Lecture 11 - MOSFETs II; Large Signal Models - Outline

- **Announcements**

  On Stellar - 2 write-ups on MOSFET models

- **The Gradual Channel Approximation** (review and more)

  MOSFET model: gradual channel approximation **(Example: n-MOS)**

$$i_D \approx \begin{cases} 0 & \text{for } (v_{GS} - V_T)/\alpha \le 0 \le v_{DS} \quad \textbf{(cutoff)} \\ K(v_{GS} - V_T)^2/2\alpha & \text{for } 0 \le (v_{GS} - V_T)/\alpha \le v_{DS} \quad \textbf{(saturation)} \\ K(v_{GS} - V_T - \alpha v_{DS}/2)v_{DS} & \text{for } 0 \le v_{DS} \le (v_{GS} - V_T)/\alpha \quad \textbf{(linear)} \end{cases}$$

  with $K \equiv (W/L)\mu_e C_{ox}^*$, $V_T = V_{FB} - 2\phi_{p\text{-}Si} + [2\varepsilon_{Si}\, qN_A(|2\phi_{p\text{-}Si}| - v_{BS})]^{1/2}/C_{ox}^*$

  and $\alpha = 1 + [(\varepsilon_{Si}\, qN_A/2(|2\phi_{p\text{-}Si}| - v_{BS})]^{1/2}/C_{ox}$ **(frequently $\alpha \approx 1$)**

- **Refined device models for transistors** (MOS and BJT)

  Other flavors of MOSFETS: p-channel, depletion mode

  The Early Effect:

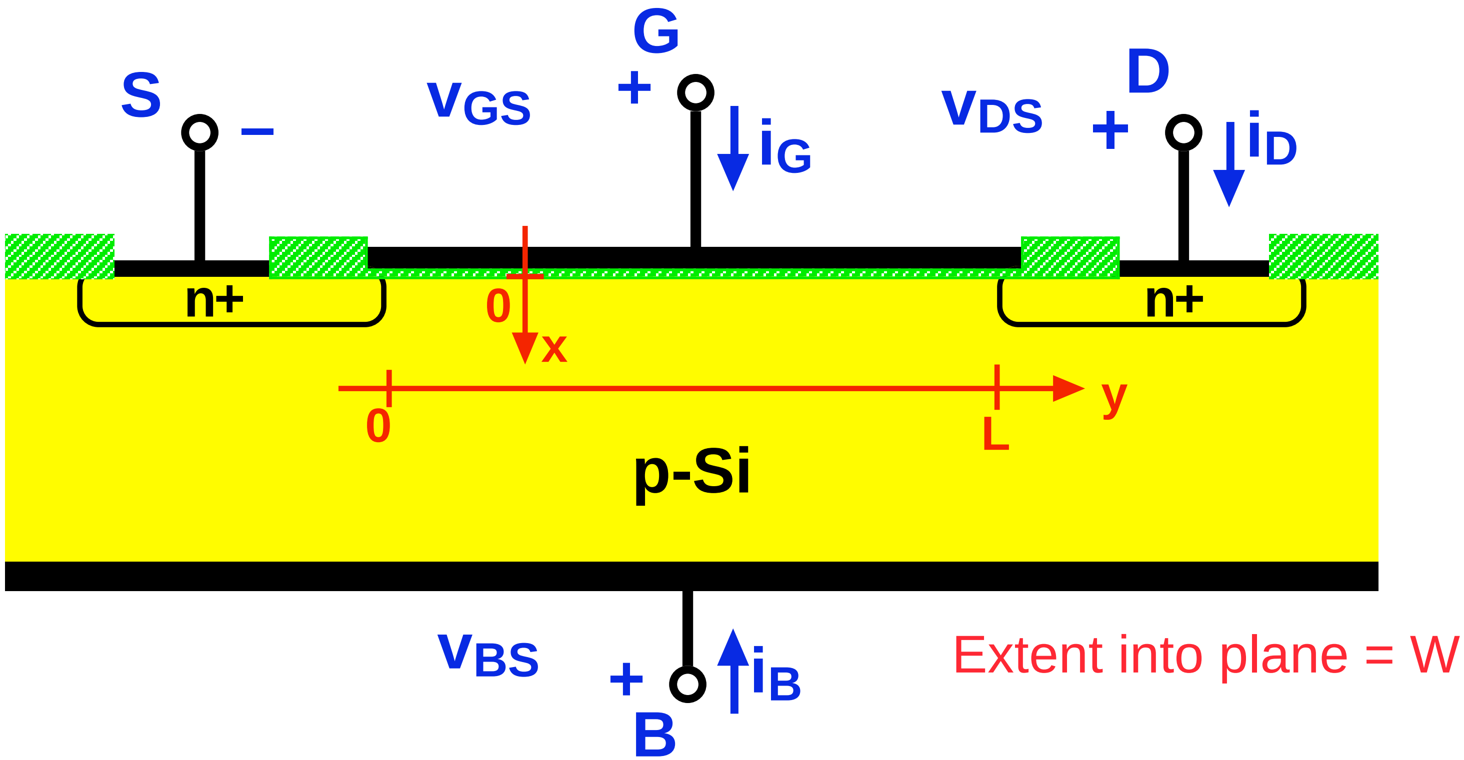
  1. Base-width modulation in BJTs: $w_B(v_{CE})$
  2. Channel-length modulation in MOSFETs: $L(v_{DS})$

