

# **Lecture 5**

## **PN Junction and MOS Electrostatics(II)**

### **PN JUNCTION IN THERMAL EQUILIBRIUM**

#### **Outline**

1. Introduction
2. Electrostatics of pn junction in thermal equilibrium
3. The depletion approximation
4. Contact potentials

#### **Reading Assignment:**

Howe and Sodini, Chapter 3, Sections 3.3-3.6

# 1. Introduction

- pn junction
  - p-region and n-region in intimate contact

## Why is the p-n junction worth studying?

It is present in virtually every semiconductor device!

### Example: CMOS cross-section

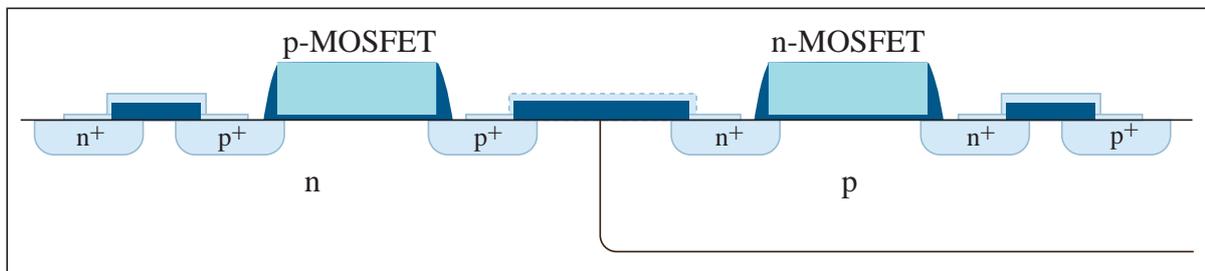
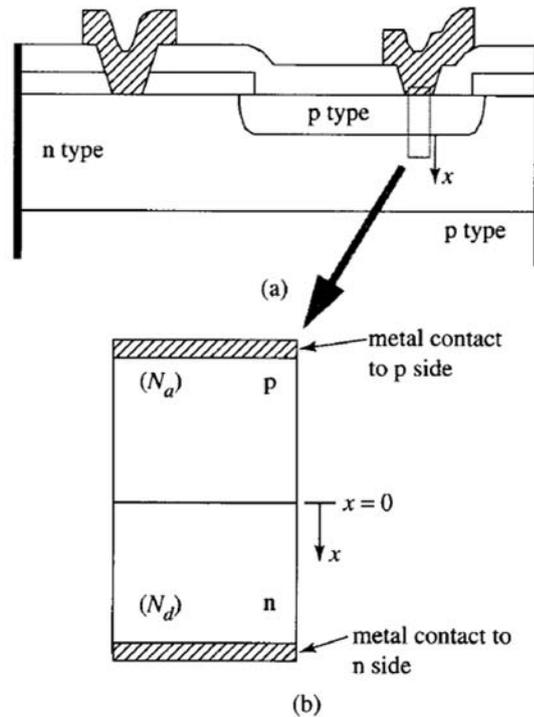


Figure by MIT OpenCourseWare.

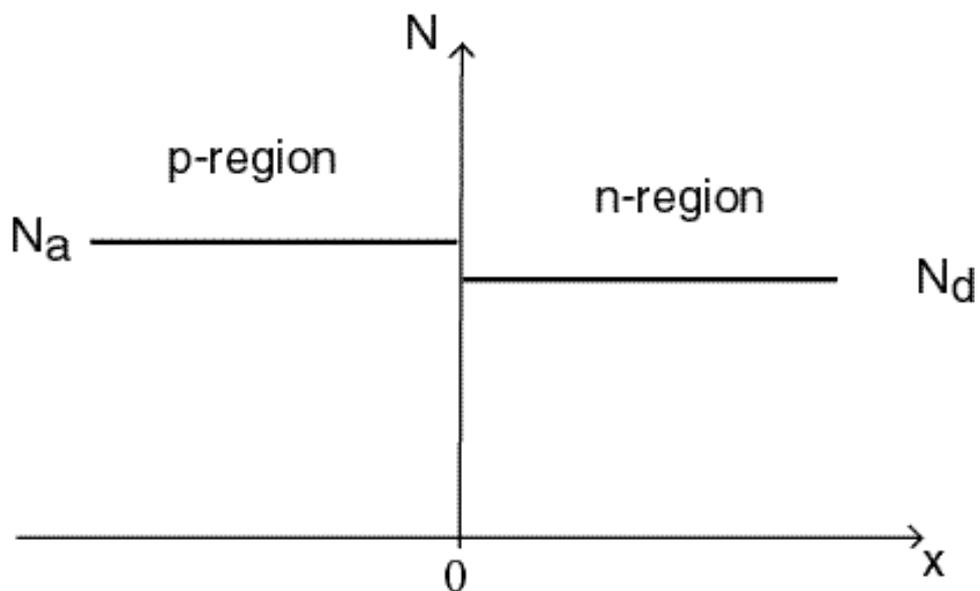
**Understanding the pn junction is essential to understanding transistor operation**

## 2. Electrostatics of p-n junction in equilibrium

Focus on intrinsic region:

