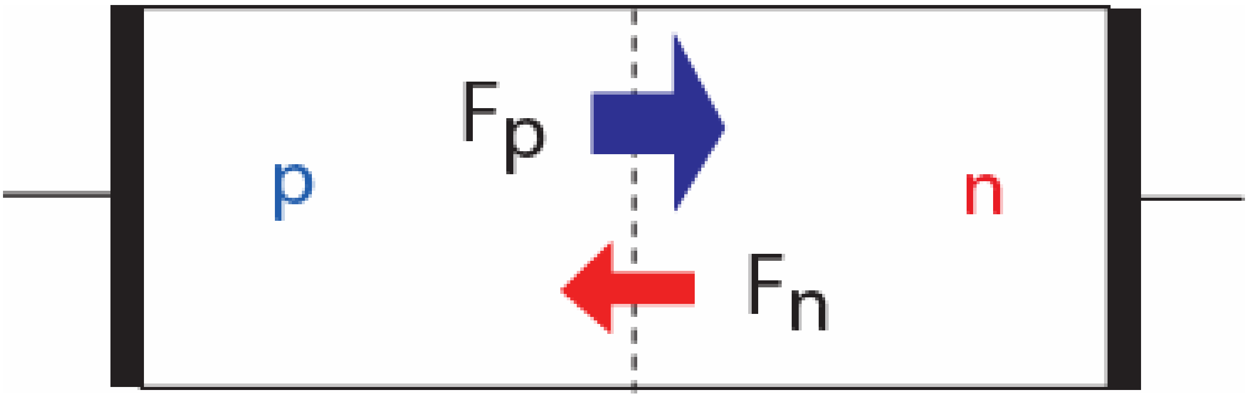
$$I_D = I_o \left[e^{\left(\frac{V_D}{V_{th}}\right)} - 1 \right]$$



Physics of forward bias:

Diode Current equation:

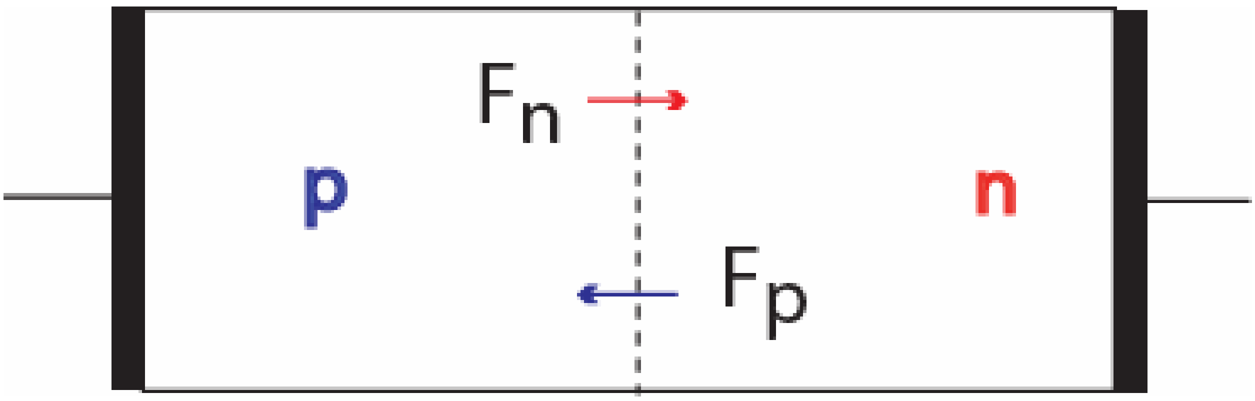
$$I_D = I_o\left[\exp\left(\frac{qV_D}{kT}\right)-1\right]$$



- Junction potential ϕ_J (potential drop across SCR) reduced by $|V_D|$
  - ⇒ **minority carrier injection into QNRs**
- Minority carrier diffusion through QNRs
- Minority carrier recombination at contacts to the QNRs (and surfaces)
- Large supply of carriers injected into the QNRs
  - $\Rightarrow \quad I_D \propto \exp\left[\frac{qV_D}{kT}\right]$