  Charge stores:

  1. Junction diodes
  2. BJTs
  3. MOSFETs

  Extrinsic parasitics: Lead resistances, capacitances, and inductances

An n-channel MOSFET showing gradual channel axes



Gradual Channel Approximation:

- A one-dimensional electrostatics problem in the x direction is solved to find the channel charge, $q_N^*(y)$; this charge depends on v_{GS}, $v_{CS}(y)$ and v_{BS}.
- A one-dimensional drift problem in the y direction then gives the channel current, i_D, as a function of v_{GS}, v_{DS}, and v_{BS}.

Gradual Channel Approximation i-v Modeling

(n-channel MOS used as the example)

The Gradual Channel Approximation is the approach typically used to model the drain current in field effect transistors.*

It assumes that the drain current, i_D, consists entirely of carriers flowing in the channel of the device, and is thus proportional to the sheet density of carriers at any point and their net average velocity. It is not a function of y, but its components in general are:

$$i_D = -W \cdot -q \cdot n^*_{ch}(y) \cdot \bar{s}_{ey}(y)$$

In this expression, W is the width of the device, $-q$ is the charge on each electron, $n^*_{ch}(y)$ is sheet electron concentration in the channel (i.e. electrons/cm^2) at y, and $\bar{s}_{ey}(y)$ is the net electron velocity in the y-direction.

If the electric field is not too large, $\bar{s}_{ey}(y) = -\mu_e E_y(y)$, and

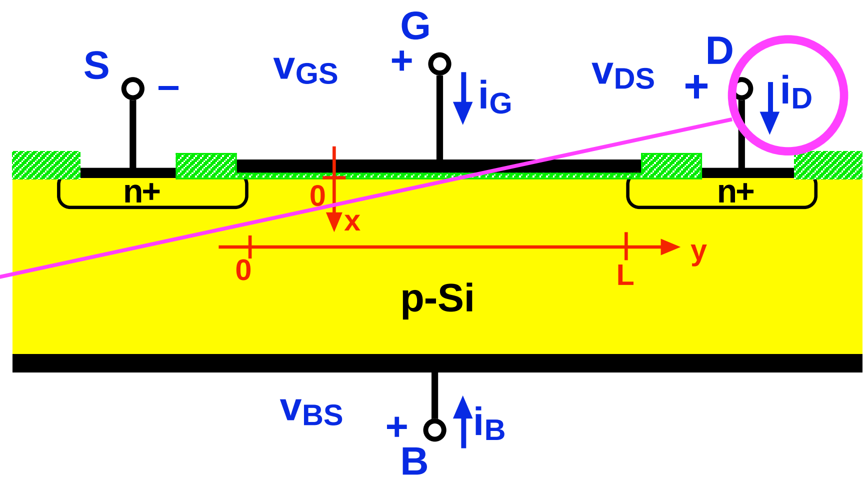
$$i_D = -W \cdot q \cdot n^*_{ch}(y) \cdot \mu_e E_y(y) = W \cdot q \cdot n^*_{ch}(y) \cdot \mu_e \frac{dv_{CS}(y)}{dy}$$

Cont.

