

# Lecture 10

## MOSFET (III)

### MOSFET Equivalent Circuit Models

#### Outline

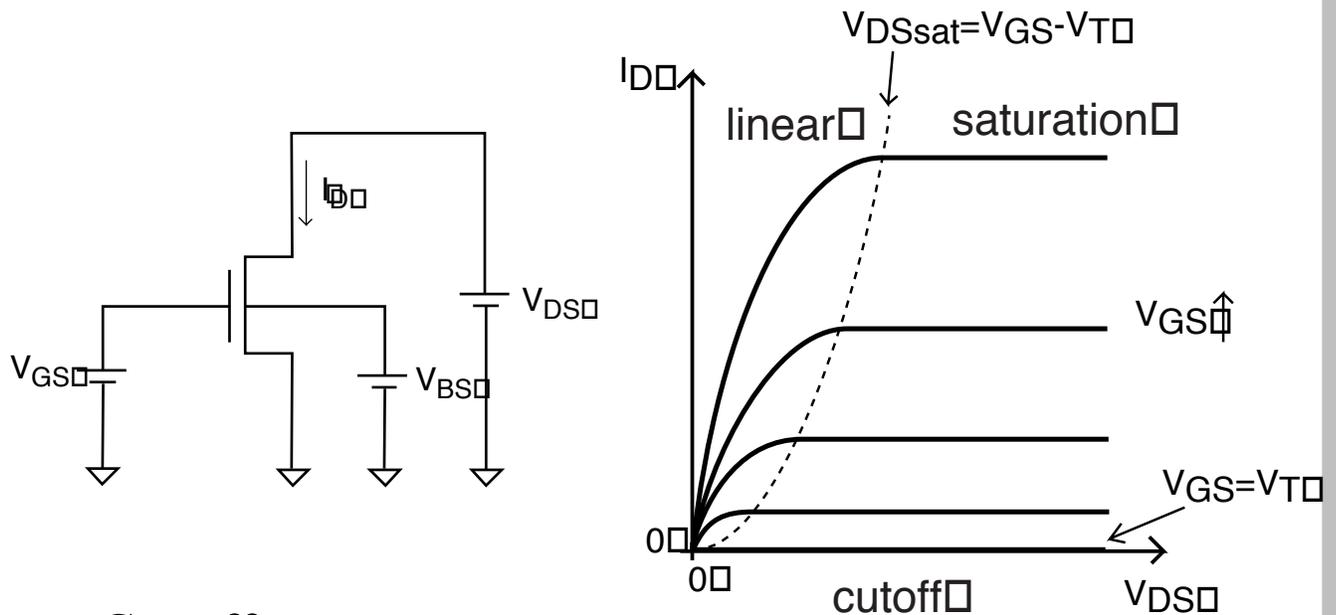
- Low-frequency small-signal equivalent circuit model
- High-frequency small-signal equivalent circuit model

#### **Reading Assignment:**

Howe and Sodini; Chapter 4, Sections 4.5-4.6

# Large Signal Model for NMOS Transistor

## Regimes of operation:



- Cut-off

$$I_D = 0$$

- Linear / Triode:

$$I_D = \frac{W}{L} \mu_n C_{ox} \left[ V_{GS} - \frac{V_{DS}}{2} - V_T \right] \cdot V_{DS}$$