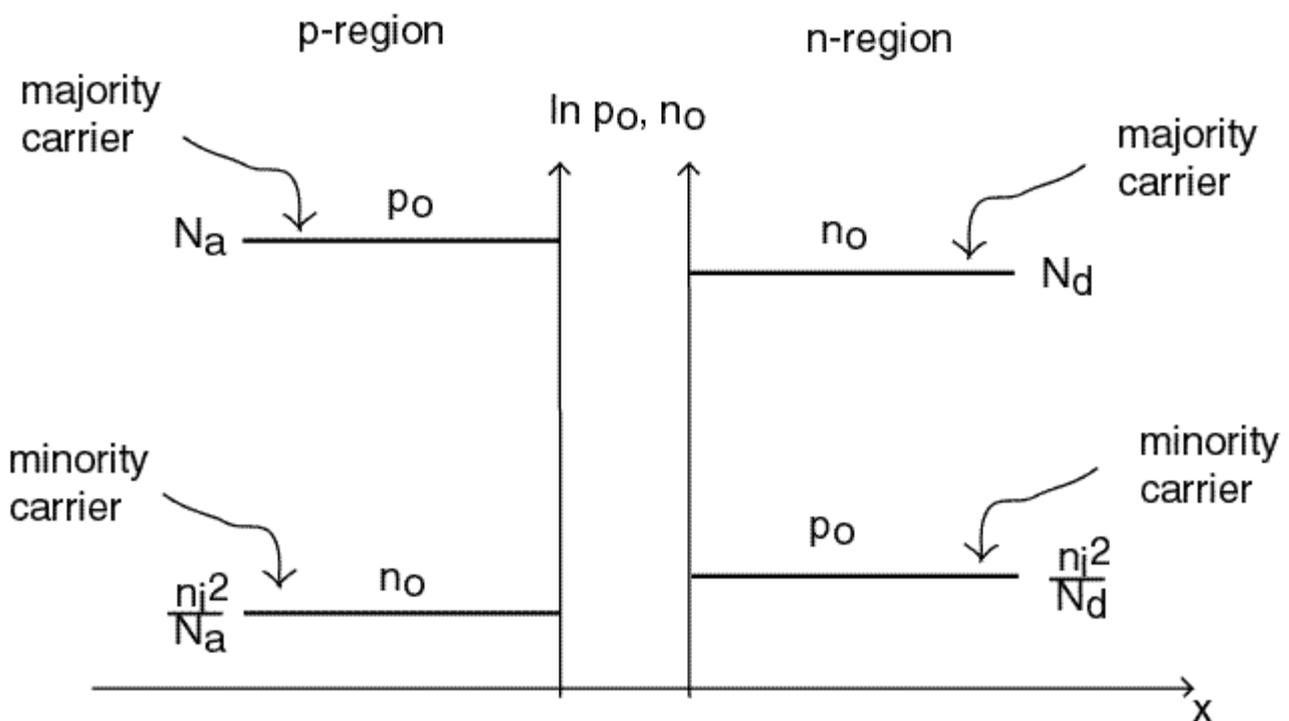


Doping distribution of an **abrupt** p-n junction



## What is the carrier concentration distribution in thermal equilibrium?

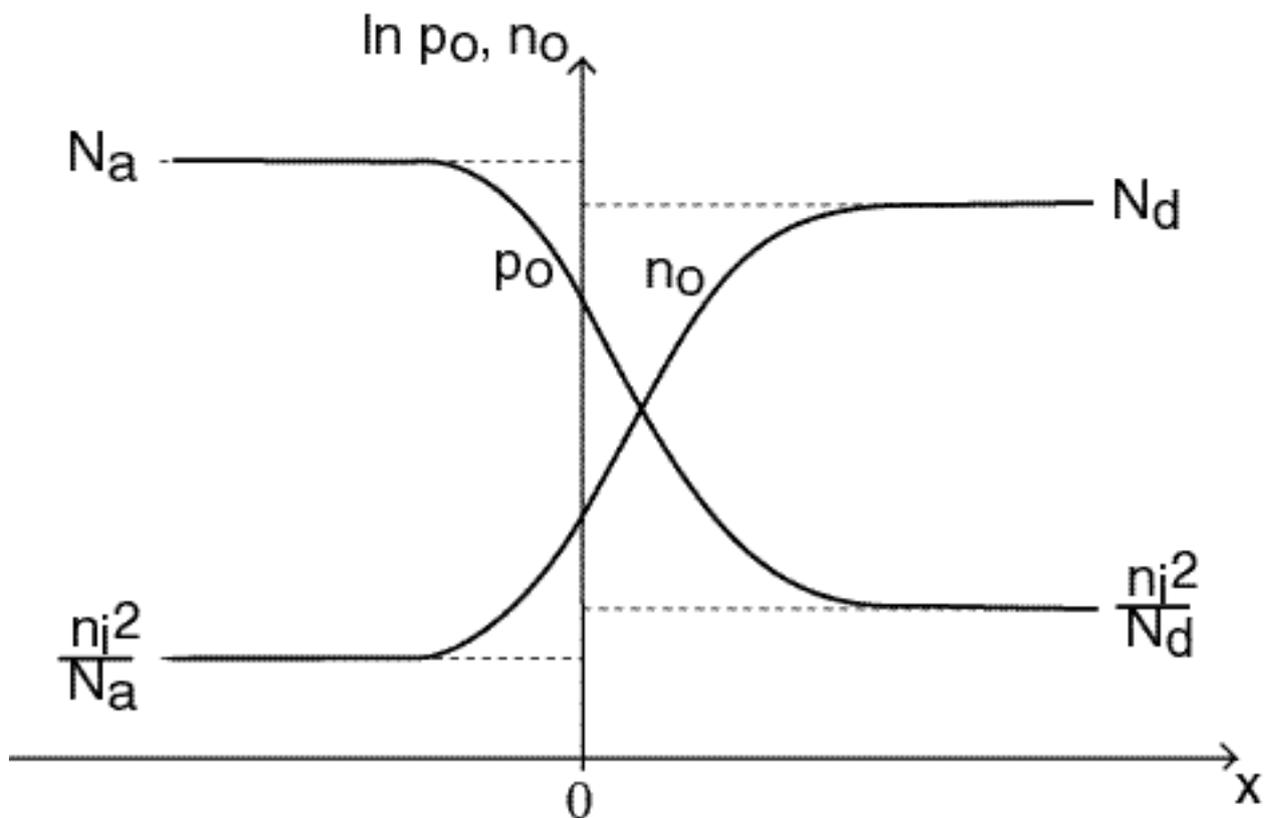
First think of the two sides separately:



Now bring the two sides together.

