Lecture 4

PN Junction and MOS Electrostatics(I)

Semiconductor Electrostatics in Thermal Equilibrium

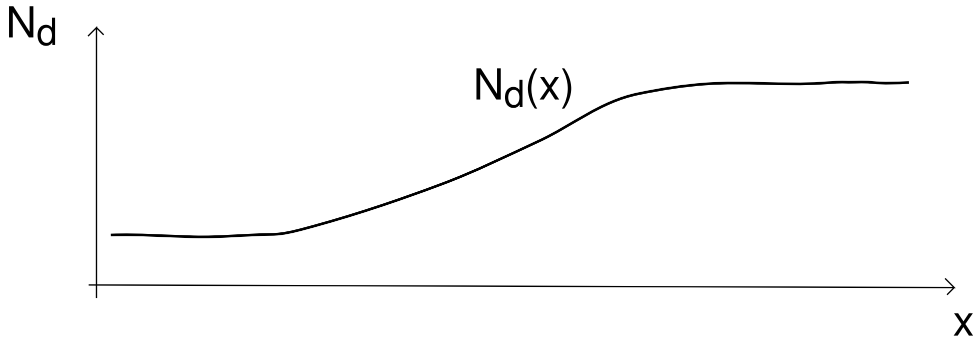
Outline

- Non-uniformly doped semiconductor in thermal equilibrium
- Relationships between potential, $\phi(x)$ and equilibrium carrier concentrations, $p_o(x)$, $n_o(x)$
  - –Boltzmann relations & "60 mV Rule"
- Quasi-neutral situation

Reading Assignment:
Howe and Sodini; Chapter 3, Sections 3.1-3.2

1. Non-uniformly doped semiconductor in thermal equilibrium

Consider a piece of n-type Si in thermal equilibrium with non-uniform dopant distribution:



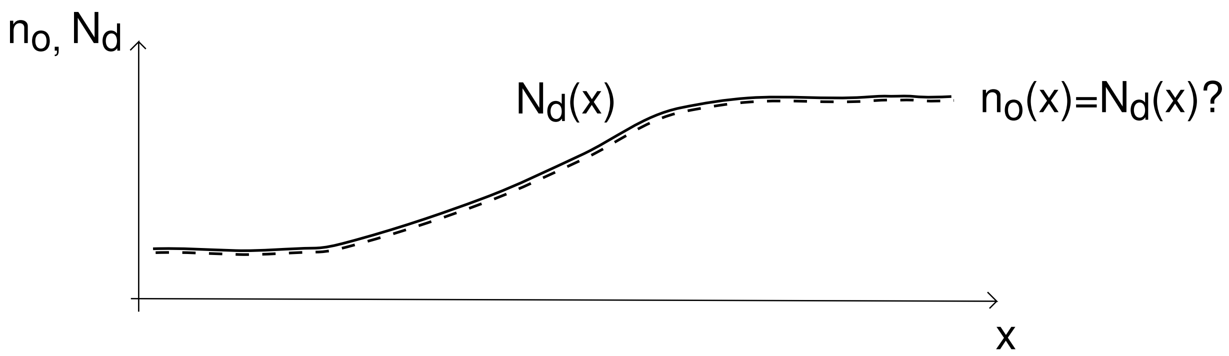
What is the resulting electron concentration in thermal equilibrium?

n-type $\Rightarrow$ lots of electrons, few holes

$\Rightarrow$ focus on electrons

OPTION 1: electron concentration follows doping concentration EXACTLY ⇒

$$n_o(x) = N_d(x)$$



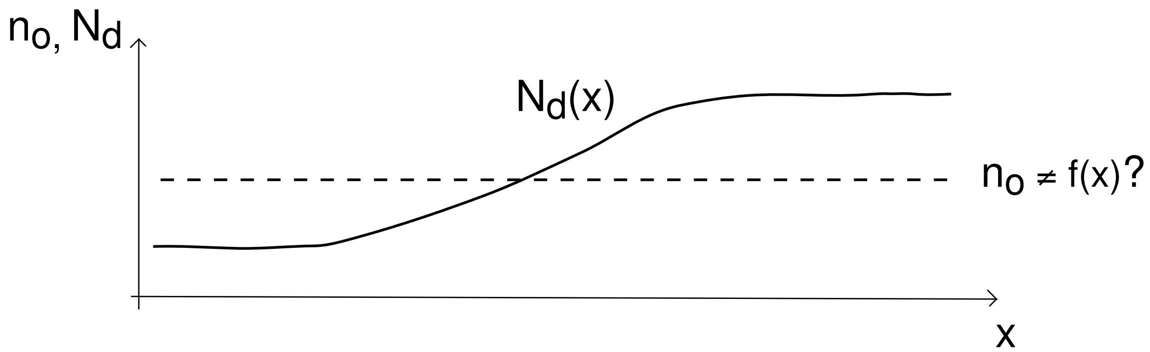
Gradient of electron concentration

⇒ **net electron diffusion**

⇒ **not in thermal equilibrium**!