Physics of reverse bias:

$$\mathbf{I_D} = \mathbf{I_o}\left[\exp\left(\frac{\mathbf{qV_D}}{\mathbf{kT}}\right) - 1\right]$$



- Junction potential ϕ_J (potential drop across SCR) increased by $|V_D|$
  - $\Rightarrow$ **minority carrier extraction from QNRs**
- Minority carrier diffusion through QNRs
- Minority carrier generation at surfaces & contacts of QNRs
- Very small supply of carriers available for extraction
  - $\Rightarrow$ I_D saturates to small value
  - $\Rightarrow \quad I_D \approx -I_o$

2. Small-signal equivalent circuit model

Examine effect of small signal overlapping bias:

$$\mathbf{i_D} = \mathbf{I_D} + \mathbf{i_d} = \mathbf{I_o}\left[\exp\left(\frac{\mathbf{q}(\mathbf{V_D} + \mathbf{v_d})}{\mathbf{kT}}\right) - 1\right]$$

If v_d small enough, linearize exponential characteristics:

$$I_D + i_d = I_o\left[\exp\left(\frac{qV_D}{kT}\right)\exp\left(\frac{qv_d}{kT}\right) - 1\right]$$

$$= I_o\left[\exp\left(\frac{qV_D}{kT}\right)\left(1 + \frac{qv_d}{kT}\right) - 1\right]$$

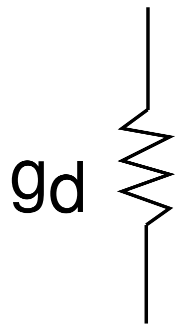
$$= \mathbf{I_o}\left[\exp\left(\frac{\mathbf{qV_D}}{\mathbf{kT}}\right) - 1\right] + \mathbf{I_o}\exp\left(\frac{\mathbf{qV_D}}{\mathbf{kT}}\right)\frac{\mathbf{qv_d}}{\mathbf{kT}}$$

Then: $$\mathbf{i_d} = \frac{\mathbf{q}(\mathbf{I_D} + \mathbf{I_o})}{\mathbf{kT}} \bullet \mathbf{v_d}$$

From a small signal point of view. Diode behaves as ***conductance*** of value:

$$g_d = \frac{q\left(I_D + I_o\right)}{kT} \approx \frac{qI_D}{kT}$$

Small-signal equivalent circuit model

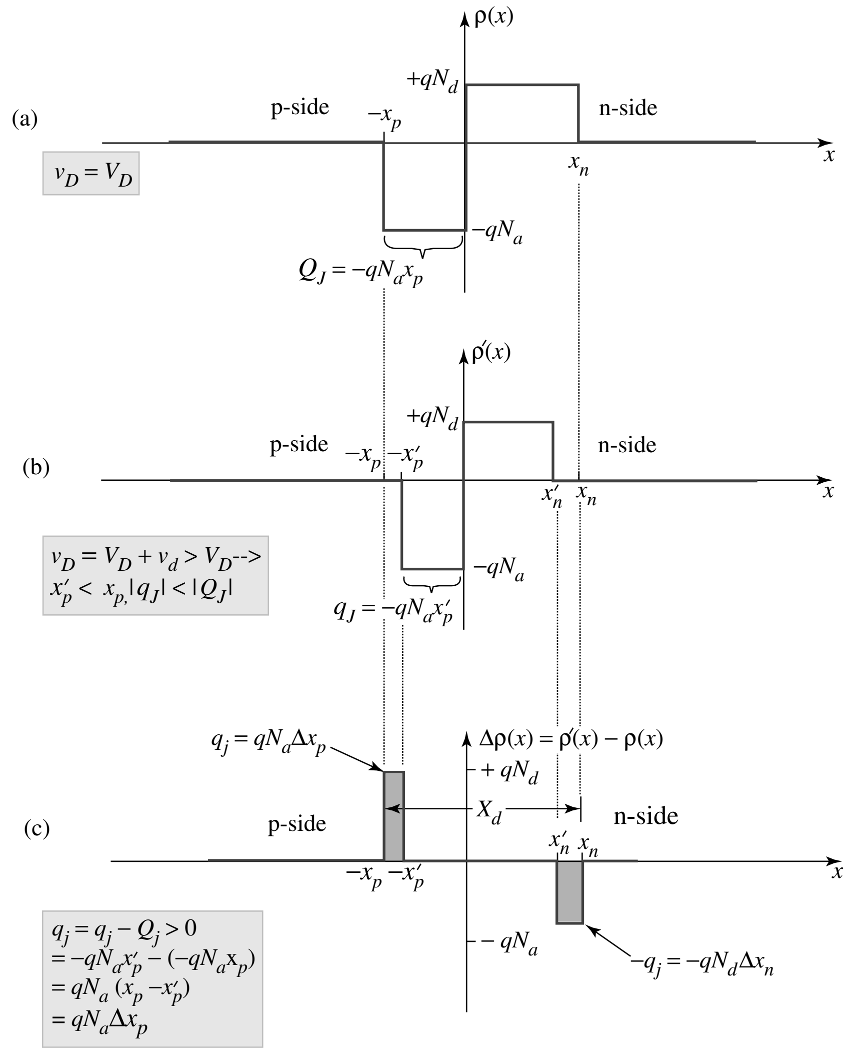


g_d depends on bias. In forward bias:

$$g_d = \frac{qI_D}{kT}$$

g_d is linear in diode current.

Capacitance associated with depletion region:

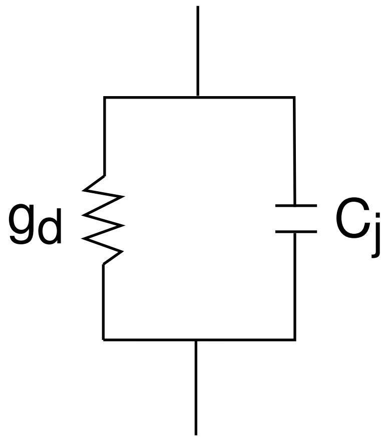


Depletion or junction capacitance:

$$C_j = C_j(V_D) = \left.\frac{dq_J}{dv_D}\right|_{V_D}$$

$$C_j = A\sqrt{\frac{q\varepsilon_s N_a N_d}{2(N_a + N_d)(\phi_B - V_D)}}$$

Small-signal equivalent circuit model



can rewrite as:

$$C_j = A\sqrt{\frac{q\varepsilon_s N_a N_d}{2(N_a + N_d)\phi_B}} \bullet \sqrt{\frac{\phi_B}{(\phi_B - V_D)}}$$