- Saturation

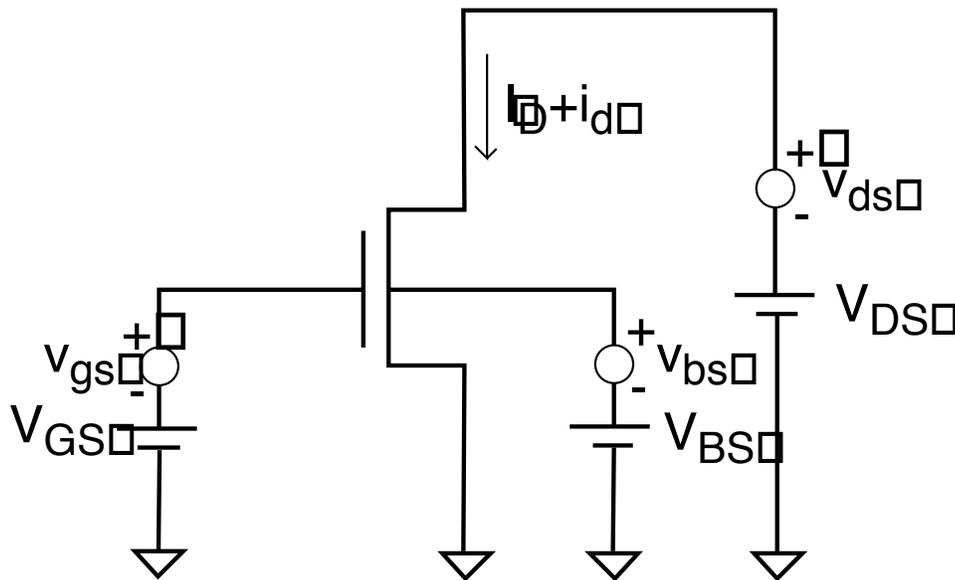
$$I_D = I_{Dsat} = \frac{W}{2L} \mu_n C_{ox} [V_{GS} - V_T]^2 \cdot [1 + \lambda V_{DS}]$$

## Effect of back bias

$$V_T(V_{BS}) = V_{To} + \gamma \left[ \sqrt{-2\phi_p - V_{BS}} - \sqrt{-2\phi_p} \right]$$

# Small-signal device modeling

In many applications, we are only interested in the response of the device to a *small-signal* applied on top of a bias.



## Key Points:

- Small-signal is *small*
  - $\Rightarrow$  response of non-linear components becomes linear
- Since response is linear, lots of linear circuit techniques such as *superposition* can be used to determine the circuit response.
- Notation:  $i_D = I_D + i_d$  --- Total = DC + Small Signal

Mathematically:

$$i_D(V_{GS}, V_{DS}, V_{BS}; v_{gs}, v_{ds}, v_{bs}) \approx$$

$$I_D(V_{GS}, V_{DS}, V_{BS}) + i_d(v_{gs}, v_{ds}, v_{bs})$$

With  $i_d$  linear on small-signal drives:

$$i_d = g_m v_{gs} + g_o v_{ds} + g_{mb} v_{bs}$$

Define:

$$g_m \equiv \textit{transconductance} \text{ [S]}$$

$$g_o \equiv \textit{output or drain conductance} \text{ [S]}$$

$$g_{mb} \equiv \textit{backgate transconductance} \text{ [S]}$$

Approach to computing  $g_m$ ,  $g_o$ , and  $g_{mb}$ .

$$g_m \approx \left. \frac{\partial i_D}{\partial v_{GS}} \right|_Q$$

$$g_o \approx \left. \frac{\partial i_D}{\partial v_{DS}} \right|_Q$$

$$g_{mb} \approx \left. \frac{\partial i_D}{\partial v_{BS}} \right|_Q$$

$$Q \equiv [v_{GS} = V_{GS}, v_{DS} = V_{DS}, v_{BS} = V_{BS}]$$

## Transconductance

In saturation regime:

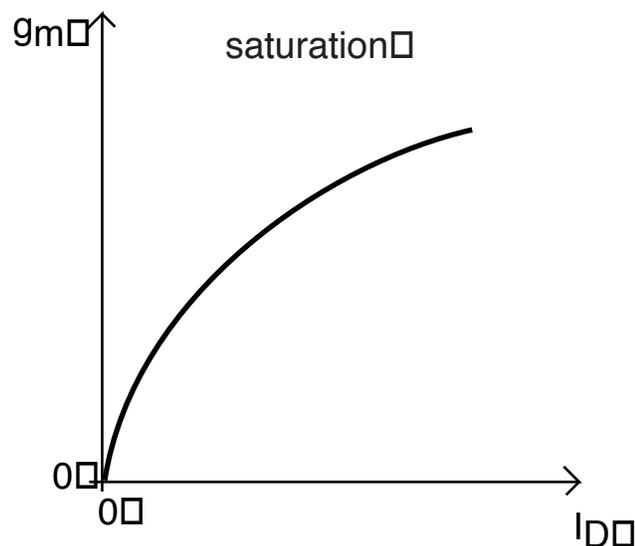
$$i_D = \frac{W}{2L} \mu_n C_{ox} [v_{GS} - V_T]^2 \cdot [1 + \lambda V_{DS}]$$