OPTION 2: electron concentration uniform in space

$$n_o(x) = n_{ave} \neq f(x)$$



Think about space charge density:

$$\rho(x) \approx q\left[N_d(x) - n_o(x)\right]$$

If $N_d(x) \neq n_o(x)$

$\Rightarrow \rho(x) \neq 0$

$\Rightarrow$ electric field

$\Rightarrow$ **net electron drift**

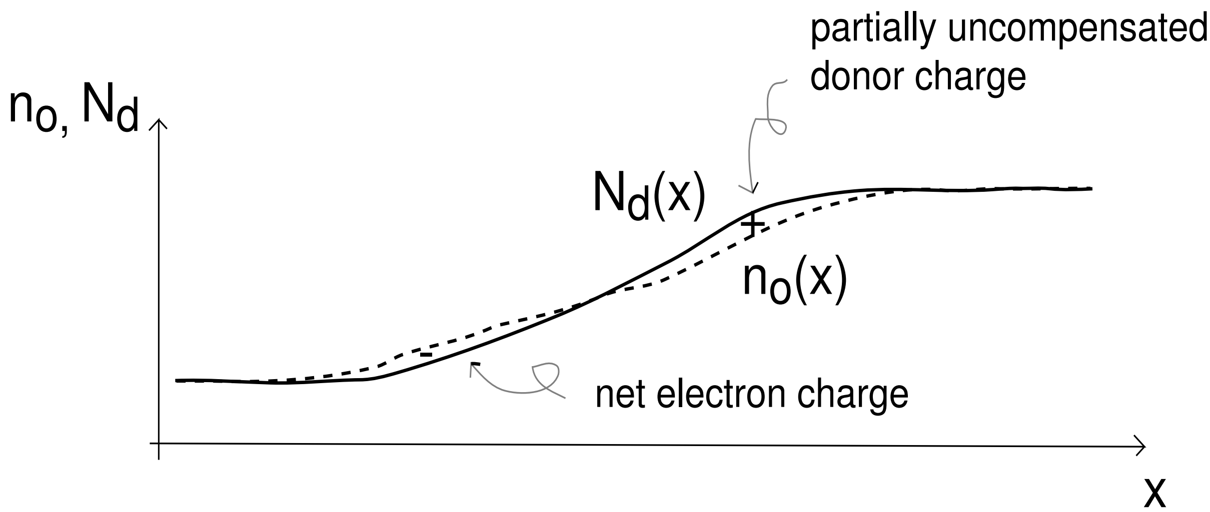
$\Rightarrow$ not in thermal equilibrium!

OPTION 3: Demand that $J_n = 0$ in thermal equilibrium at every x ($J_p = 0$ too)