* Junction FETs (JFETs), MEtal Semiconductor FETs (MESFETs[1]), and Heterojunction FETs (HJFETs[2]), as well as Metal Oxide Semiconductor FETs (MOSFETs).

1. Also called Shottky Barrrier FETs (SBFETs). 2. Includes HEMTs, TEGFETs, MODFETs, SDFETs, HFETs, PHEMTs, MHEMTs, etc.

GCA i-v Modeling, cont.



We have:

$$i_D = W \cdot q \cdot n^*_{ch}(y) \cdot \mu_e \frac{dv_{CS}(y)}{dy}$$

To eliminate the derivative from this equation we integrate both sides with respect to y from the source (y = 0) to the drain (y = L). This corresponds to integrating the right hand side with respect to v_{CS} from 0 to v_{DS}, because $v_{CS}(0) = 0$ to $v_{CS}(L) = v_{DS}$:

$$\int_0^L i_D\, dy = W \cdot \mu_e \cdot q \cdot \int_0^L n^*_{ch}(y) \frac{dv_{CS}(y)}{dy} dy = W \cdot \mu_e \cdot q \cdot \int_0^{v_{DS}} n^*_{ch}(v_{CS})\, dv_{CS}$$

The left hand integral is easy to evaluate; it is simply $i_D L$. Thus we have:

$$\int_0^L i_D\, dy = i_D L \quad \Rightarrow \quad \boxed{i_D = \frac{W}{L} \cdot \mu_e \cdot q \cdot \int_0^{v_{DS}} n^*_{ch}(v_{CS})\, dv_{CS}}$$

GCA i-v Modeling, cont.

The various FETs differ primarily in the nature of their channels and thereby, the expressions for $n^*_{ch}(y)$.

For a <u>MOSFET</u> we speak in terms of the inversion layer charge, $q_n^*(y)$, which is equivalent to - $q \cdot n^*_{ch}(y)$. Thus we have:

$$i_D = -\frac{W}{L}\mu_e \int_0^{v_{DS}} q_n^*(v_{GS}, v_{CS}, v_{BS})\, dv_{CS}$$

We derived q_n^* earlier by solving the vertical electrostatics problem, and found:

$$q_n^*(v_{GS}, v_{CS}, v_{BS}) = -C_{ox}^*\left[v_{GS} - v_{CS} - V_T(v_{CS}, v_{BS})\right]$$

$$\text{with} \quad V_T(v_{CS}, v_{BS}) = V_{FB} - 2\phi_{p-Si} + \left\{2\varepsilon_{Si} q N_A\left[\left|2\phi_{p-Si}\right| - v_{BS} + v_{CS}\right]\right\}^{1/2} \Big/ C_{ox}^*$$

Using this in the equation for i_D, we obtain:

$$i_D(v_{GS}, v_{DS}, v_{BS}) = \frac{W}{L}\mu_e \int_0^{v_{DS}} \left\{C_{ox}^*\left[v_{GS} - v_{CS} - V_T(v_{CS}, v_{BS})\right]\right\} dv_{CS}$$

At this point we can do the integral, but it is common to simplify the expression of $V_T(v_{CS}, v_{BS})$ first, to get a more useful result.

GCA - dealing with the non-linear dependence of V_T on v_{CS}

Approach #1 - Live with it

Even though $V_T(v_{CS},v_{BS})$ is a non-linear function of v_{CS}, we can still put it in this last equation for i_D:

$$i_D = \frac{W}{L}\mu_e \int_0^{v_{DS}} \left\{ C_{ox}^* \left[v_{GS} - v_{CS} - V_{FB} + 2\phi_{p-Si} - \frac{t_{ox}}{\varepsilon_{ox}} \sqrt{2\varepsilon_{Si} q N_A \left[\left|2\phi_{p-Si}\right| - v_{BS} + v_{CS} \right]} \right] \right\} dv_{CS}$$

and do the integral, obtaining:

$$i_D(v_{DS},v_{GS},v_{BS}) = \frac{W}{L}\mu_e C_{ox}^* \left\{ \left(v_{GS} - \left|2\phi_p\right| - V_{FB} - \frac{v_{DS}}{2} \right) v_{DS} + \frac{3}{2}\sqrt{2\varepsilon_{Si} q N_A}\left[\left(\left|2\phi_p\right| + v_{DS} - v_{BS} \right)^{3/2} - \left(\left|2\phi_p\right| - v_{BS} \right)^{3/2} \right] \right\}$$

The problem is that this result is very unwieldy, and difficult to work with. More to the point, we don't have to live with it because it is easy to get very good, approximate solutions that are much simpler to work with.