Then (neglecting channel length modulation) the transconductance is:

$$g_m = \left. \frac{\partial i_D}{\partial v_{GS}} \right|_Q \approx \frac{W}{L} \mu_n C_{ox} (V_{GS} - V_T)$$

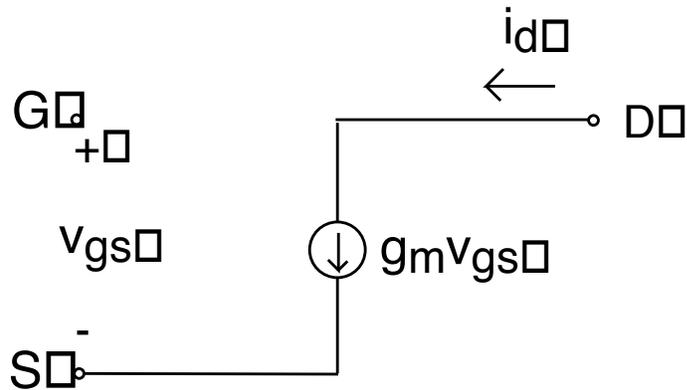
Rewrite in terms of  $I_D$ :

$$g_m = \sqrt{2 \frac{W}{L} \mu_n C_{ox} I_D}$$

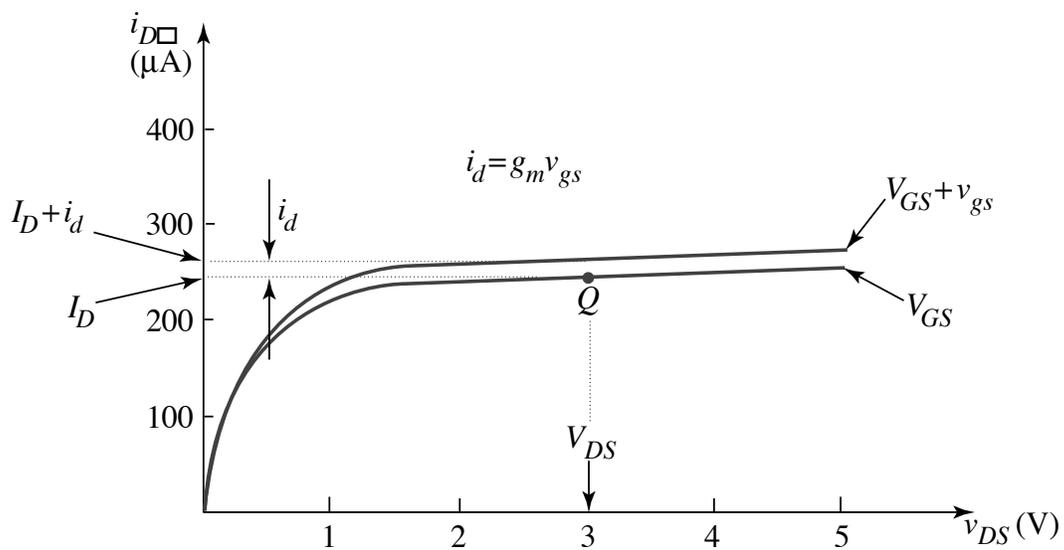


# Transconductance (contd.)

Equivalent circuit model representation of  $g_m$ :



B



# Output conductance

In saturation regime:

$$i_D = \frac{W}{2L} \mu_n C_{ox} [v_{GS} - V_T]^2 \cdot [1 + \lambda v_{DS}]$$

Then:

$$g_o = \left. \frac{\partial i_D}{\partial v_{DS}} \right|_Q = \frac{W}{2L} \mu_n C_{ox} (V_{GS} - V_T)^2 \cdot \lambda \approx \lambda I_D$$

Output resistance is the inverse of output conductance:

$$r_o = \frac{1}{g_o} = \frac{1}{\lambda I_D}$$

Remember also:

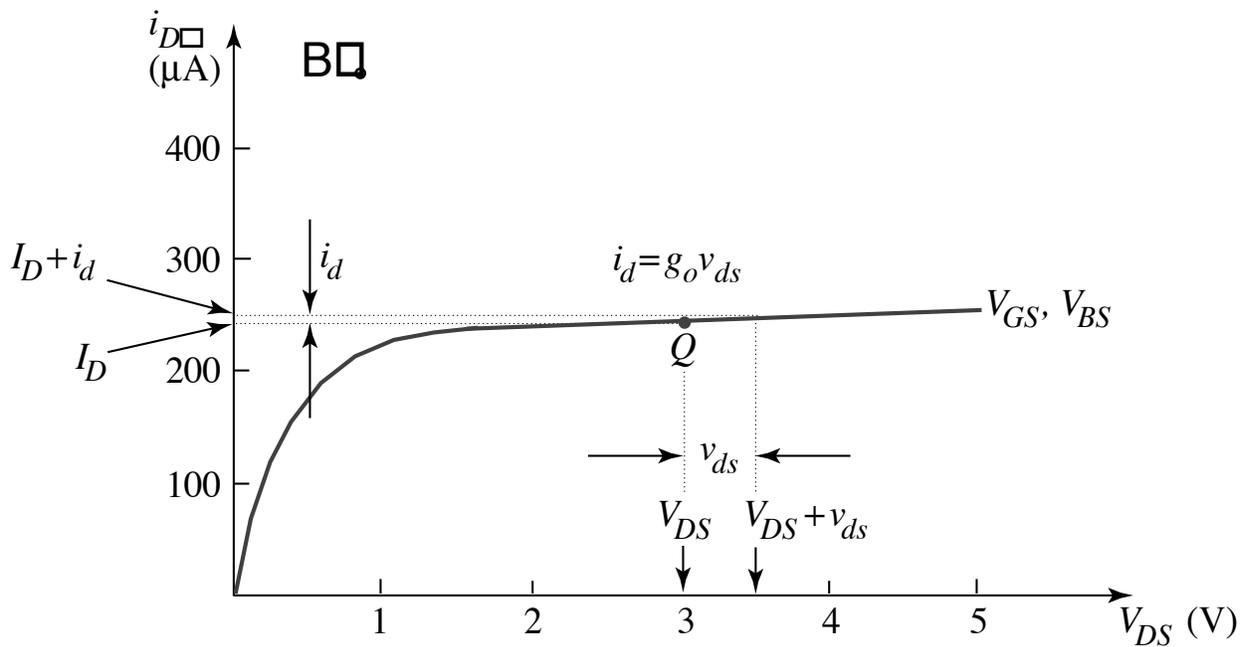
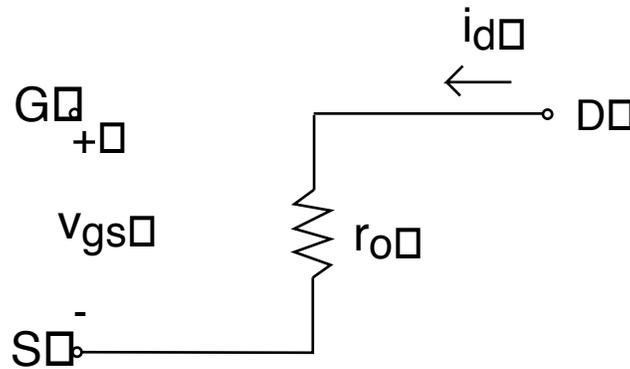
$$\lambda \propto \frac{1}{L}$$

Hence:

$$r_o \propto L$$

## Output conductance (contd.)

Equivalent circuit model representation of  $g_o$ :



## Backgate transconductance

In saturation regime (neglect channel length modulation):

$$i_D \approx \frac{W}{2L} \mu_n C_{ox} [v_{GS} - V_T]^2$$

Then:

$$g_{mb} = \left. \frac{\partial i_D}{\partial v_{BS}} \right|_Q = -\frac{W}{L} \mu_n C_{ox} (V_{GS} - V_T) \cdot \left( \left. \frac{\partial V_T}{\partial v_{BS}} \right|_Q \right)$$