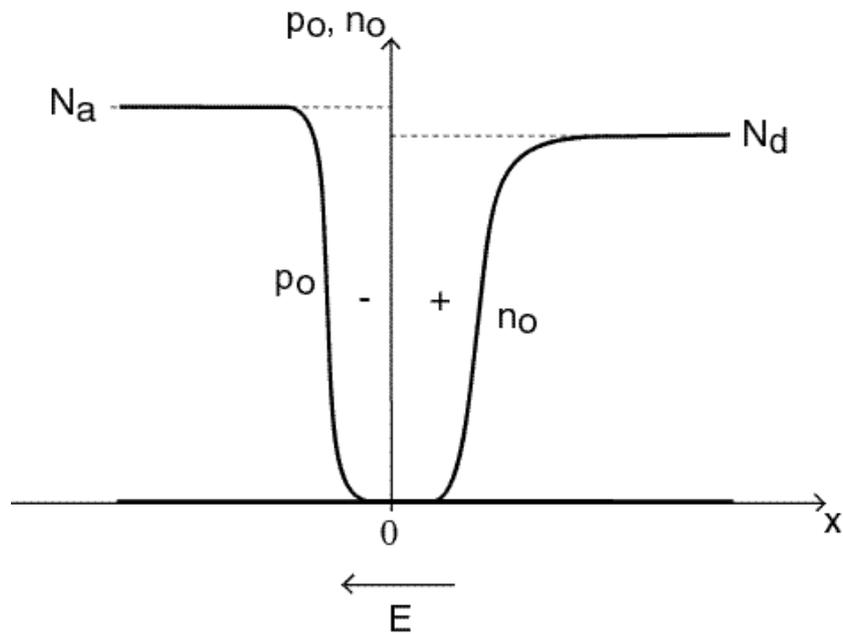
What happens?

## Resulting carrier concentration profile in thermal equilibrium:



- Far away from the metallurgical junction: nothing happens
  - Two *quasi-neutral regions*
- Around the metallurgical junction: diffusion of carriers must counter-balance drift
  - *Space-charge region*

On a linear scale:



**Thermal equilibrium:** balance between drift and diffusion

$$J_n(x) = J_n^{drift}(x) + J_n^{diff}(x) = 0$$

$$J_p(x) = J_p^{drift}(x) + J_p^{diff}(x) = 0$$

We can divide semiconductor into three regions

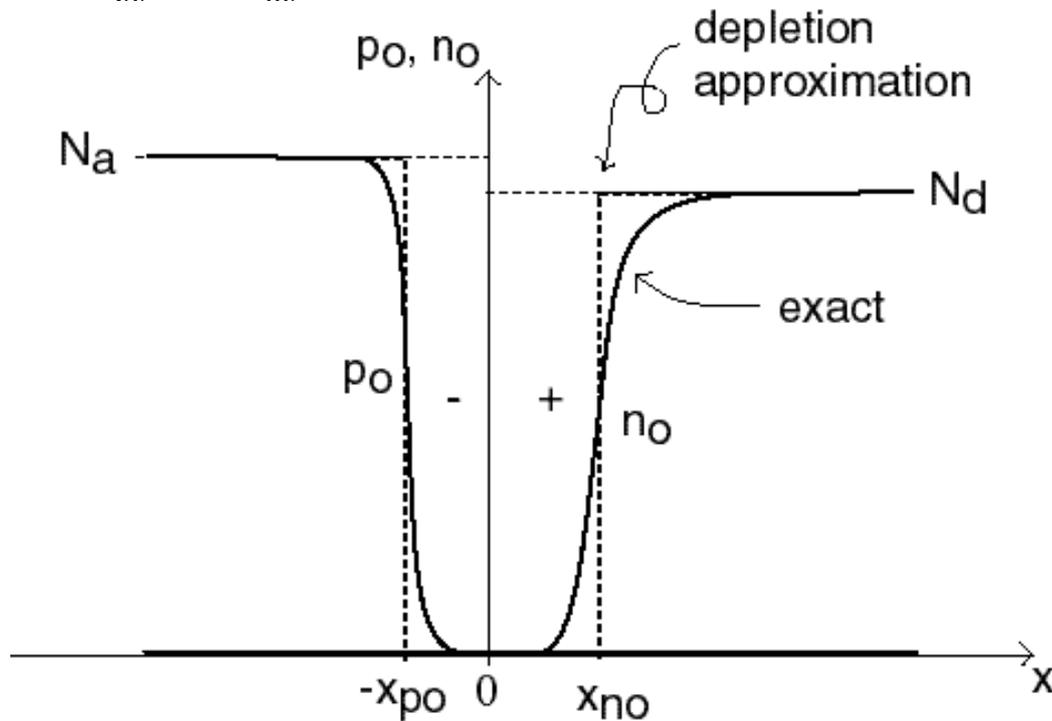
- Two quasi-neutral n- and p-regions (QNR's)
- One space-charge region (SCR)

Now, we want to know  $n_0(x)$ ,  $p_0(x)$ ,  $\rho(x)$ ,  $E(x)$  and  $\phi(x)$ .