or,

$$C_j = \frac{C_{jo}}{\sqrt{1 - \frac{V_D}{\phi_B}}}$$

Under Forward Bias assume $V_D \approx \frac{\phi_B}{2}$

$$C_j = \sqrt{2}\, C_{jo}$$

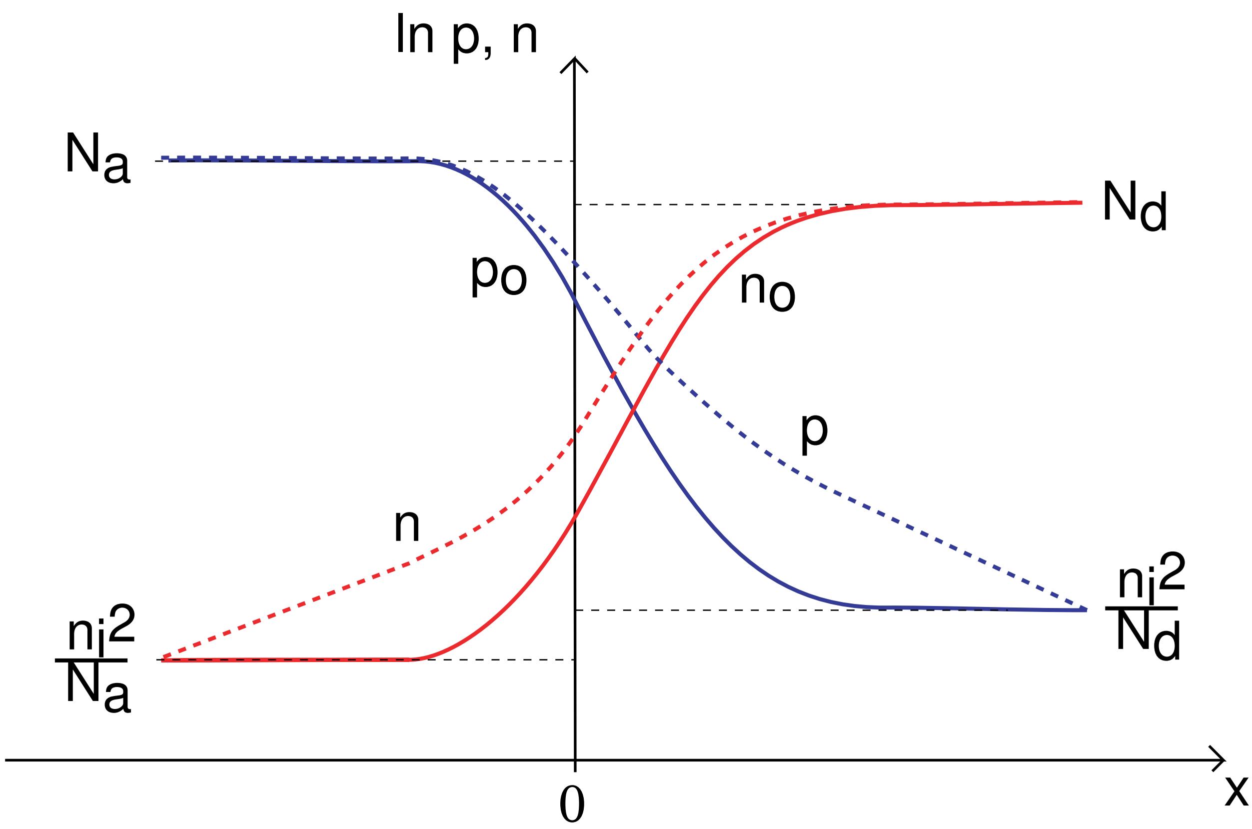
C_{jo} ≡ ***zero-voltage junction capacitance***

3. Charge Carrier Storage:

diffusion capacitance

What happens to majority carriers?

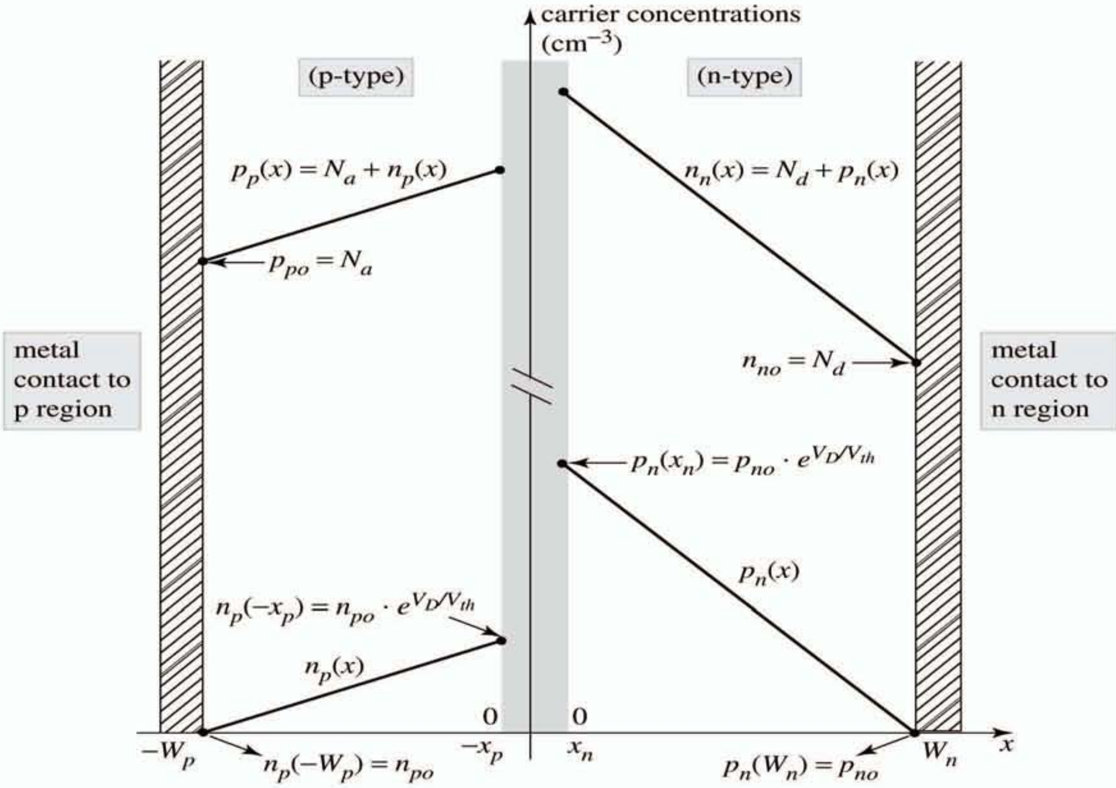
Carrier picture thus far:



If QNR minority carrier concentration ↑ but majority carrier concentration **unchanged?** ⇒ quasi-neutrality is **violated**.

Quasi-neutrality demands that at every point in QNR:

excess minority carrier concentration
= excess majority carrier concentration



In n-type Si, at every x:

$$p_n(x) - p_{no} = n_n(x) - n_{no}$$

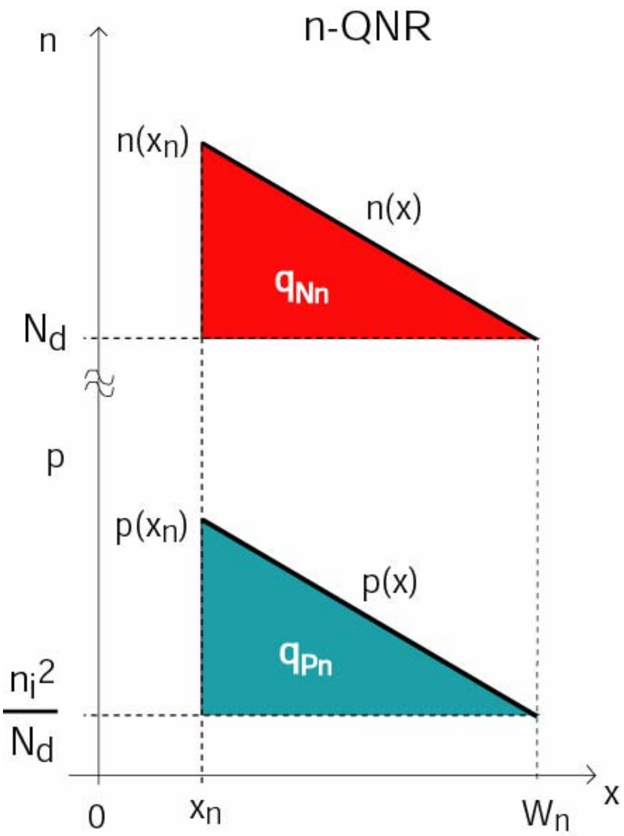
In p-type Si, at every x:

$$n_p(x) - n_{po} = p_p(x) - p_{po}$$

Quasi-neutrality demands that at every point in QNR:

excess minority carrier concentration

= excess majority carrier concentration



Mathematically:

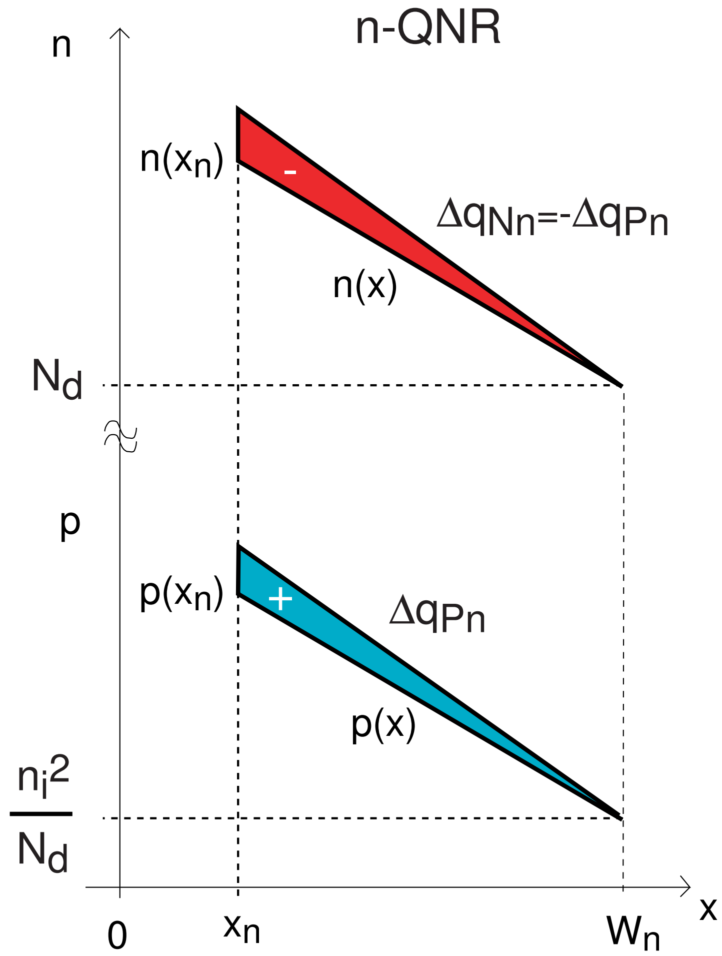
$$\mathbf{p'_n(x) = p_n(x) - p_{no} \approx n'_n(x) = n_n(x) - n_{no}}$$

Define integrated carrier charge:

$$q_{Pn} = qA\frac{1}{2}p'(x_n)\bullet(W_n - x_n)$$

$$= qA\frac{W_n - x_n}{2}\frac{n_i^2}{N_d}\exp\left[\frac{qV_D}{kT} - 1\right] = -q_{Nn}$$

Now examine small increase in V_D:



Small increase in $V_D \Rightarrow$ small increase in $q_{Pn} \Rightarrow$ small increase in $|q_{Nn}|$

Behaves as capacitor of capacitance:

$$\mathbf{C_{dn}} = \left.\frac{\mathbf{dq_{Pn}}}{\mathbf{dv_D}}\right|_{\mathbf{v_D=V_D}} = \mathbf{qA}\frac{\mathbf{W_n - x_n}}{2}\frac{\mathbf{n_i^2}}{\mathbf{N_d}}\frac{\mathbf{q}}{\mathbf{kT}}\exp\left[\frac{\mathbf{qV_D}}{\mathbf{kT}}\right]$$

Can write in terms of I_{Dp} (portion of diode current due to holes in n-QNR):

$$\mathbf{C_{dn} = \frac{q}{kT}\frac{(W_n - x_n)^2}{2D_p} qA \frac{n_i^2}{N_d}\frac{D_p}{W_n - x_n}} \exp\left[\mathbf{\frac{qV_D}{kT}}\right]$$

$$\mathbf{\approx \frac{q}{kT}\frac{(W_n - x_n)^2}{2D_p} I_{Dp}}$$

Define ***transit time*** of holes through n-QNR:

$$\tau_{Tp} = \frac{(W_n - x_n)^2}{2D_p}$$

Transit time is the ***average time for a hole to diffuse through n-QNR*** [will discuss in more detail in BJT]