Diffusion precisely balances Drift

$$J_n(x) = J_n^{drift}(x) + J_n^{diff}(x) = 0$$

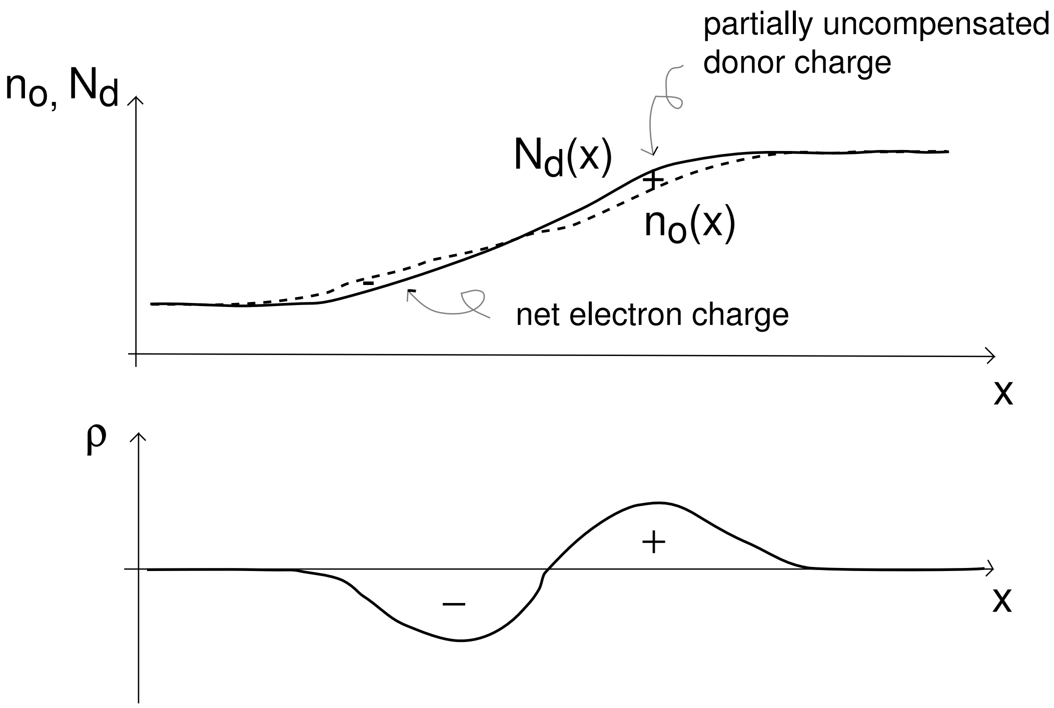
What is $n_o(x)$ that satisfies this condition?



Let us examine the electrostatics implications of $n_o(x) \neq N_d(x)$

Space charge density

$$\rho(x) = q\left[N_d(x) - n_o(x)\right]$$



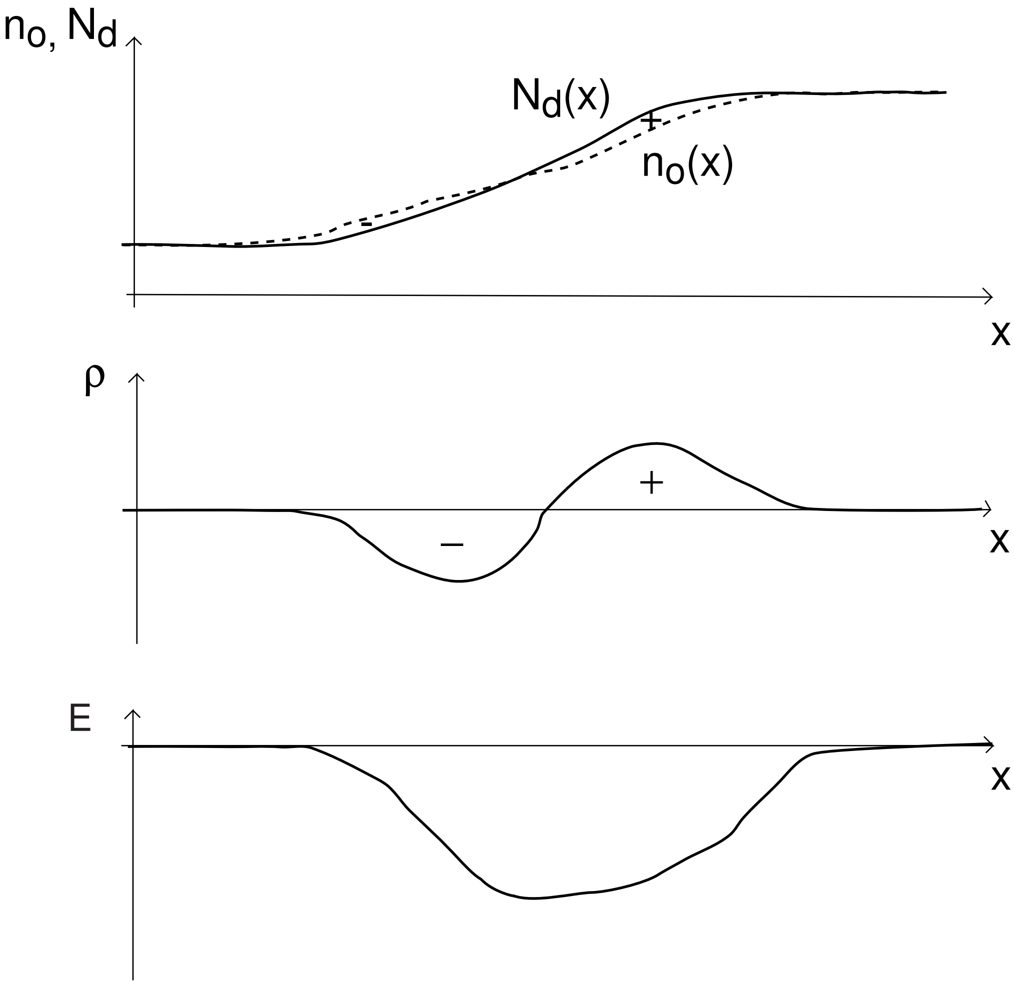
Electric Field

Poisson's equation:

$$\frac{\mathbf{dE}}{\mathbf{dx}} = \frac{\rho(\mathbf{x})}{\varepsilon_s}$$

Integrate from x = 0:

$$E(x) - E(0) = \frac{1}{\varepsilon_s}\int_0^x \rho(x')\,dx'$$



Electrostatic Potential

$$\frac{d\phi}{dx} = -\mathbf{E}(\mathbf{x})$$

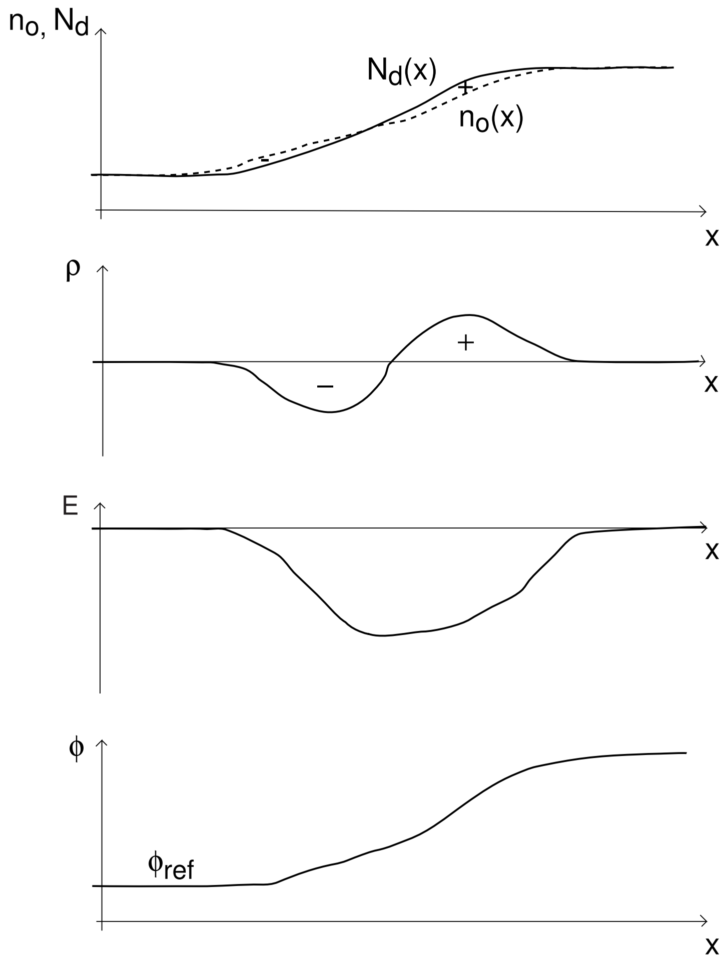
Integrate from x=0:

$$\phi(x) - \phi(0) = -\int_0^x E(x')dx'$$

Need to select reference

(**physics is in the potential difference, not in absolute value!**);

Select $\phi(x = 0) = \phi_{ref}$



2. Relationships between potential, ϕ(x) and equilibrium carrier concentrations, $p_o(x)$, $n_o(x)$ (Boltzmann relations)

$$J_n = 0 = q n_o \mu_n E + q D_n \frac{dn_o}{dx}$$

$$\frac{\mu_n}{D_n} \bullet \frac{d\phi}{dx} = \frac{1}{n_o} \bullet \frac{dn_o}{dx}$$

Using Einstein relation:

$$\frac{q}{kT} \bullet \frac{d\phi}{dx} = \frac{d(\ln n_o)}{dx}$$

Integrate:

$$\frac{q}{kT}\left(\phi - \phi_{ref}\right) = \ln n_o - \ln n_{o,ref} = \ln \frac{n_o}{n_{o,ref}}$$