**We need to solve Poisson's equation using a simple but powerful approximation**

### 3. The Depletion Approximation

- Assume the QNR's are perfectly charge neutral
- Assume the SCR is depleted of carriers
  - *depletion region*
- Transition between SCR and QNR's sharp at
  - $-x_{no}$  and  $x_{no}$  (**must calculate where to place these**)



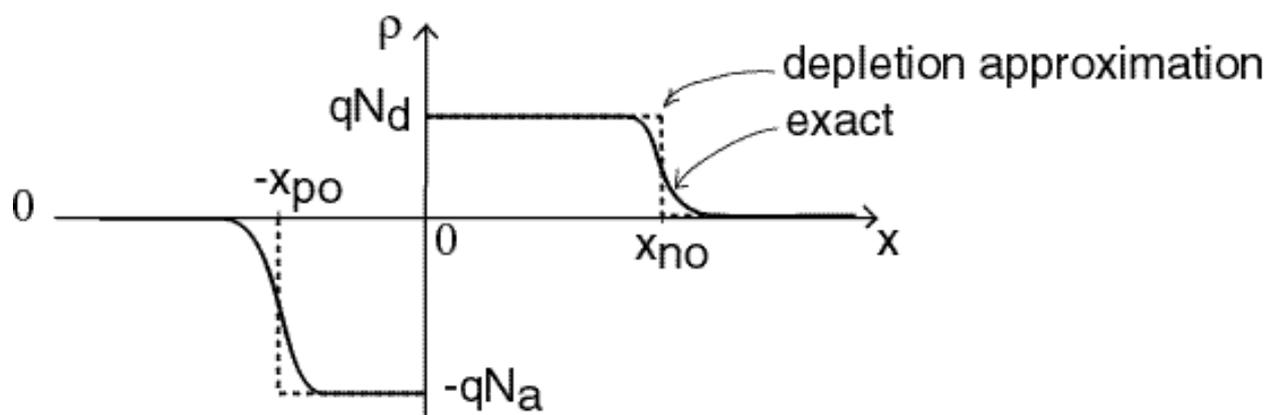
$$x < -x_{po}; \quad p_o(x) = N_a, \quad n_o(x) = \frac{n_i^2}{N_a}$$

$$-x_{po} < x < 0; \quad p_o(x), \quad n_o(x) \ll N_a$$

$$0 < x < x_{no}; \quad n_o(x), \quad p_o(x) \ll N_d$$

$$x > x_{no}; \quad n_o(x) = N_d, \quad p_o(x) = \frac{n_i^2}{N_d}$$

## Space Charge Density

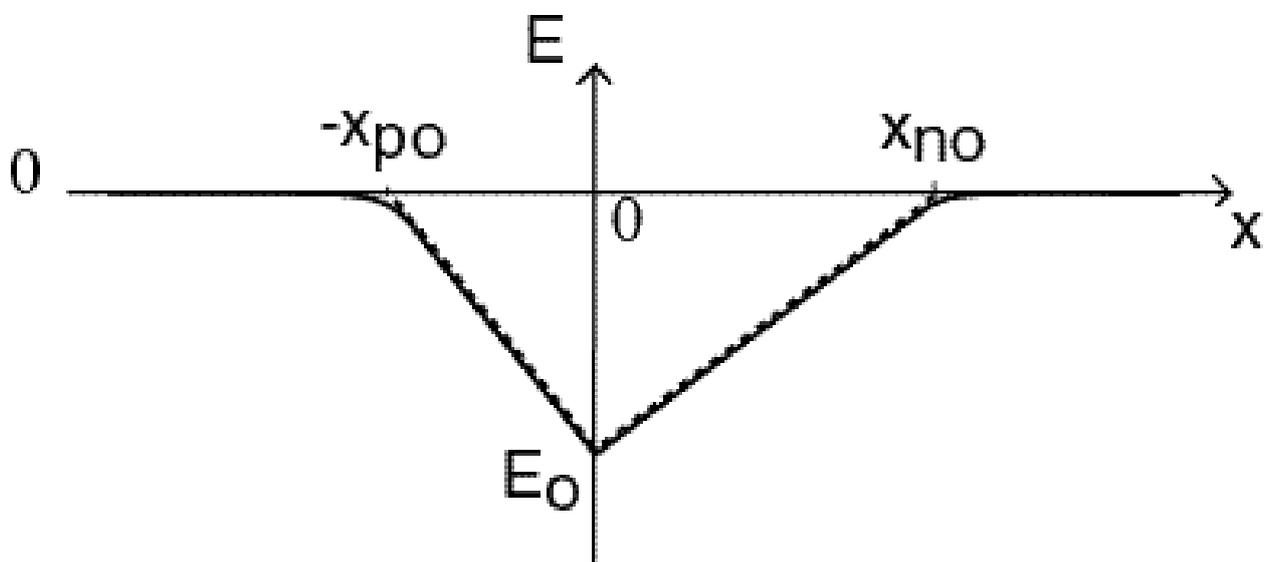


$$\begin{aligned}\rho(x) &= 0; & x < -x_{po} \\ &= -qN_a; & -x_{po} < x < 0 \\ &= qN_d; & 0 < x < x_{no} \\ &= 0; & x > x_{no}\end{aligned}$$