Then:

$$\mathbf{C_{dn} \approx \frac{q}{kT} \bullet \tau_{Tp} \bullet I_{Dp}}$$

Similarly for p-QNR:

$$\mathbf{C_{dp} \approx \frac{q}{kT} \bullet \tau_{Tn} \bullet I_{Dn}}$$

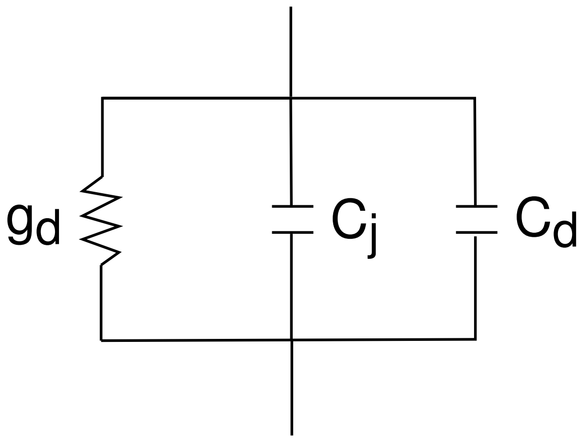
where τ_{Tn} is ***transit time*** of electrons through p-QNR:

$$\tau_{Tn} = \frac{\left(W_p - x_p\right)^2}{2D_n}$$

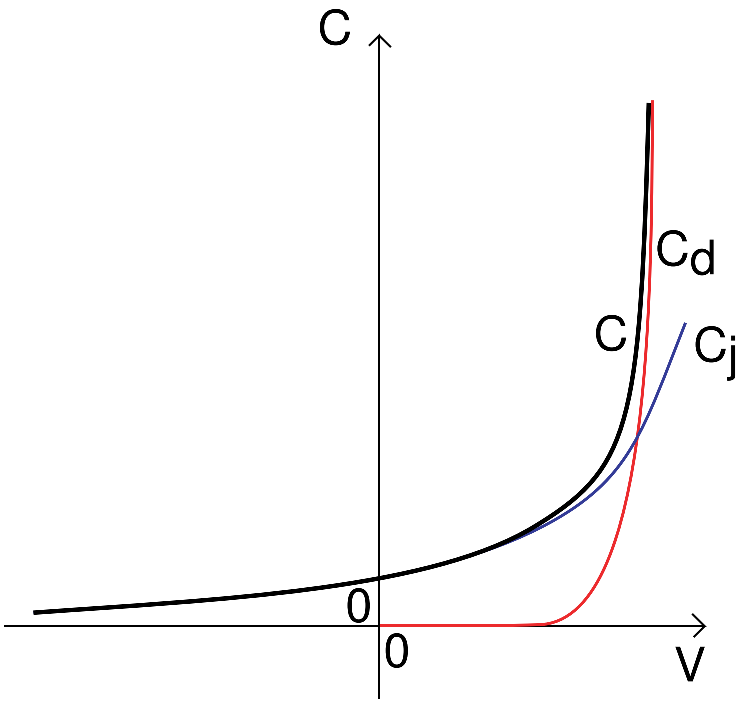
Both capacitors sit in ***parallel*** ⇒ total diffusion capacitance:

$$\mathbf{C_d = C_{dn} + C_{dp} = \frac{q}{kT}\left(\tau_{Tn} I_{Dn} + \tau_{Tp} I_{Dp}\right)}$$

Complete small-signal equivalent circuit model for diode:



Bias dependence of C_j and C_d:



- C_j dominates in reverse bias and small forward bias

$$\propto \frac{1}{\sqrt{\phi_B - V_D}}$$

- C_d dominates in strong forward bias

$$\propto \exp\left[\frac{qV_D}{kT}\right]$$

What did we learn today?

Summary of Key Concepts

Large and Small-signal behavior of diode:

- ***Diode Current:***

$$I = I_o \left(e^{\left[\frac{qV_D}{kT}\right]} - 1 \right)$$

- ***Conductance***: associated with current-voltage characteristics
  - $g_d \propto I$ in forward bias,
  - g_d negligible in reverse bias

- ***Junction capacitance***: associated with charge modulation in depletion region

$$C_j \propto \frac{1}{\sqrt{\phi_B - V_D}}$$

- ***Diffusion capacitance***: associated with charge storage in QNRs to maintain quasi-neutrality.

$$C_d \propto e^{\left[\frac{qV_D}{kT}\right]}$$

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