Then:

$$n_o = n_{o,ref} \exp\left[\frac{q\left(\phi - \phi_{ref}\right)}{kT}\right]$$

Any reference is good

In 6.012, $\phi_{ref} = 0$ at $n_{o,ref} = n_i$

Then:

$$n_o = n_i e^{q\phi/kT}$$

If we do same with holes (starting with $J_p = 0$ in thermal equilibrium, or simply using $n_o p_o = n_i^2$);

$$p_o = n_i e^{-q\phi/kT}$$

We can re-write as:

$$\phi = \frac{kT}{q} \bullet \ln\frac{n_o}{n_i}$$

and

$$\phi = -\frac{kT}{q} \bullet \ln\frac{p_o}{n_i}$$

"60 mV" Rule

At room temperature for Si:

$$\phi = (25\,\mathrm{m}\quad)\bullet \ln\frac{n_o}{n_i} = (25\,\mathrm{mV})\bullet \ln(10)\bullet \log\frac{n_o}{n_i}$$

Or

$$\phi \approx (60\,\mathrm{m}\quad)\bullet \log\frac{n_o}{n_i}$$

EXAMPLE 1:

$$n_o = 10^{18}\,\mathrm{cm}^{-3} \quad \Rightarrow \quad \phi = (60\,\mathrm{m}\quad)\times 8 = 480\,\mathrm{mV}$$

"60 mV" Rule: contd.

With holes:

$$\phi = -(25\,\mathrm{m}\quad)\bullet \ln\frac{p_o}{n_i} = -(25\,\mathrm{m}\quad)\bullet \ln(10)\bullet \log\frac{p_o}{n_i}$$

Or

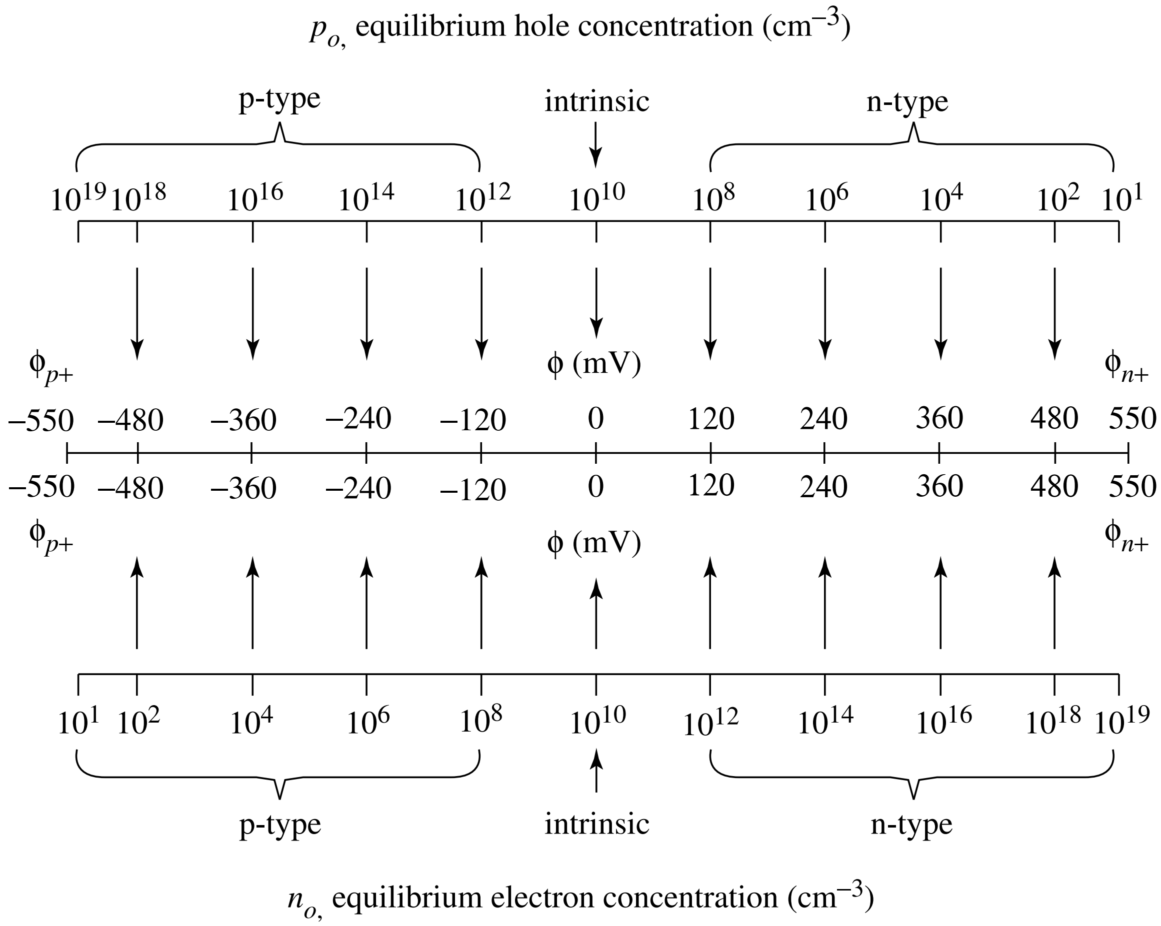
$$\phi \approx -(60\,\mathrm{m}\quad)\bullet \log\frac{p_o}{n_i}$$