# Electric Field

Integrate Poisson's equation

$$E(x_2) - E(x_1) = \frac{1}{\epsilon_s} \int_{x_1}^{x_2} \rho(x) dx$$



$$x < -x_{po}; \quad E(x) = 0$$

$$-x_{po} < x < 0; \quad E(x) - E(-x_{po}) = \frac{1}{\epsilon_s} \int_{-x_{po}}^x -qN_a dx'$$

$$= \left[ -\frac{qN_a}{\epsilon_s} x \right]_{-x_{po}}^x = \frac{-qN_a}{\epsilon_s} (x + x_{po})$$

$$0 < x < x_{no}; \quad E(x) = \frac{qN_d}{\epsilon_s} (x - x_{no})$$

$$x > x_{no}; \quad E(x) = 0$$

# Electrostatic Potential

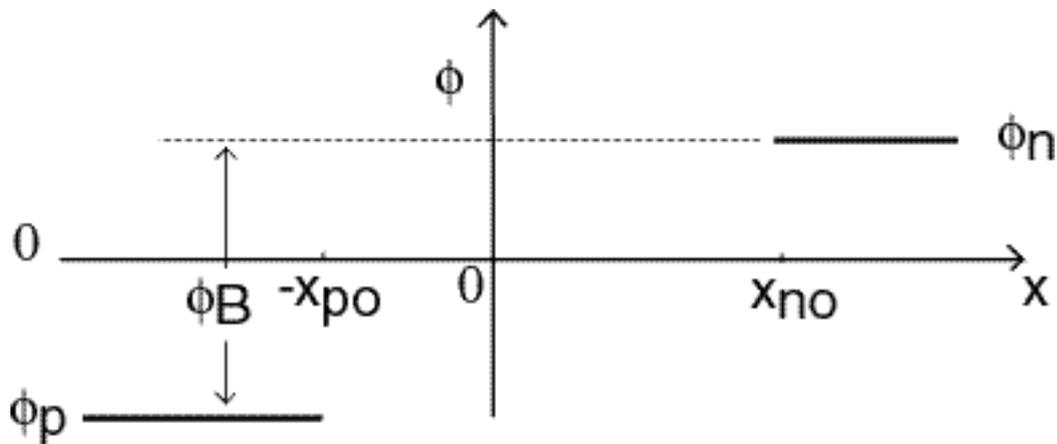
(with  $\phi=0$  @  $n_o=p_o=n_i$ )

$$\phi = \frac{kT}{q} \cdot \ln \frac{n_o}{n_i} \quad \phi = -\frac{kT}{q} \cdot \ln \frac{p_o}{n_i}$$

In QNRs,  $n_o$  and  $p_o$  are known  $\Rightarrow$  can determine  $\phi$

$$\text{in p-QNR: } p_o=N_a \Rightarrow \phi_p = -\frac{kT}{q} \cdot \ln \frac{N_a}{n_i}$$

$$\text{in n-QNR: } n_o=N_d \Rightarrow \phi_n = \frac{kT}{q} \cdot \ln \frac{N_d}{n_i}$$



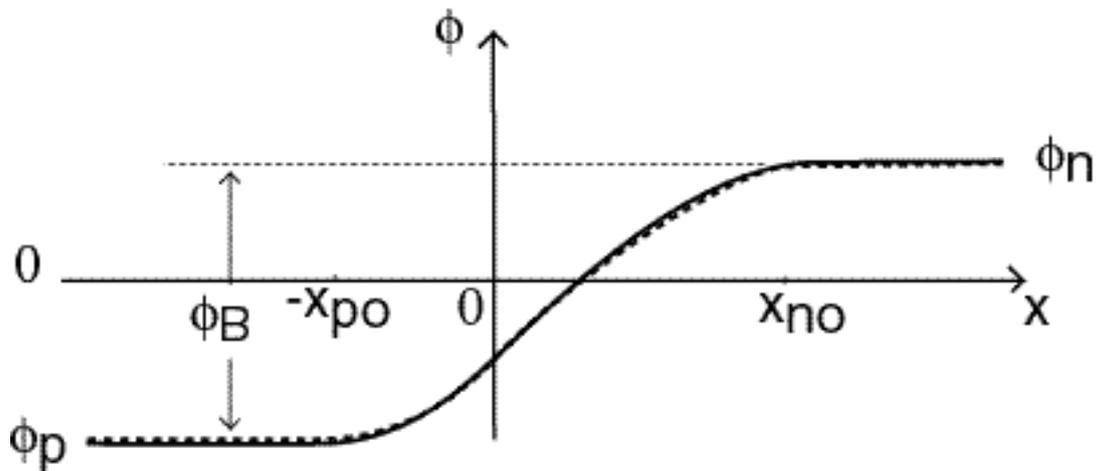
Built-in potential:

$$\phi_B = \phi_n - \phi_p = \frac{kT}{q} \cdot \ln \frac{N_d N_a}{n_i^2}$$

**This expression is always correct in TE!  
We did not use depletion approximation.**

**To obtain  $\phi(x)$  in between, integrate  $E(x)$**

$$\phi(x_2) - \phi(x_1) = - \int_{x_1}^{x_2} E(x') dx'$$



$$x < -x_{po}; \quad \phi(x) = \phi_p$$

$$\begin{aligned} -x_{po} < x < 0; \quad \phi(x) - \phi(-x_{po}) &= - \int_{-x_{po}}^x -\frac{qN_a}{\epsilon_s} (x' + x_{po}) dx' \\ &= \frac{qN_a}{2\epsilon_s} (x + x_{po})^2 \end{aligned}$$

$$\phi(x) = \phi_p + \frac{qN_a}{2\epsilon_s} (x + x_{po})^2$$

$$0 < x < x_{no}; \quad \phi(x) = \phi_n - \frac{qN_d}{2\epsilon_s} (x - x_{no})^2$$

$$x > x_{no}; \quad \phi(x) = \phi_n$$

**Almost done ....**

**Still do not know  $x_{no}$  and  $x_{po} \Rightarrow$  need two more equations**

1. Require overall charge neutrality:

$$qN_a x_{po} = qN_d x_{no}$$

2. Require  $\phi(x)$  to be continuous at  $x=0$ ;

$$\phi_p + \frac{qN_a}{2\epsilon_s} x_{po}^2 = \phi_n - \frac{qN_d}{2\epsilon_s} x_{no}^2$$