GCA - dealing with the non-linear dependence of V_T on v_{CS}

Approach #2 - Ignore it

Early on researchers noticed that the difference between V_T at 0 and at y, i.e. $V_T(0,v_{BS})$ and $V_T(v_{DS},v_{BS})$, is small, and that using $V_T(0,v_{BS})$ alone gives a result that is still quite accurate and is very easy to use:

$$i_D(v_{GS},v_{DS},v_{BS}) = \frac{W}{L}\mu_e \int_0^{v_{DS}} \left\{ C_{ox}^* \left[v_{GS} - v_{CS} - V_T(0,v_{BS}) \right] \right\} dv_{CS}$$

$$= \frac{W}{L}\mu_e C_{ox}^* \left\{ \left[v_{GS} - V_T(0,v_{BS}) \right] v_{DS} - \frac{v_{DS}^2}{2} \right\}$$

The variable, v_{CS}, is set to 0 in V_T.

This result looks much simpler than the result of Approach #1, and it is much easier to use in hand calculations. It is, in fact, the one most commonly used by the vast majority of engineers. At the same time, the fact that it was obtained by ignoring the dependence of V_T on v_{CS} is cause for concern, unless we have a way to judge the validity of our approximation. We can get the necessary metric through Approach #3.

GCA - dealing with the non-linear dependence of V_T on v_{CS}

Approach #3 - Linearize it (i.e. expand it, keep first order term)

In this approach we leave the variation of V_T with v_{CS} in, but linearize it by doing a Taylor's series expansion about $v_{CS} = 0$:

$$V_T[v_{CS}, v_{BS}] \approx V_T(0, v_{BS}) + \left.\frac{\partial V_T}{\partial v_{CS}}\right|_{v_{CS}=0} \cdot v_{CS}$$

Taking the derivative and evaluating it at $v_{CS} = 0$ yields:

$$V_T[v_{CS}, v_{BS}] \approx V_T(0, v_{BS}) + \frac{t_{ox}}{\varepsilon_{ox}} \sqrt{\frac{\varepsilon_{Si} q N_A}{2\left(\left|2\phi_p\right| - v_{BS}\right)}} \cdot v_{CS}$$

With this q_n^* is

$$q_n^*(v_{GS}, v_{CS}, v_{BS}) \approx -C_{ox}^*\left[v_{GS} - v_{CS} + V_T(0, v_{BS}) - \frac{t_{ox}}{\varepsilon_{ox}} \sqrt{\frac{\varepsilon_{Si} q N_A}{2\left(\left|2\phi_p\right| - v_{BS}\right)}} \cdot v_{CS}\right]$$

$$= -C_{ox}^*\left[v_{GS} - \alpha v_{CS} + V_T(v_{BS})\right]$$

where

$$\alpha \equiv \left[1 + \frac{t_{ox}}{\varepsilon_{ox}} \sqrt{\varepsilon_{Si} q N_A \Big/ 2\left(\left|2\phi_p\right| - v_{BS}\right)}\right] \quad \text{and} \quad V_T(v_{BS}) \equiv V_T(0, v_{BS})$$

Cont.