EXAMPLE 2:

$$n_o = 10^{18}\,\mathrm{cm}^{-3} \Rightarrow p_o = 10^{2}\,\mathrm{cm}^{-3}$$

$$\Rightarrow \phi = -(60\,\mathrm{m}\quad)\times -8 = 480\,\mathrm{mV}$$

Relationship between ϕ, n_o and p_o:



Note: ϕ cannot exceed 550 mV or be smaller than -550 mV. (Beyond this point different physics come into play.)

Example 3: Compute potential difference in thermal equilibrium between region where $n_o = 10^{17}$ cm^{-3} and $n_o = 10^{15}$ cm^{-3}.

$$\phi(n_o = 10^{17}\ cm^{-3}) = 60 \times 7 = 420\ mV$$

$$\phi(n_o = 10^{15}\ cm^{-3}) = 60 \times 5 = 300\ mV$$

$$\phi(n_o = 10^{17}\ cm^{-3}) - \phi(n_o = 10^{15}\ cm^{-3}) = 120\ mV$$

Example 4: Compute potential difference in thermal equilibrium between region where $p_o = 10^{20}$ cm^{-3} and $p_o = 10^{16}$ cm^{-3}.

$$\phi(p_o = 10^{20}\ cm^{-3}) = \phi_{max} = -550\ mV$$

$$\phi(p_o = 10^{16}\ cm^{-3}) = -60 \times 6 = -360\ mV$$

$$\phi(p_o = 10^{20}\ cm^{-3}) - \phi(p_o = 10^{16}\ cm^{-3}) = -190\ mV$$

3. Quasi-neutral situation

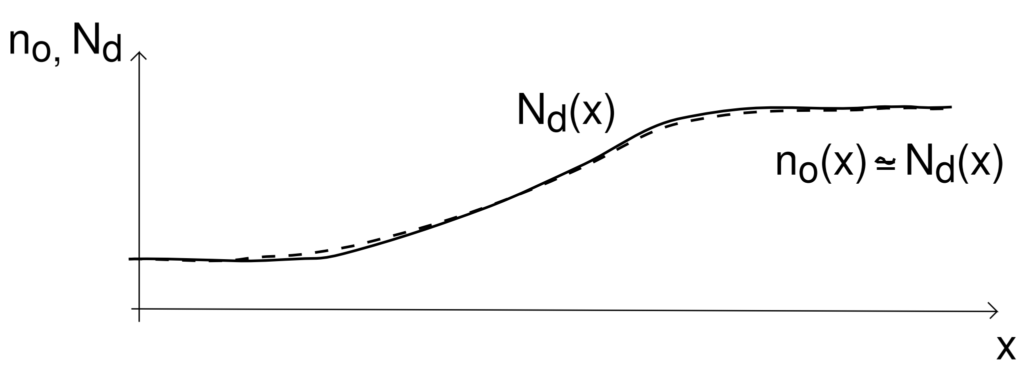
If $N_d(x)$ changes slowly with x ⇒ $n_o(x)$ also changes slowly with x. WHY?

Small dn_o/dx implies a small diffusion current. We do not need a large drift current to balance it.

Small drift current implies a small electric field and therefore a small space charge

Then: $$n_o(x) \approx N_d(x)$$

$n_o(x)$ tracks $N_d(x)$ well ⇒ minimum space charge
⇒ semiconductor is quasi-neutral



What did we learn today?

Summary of Key Concepts

- It is possible to have an electric field inside a semiconductor in thermal equilibrium
  - $\Rightarrow$ ***Non-uniform doping distribution.***
- In thermal equilibrium, there is a fundamental relationship between the $\phi(x)$ and the equilibrium carrier concentrations $n_o(x)$ & $p_o(x)$
  - **Boltzmann relations (or "60 mV Rule").**
- In a slowly varying doping profile, majority carrier concentration tracks well the doping concentration.