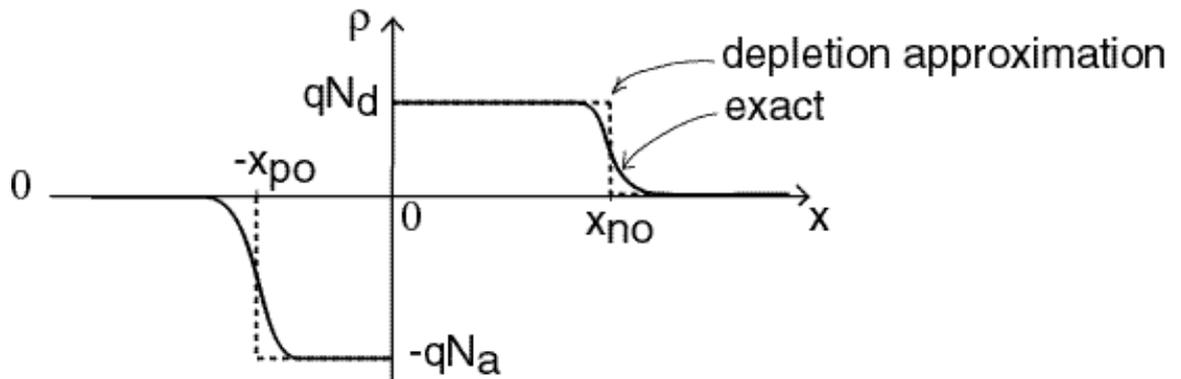
Two equations with two unknowns — obtain solution:

$$x_{no} = \sqrt{\frac{2\epsilon_s \phi_B N_a}{q(N_a + N_d)N_d}} \quad x_{po} = \sqrt{\frac{2\epsilon_s \phi_B N_d}{q(N_a + N_d)N_a}}$$

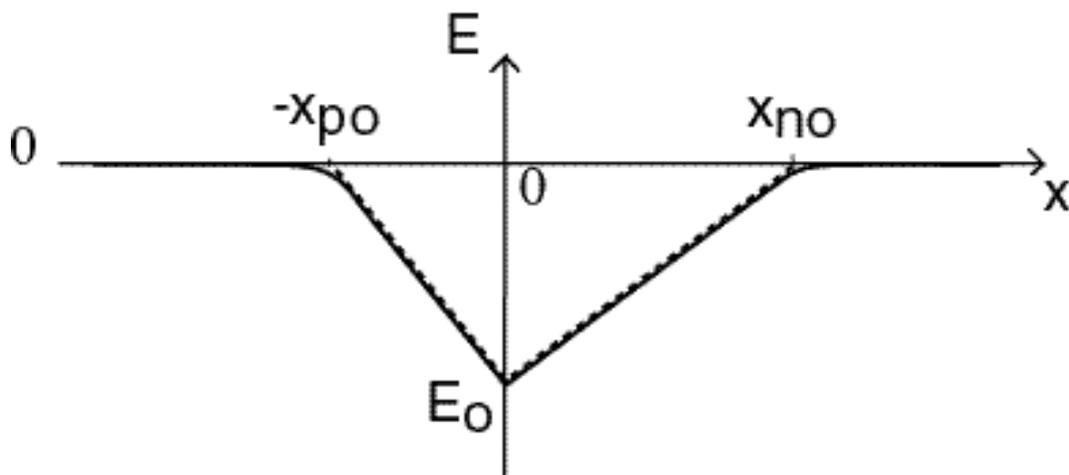
**Now problem is completely solved!**

# Solution Summary

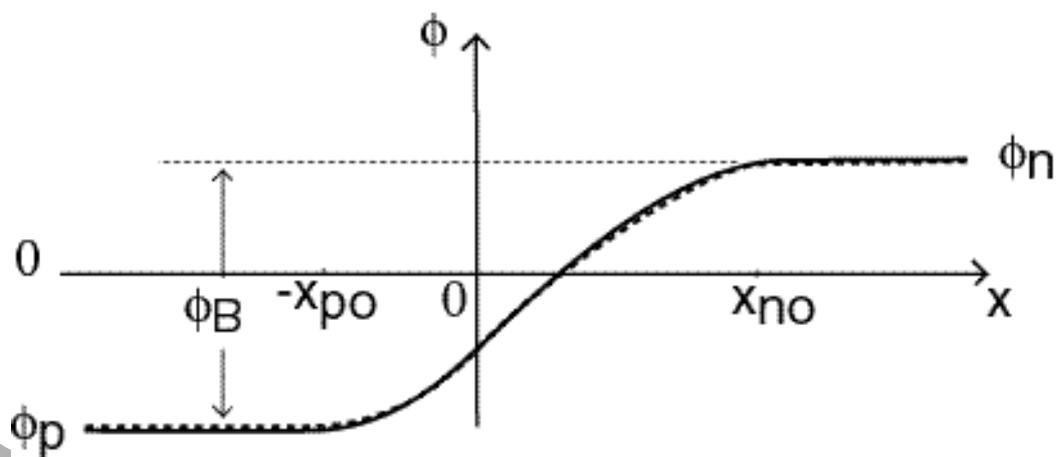
## Space Charge Density



## Electrostatic Field



## Electrostatic Potential



## Other results:

Width of the space charge region:

$$x_{do} = x_{po} + x_{no} = \sqrt{\frac{2\epsilon_s \phi_B (N_a + N_d)}{q N_a N_d}}$$