Since:

$$V_T(v_{BS}) = V_{T0} + \gamma \left[ \sqrt{-2\phi_p - v_{BS}} - \sqrt{-2\phi_p} \right]$$

Then :

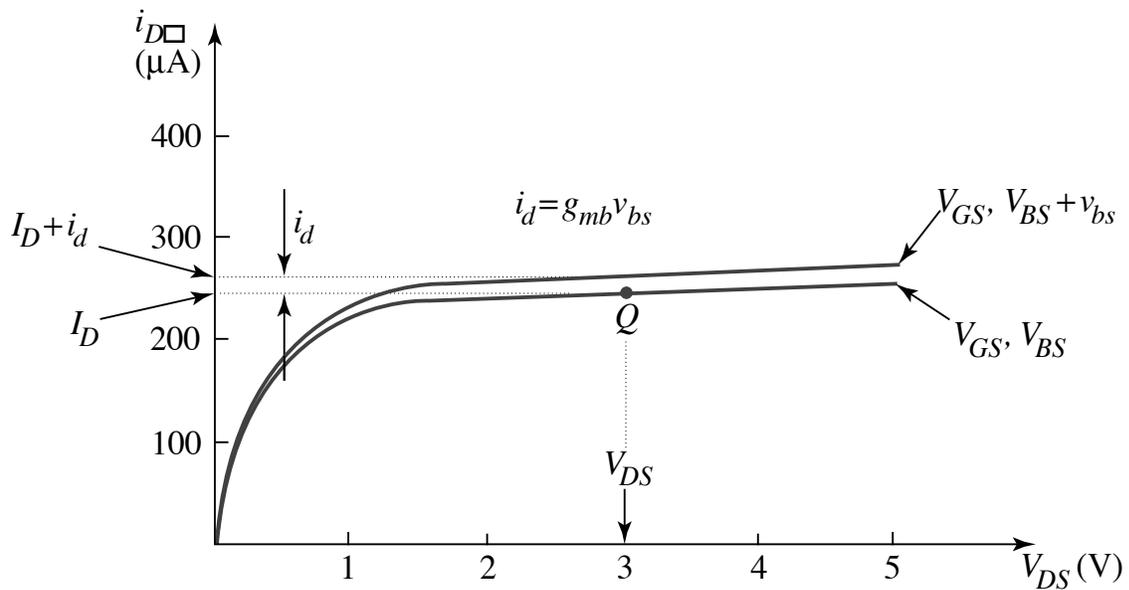
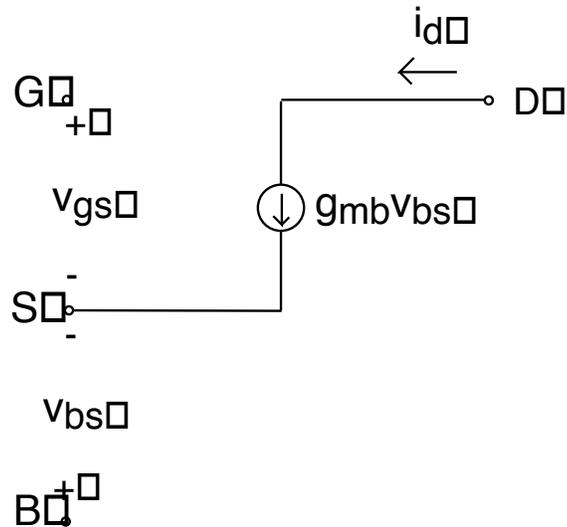
$$\left. \frac{\partial V_T}{\partial v_{BS}} \right|_Q = \frac{-\gamma}{2\sqrt{-2\phi_p - V_{BS}}}$$

Hence:

$$g_{mb} = \frac{\gamma g_m}{2\sqrt{-2\phi_p - V_{BS}}}$$

## Backgate transconductance (contd.)

Equivalent circuit representation of  $g_{mb}$ :



