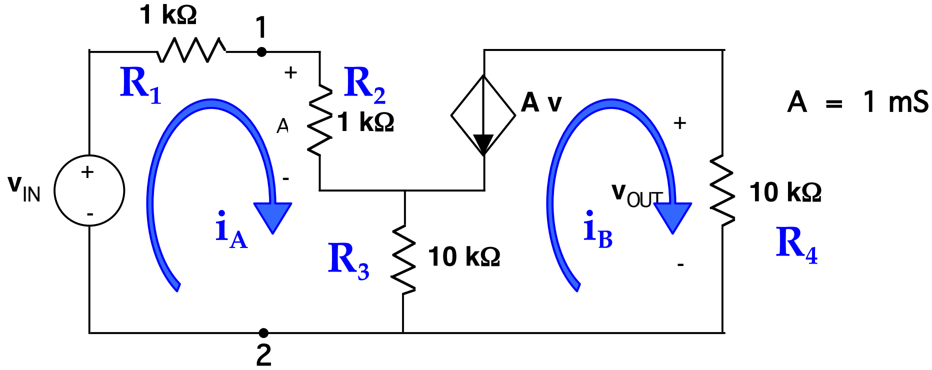
6.012 - Electronic Devices and Circuits - Take Home Diagnostic Exercise

The four problems in this exercise deal with material you will use in 6.012 from the prerequisite courses. Give yourself one hour to work it. Solutions will be distributed at the next recitation and you can then judge for yourself which material you need to review.

Problem 1 - Basic linear circuit analysis

Consider the circuit illustrated below:



(a) What is the voltage v_{OUT} when the input voltage, v_{IN}, is 2 Volts?

First notice that $v_{OUT} = R_4 i_B$, $i_B = -Av_A$, and $v_A = R_2 i_A$. Then do a loop equation for Loop A to get: $i_A(R_1 + R_2) + (i_A - i_B)R_3 = v_{IN}$. Combine the first two equations to get $v_{OUT} = -AR_4 v_A$ and then put the second and third equations into the last equation and solve for v_A in terms of v_{IN}. Put this into the expression for v_{OUT} to get: $v_{OUT} = -\{AR_2R_4/[R_1 + R_2(1+AR_3) + R_3]\}v_{IN}$. Putting values in yields: $v_{OUT} = -10v_{IN}/22$ V.

(b) What is the Thevenin equivalent resistance of the circuit to the right of Nodes 1 and 2?

The Thevenin equivalent resistance we want is v_{12}/i_A, and $v_{12} = R_2 i_A + (i_A + Av_A)R_3$. Substitute $v_A = R_2 i_A$ and solve for R_T: $R_T = v_{12}/i_A = R_2 + R_3(1 + AR_2) = 22\ k\Omega$.

Problem 2 - Linear differential equations

Consider a parameter $c_P(t)$, which varies temporally according to the equation:

$$dc_P(t)/dt + A\,c_P(t) = B$$

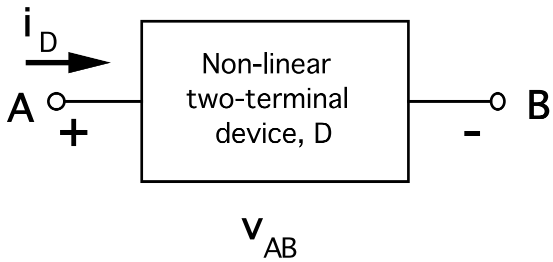
where A and B are positive constants. What is $c_P(t)$ for $t > 0$ if $c_P(0) = 0$ and $c_P(t)$ is finite?

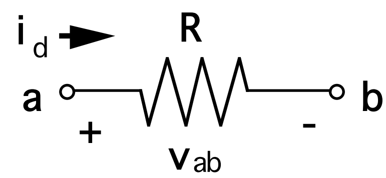
The homogeneous solution to this equation is: $c_{p,HS}(t) = C_1 e^{-At}$. The particular solution is: $c_{p,HS}(t) = B/A$. the total solution is thus: $c_p(t) = B/A + C_1 e^{-At}$, with C_1 such that $c_p(0) = 0$. Clearly $C_1 = -B/A$, and thus

$$c_p(t) = \frac{B}{A}\left(1 - C_1 e^{-At}\right)$$

Problem 3 - Linearization about an operating point

Consider the non-linear two-terminal electronic device, D, illustrated below on the left. The current through the device, $i_D(t)$, is related to the voltage across its terminals, $v_{AB}(t)$, by the equation, $i_D(t) = B\,(v_{AB} + C\,v_{AB}^3)$, where $B = 10^{-3}$ A/V and $C = 4\ V^{-2}$.. There is no charge or flux storage associated with this device.



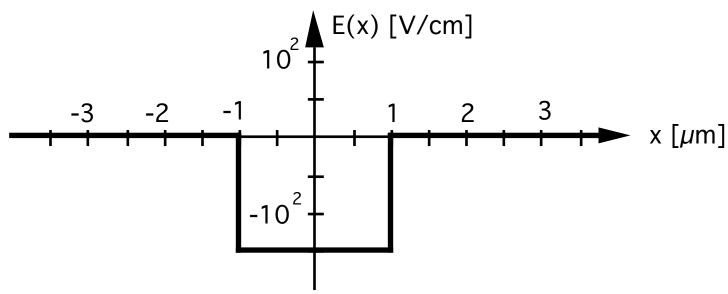


A linear equivalent circuit for the device D valid for small signal operation about a bias point, $v_{AB} = V_{AB}$, is shown to the right. What is R for the bias point $V_{AB} = 2V$?

$$R = \frac{1}{g_d},\ \text{where } g_d \equiv \left.\frac{\partial i_D}{\partial v_{AB}}\right|_{v_{AB}=V_{AB}}.\ \text{We find: } g_d = B\left(1+3CV_{AB}^2\right).\ \text{Thus: } R = 1\Big/\left[B\left(1+3CV_{AB}^2\right)\right] = 40\ \Omega.$$

Problem 4 - Simple electrostatics

In a certain sample of material the electric field, E(x,y,z), varies only in the x-direction, i.e., E(x,y,z) = E(x), where E(x) is shown below. The dielectric constant, ε, is uniform throughout the sample with a value of 10^{-10} coul/V-cm.



On the axes provided below, sketch and dimension the net charge distribution in this sample.

$$\rho(x) = \varepsilon\frac{dE}{dx} = Q^*\left[\delta(1\mu m) - \delta(-1\mu m)\right],\ \text{where } Q^* = \left(10^{-10}\ coul/V\text{-}cm\right)\left(1.5\ x\ 10^2\ V/cm\right) = 1.2x10^{-12}\ coul/cm^2.$$