Field at the metallurgical junction:

$$|E_o| = \sqrt{\frac{2q\phi_B N_a N_d}{\epsilon_s (N_a + N_d)}}$$

## Three Special Cases

- Symmetric junction:  $N_a = N_d$

$$x_{po} = x_{no}$$

- Asymmetric junction:  $N_a > N_d$

$$x_{po} < x_{no}$$

- Strongly asymmetric junction

- p<sup>+</sup>n junction:  $N_a \gg N_d$

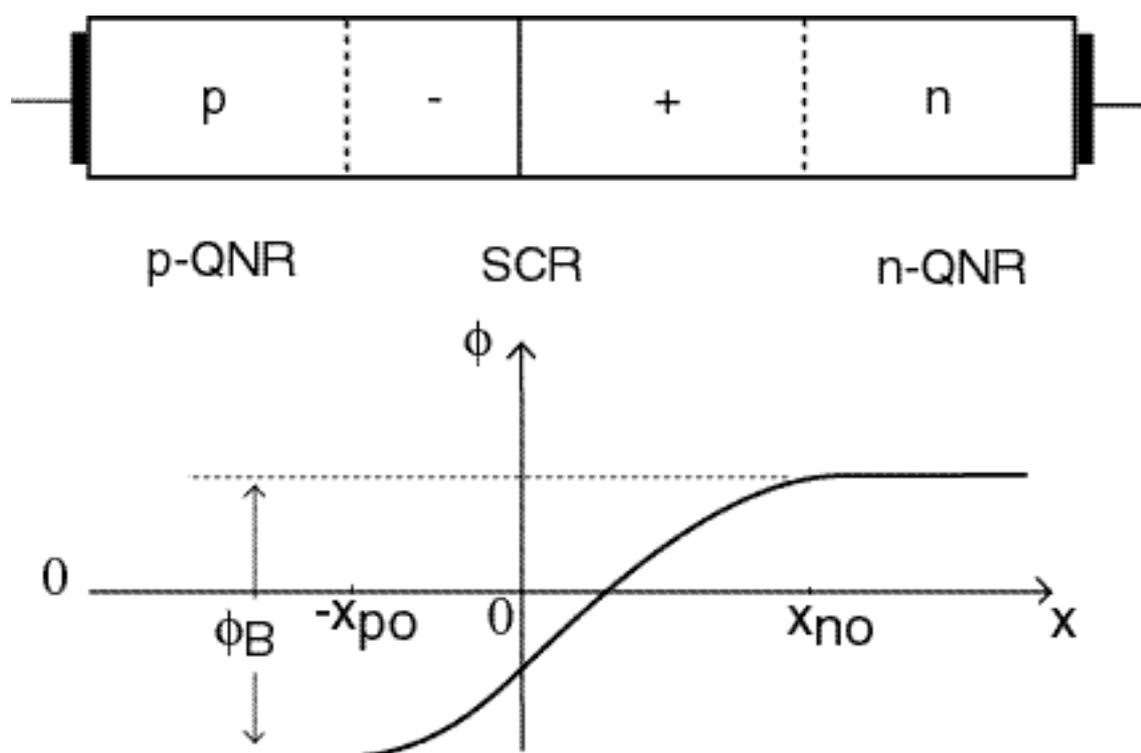
$$x_{po} \ll x_{no} \approx x_{do} \approx \sqrt{\frac{2 \epsilon_s \phi_B}{q N_d}}$$

$$|E_o| \approx \sqrt{\frac{2 q \phi_B N_d}{\epsilon_s}}$$

**The lightly-doped side controls the electrostatics of the pn junction**

## 4. Contact Potential

Potential distribution in thermal equilibrium so far:



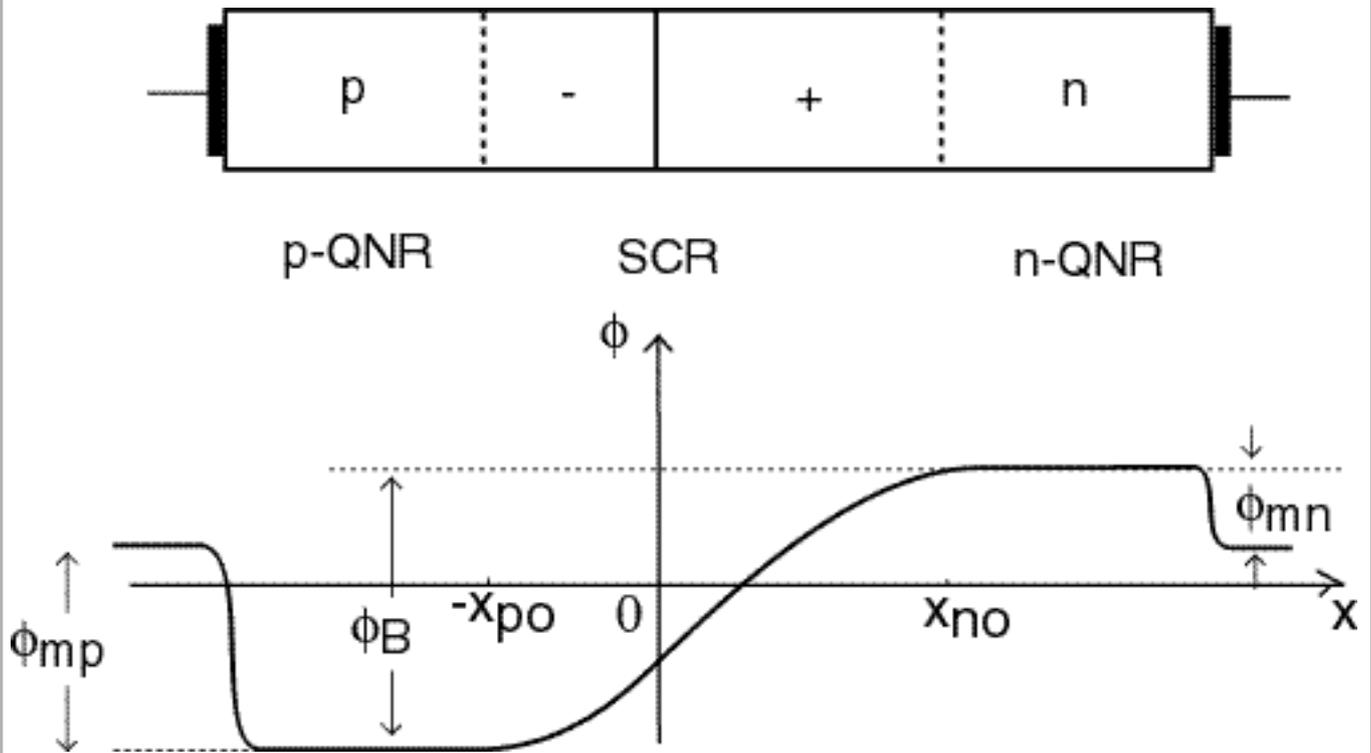
**Question 1:** *If I apply a voltmeter across the pn junction diode, do I measure  $\phi_B$ ?*

- yes                       no                       it depends

**Question 2:** *If I short terminals of pn junction diode, does current flow on the outside circuit?*

- yes                       no                       sometimes

We are missing *contact potential* at the metal-semiconductor contacts:



**Metal-semiconductor contacts:** junction of dissimilar materials

⇒ built-in potentials at contacts  $\phi_{mn}$  and  $\phi_{mp}$ .

Potential difference across structure must be zero

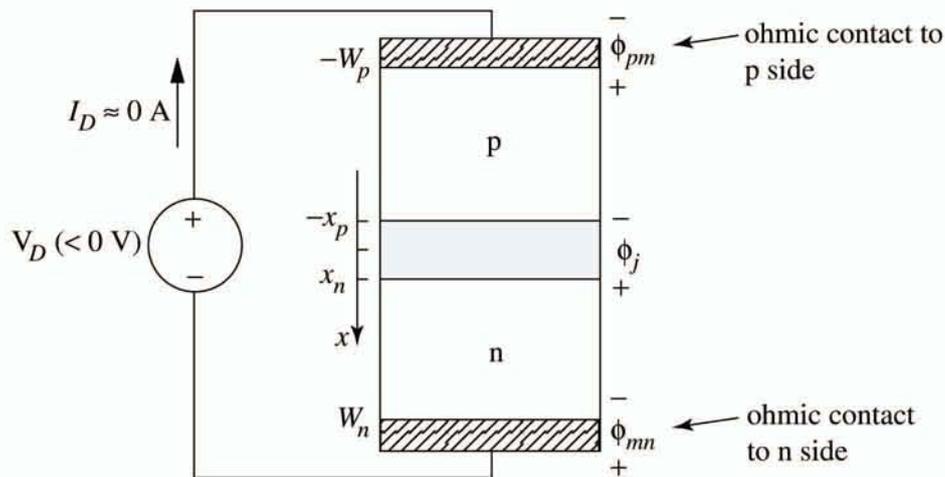
⇒ Cannot measure  $\phi_B$ .

$$\phi_B = |\phi_{mn}| + |\phi_{mp}|$$

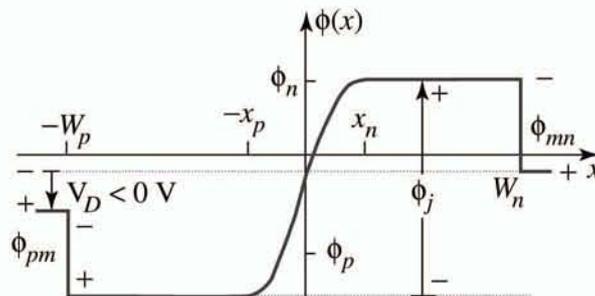
## 5. PN Junction-Reverse Bias

Assume: No Current Flows

$$\phi_j = \phi_B - V_D$$



(a)



(b)

Same Analysis applies:

Substitute

$$x_{do} = x_{po} + x_{no} = \sqrt{\frac{2 \epsilon_s (\phi_B - V_D)(N_a + N_d)}{q N_a N_d}}$$

# What did we learn today?

## Summary of Key Concepts

- Electrostatics of pn junction in equilibrium
  - A *space-charge region* surrounded by two *quasi-neutral regions* formed.
- To first order, carrier concentrations in space-charge region are much smaller than the doping level
  - $\Rightarrow$  can use *Depletion Approximation*
- From contact to contact, there is no potential build-up across the pn junction diode
  - Contact potential(s).

MIT OpenCourseWare  
<http://ocw.mit.edu>

6.012 Microelectronic Devices and Circuits  
Spring 2009

For information about citing these materials or our Terms of Use, visit: <http://ocw.mit.edu/terms>.