# Complete MOSFET small-signal equivalent circuit model for low frequency:

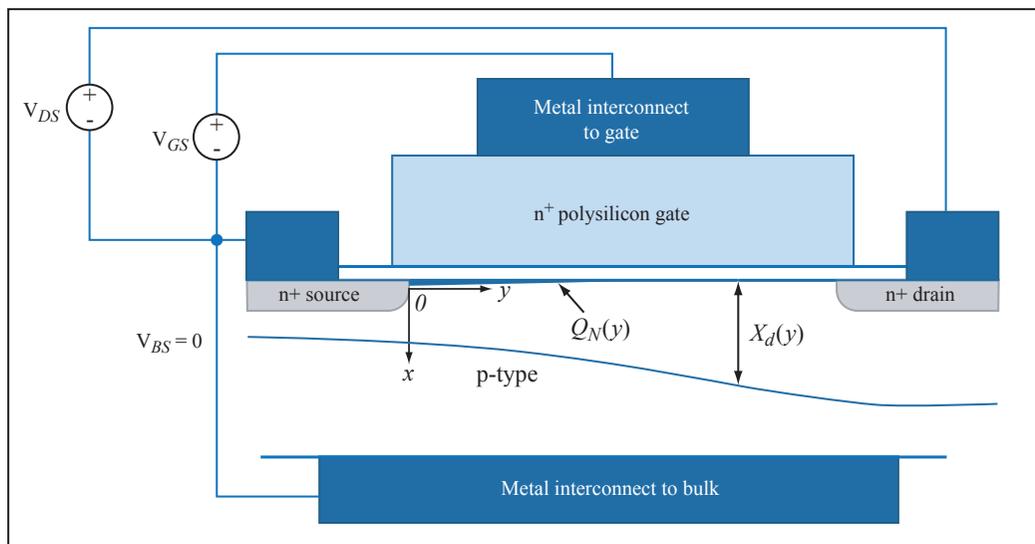
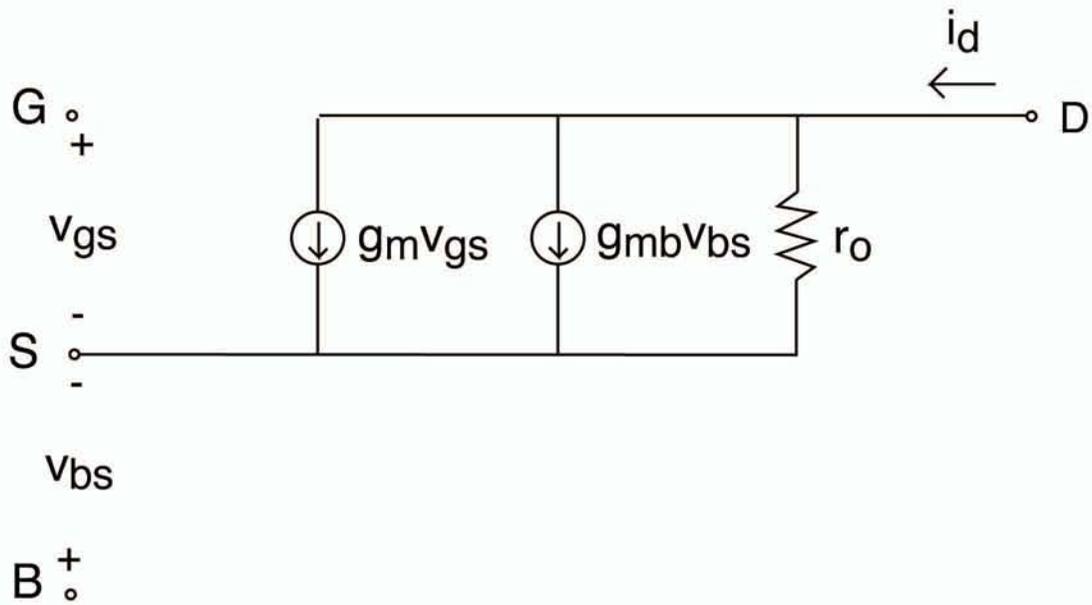
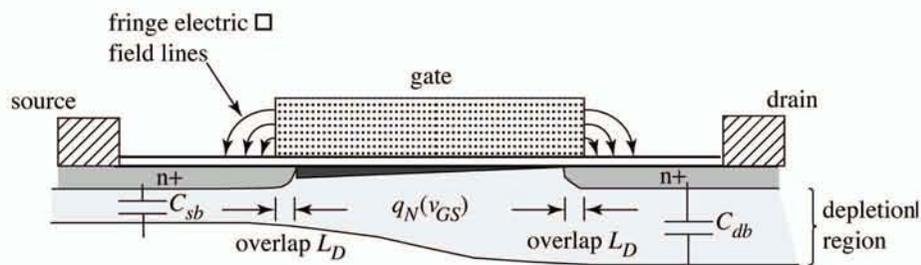


Figure by MIT OpenCourseWare.

## 2. High-frequency small-signal equivalent circuit model

Need to add capacitances. In saturation:



$C_{gs} \equiv$  channel charge + overlap capacitance,  $C_{ov}$

$C_{gd} \equiv$  overlap capacitance,  $C_{ov}$

$C_{sb} \equiv$  source junction depletion capacitance (+sidewall)

$C_{db} \equiv$  drain junction depletion capacitance (+sidewall)

ONLY Channel Charge Capacitance is intrinsic to device operation. All others are parasitic.

## Inversion layer charge in saturation

$$q_N(v_{GS}) = W \int_0^L Q_N(y) dy = W \int_0^{v_{GS}-V_T} Q_N(v_C) \cdot \frac{dy}{dv_C} \cdot dv_C$$

Note that  $q_N$  is total inversion charge in the channel &  $v_C(y)$  is the channel voltage. But:

$$\frac{dv_C}{dy} = -\frac{i_D}{W \mu_n Q_N(v_C)}$$

Then:

$$q_N(v_{GS}) = -\frac{W^2 \mu_n}{i_D} \cdot \int_0^{v_{GS}-V_T} [Q_N(v_C)]^2 \cdot dv_C$$

Remember:

$$Q_N(v_C) = -C_{ox} [v_{GS} - v_C(y) - V_T]$$

Then:

$$q_N(v_{GS}) = -\frac{W^2 \mu_n C_{ox}^2}{i_D} \cdot \int_0^{v_{GS}-V_T} [v_{GS} - v_C(y) - V_T]^2 \cdot dv_C$$

## Inversion layer charge in saturation (contd.)

Do integral, substitute  $i_D$  in saturation and get:

$$q_N(v_{GS}) = -\frac{2}{3} WLC_{ox}(v_{GS} - V_T)$$

Gate charge:

$$q_G(v_{GS}) = -q_N(v_{GS}) - Q_{B,max}$$

Intrinsic gate-to-source capacitance:

$$C_{gs,i} = \frac{dq_G}{dv_{GS}} = \frac{2}{3} WLC_{ox}$$

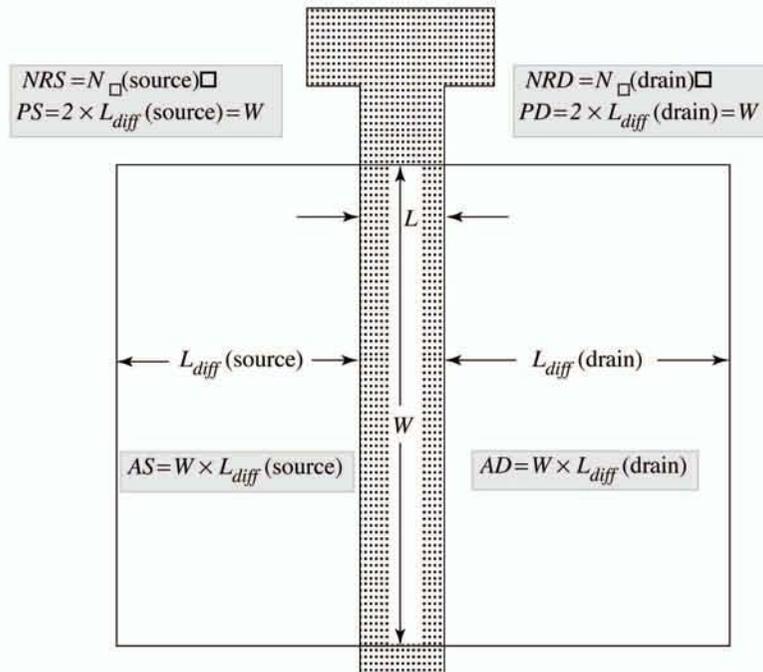
Must add overlap capacitance:

$$C_{gs} = \frac{2}{3} WLC_{ox} + WC_{ov}$$

Gate-to-drain capacitance — only overlap capacitance:

$$C_{gd} = WC_{ov}$$

# Other capacitances



Source-to-Bulk capacitance:

$$C_{sb} = WL_{diff}C_j + (2L_{diff} + W)C_{jsw}$$

where  $C_j$ : Bottom Wall at  $V_{SB}(F/cm^2)$

$C_{jsw}$ : Side Wall at  $V_{SB}(F/cm)$

Drain-to-Bulk capacitance:

$$C_{db} = WL_{diff}C_j + (2L_{diff} + W)C_{jsw}$$

where  $C_j$ : Bottom Wall at  $V_{DB}(F/cm^2)$

$C_{jsw}$ : Side Wall at  $V_{DB}(F/cm)$

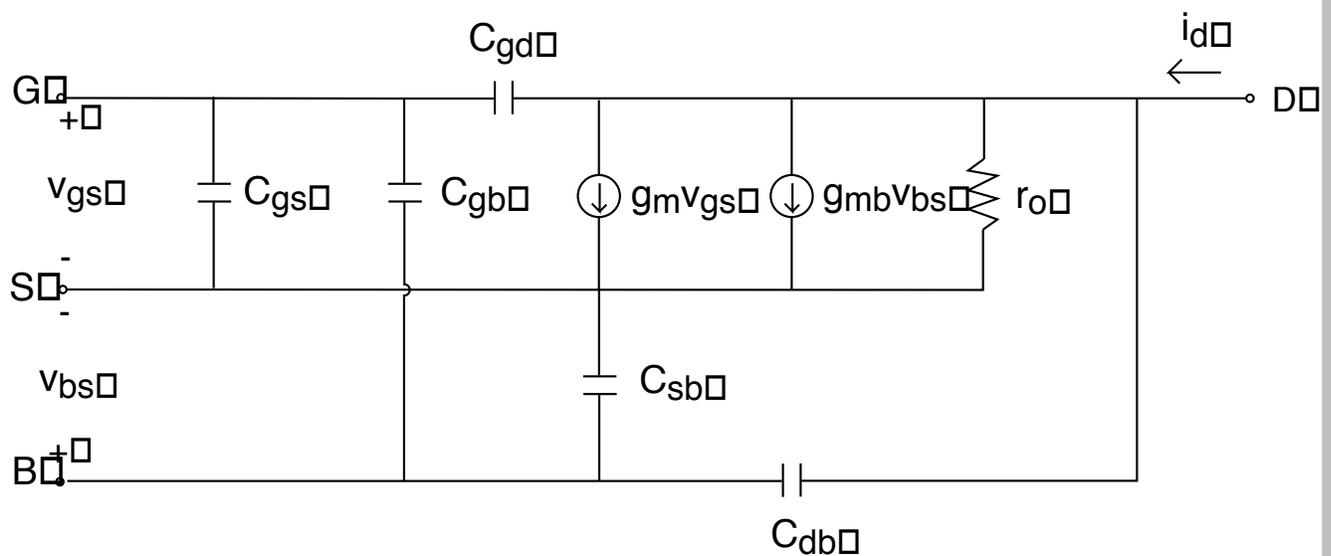
Gate-to-Bulk capacitance:

$C_{gb} \equiv$  small parasitic capacitance in most cases (ignore)

# What did we learn today?

## Summary of Key Concepts

High-frequency small-signal equivalent circuit model of MOSFET



In saturation:

$$g_m \propto \sqrt{\frac{W}{L}} I_D$$

$$r_o \propto \frac{L}{I_D}$$

$$C_{gs} \propto WLC_{ox}$$

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