GCA - dealing with the non-linear dependence of V_T on v_{CS}

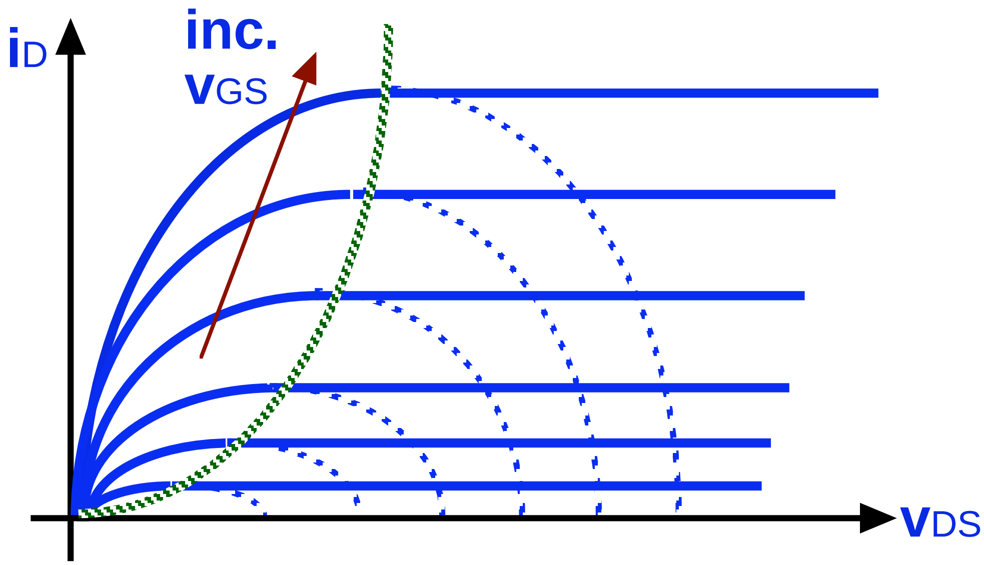
Using this result in the integral in the expression for i_D gives:

$$i_D(v_{GS},v_{DS},v_{BS}) = \frac{W}{L}\mu_e \int_0^{v_{DS}} \left\{ C_{ox}^* \left[v_{GS} - \alpha v_{CS} - V_T(0,v_{BS}) \right] \right\} dv_{CS}$$

$$= \frac{W}{L}\mu_e C_{ox}^* \left\{ \left[v_{GS} - V_T(v_{BS}) \right] v_{DS} - \alpha \frac{v_{DS}^2}{2} \right\}$$

Except for α this is the Approach 2 result.

Plotting this equation for increasing values of v_{GS} we see that it traces inverted parabolas as shown below.



Note: i_D saturates after its peak value (solid lines), rather than decreasing (dashed lines).

Cont.

Gradual Channel Approximation, cont.

The drain current expression, cont:

The point at which i_D reaches its peak value and saturates is easily found. Taking the derivative and setting it equal to zero we find:

$$\frac{\partial i_D}{\partial v_{DS}} = 0 \quad \text{when} \quad v_{DS} = \frac{1}{\alpha}\left[v_{GS} - V_T(v_{BS})\right]$$

What happens physically at this voltage is that the channel (inversion) at the drain end of the channel disappears:

$$q_n^*(L) \approx -C_{ox}^*\{v_{GS} - V_T(v_{BS}) - \alpha v_{DS}\}$$

$$= 0 \quad \text{when} \quad v_{DS} = \frac{1}{\alpha}\left[v_{GS} - V_T(v_{BS})\right]$$

For $v_{DS} > [v_{GS}\text{-}V_T(v_{BS})]/\alpha$, all the additional drain-to-source voltage appears across the high resistance region at the drain end of the channel where the mobile charge density is very small, and i_D remains constant independent of v_{DS}:

$$i_D(v_{GS}, v_{DS}, v_{BS}) = \frac{1}{2\alpha}\frac{W}{L}\mu_e C_{ox}^*\left[v_{GS} - V_T(v_{BS})\right]^2 \quad \text{for} \quad v_{DS} > \frac{1}{\alpha}\left[v_{GS} - V_T(v_{BS})\right]$$

Gradual Channel Approximation, cont.

The full model:

With this drain current expression, we now have the complete set of Gradual Channel Model expressions for the MOSFET terminal characteristics in the three regions of operation:

Valid for $v_{BS} \leq 0$, and $v_{DS} \geq 0$:

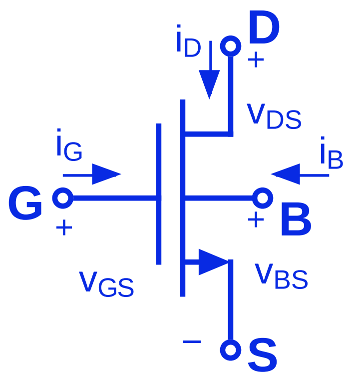
$$i_G(v_{GS}, v_{DS}, v_{BS}) = 0 \quad \text{and} \quad i_B(v_{GS}, v_{DS}, v_{BS}) = 0$$

$$i_D(v_{GS}, v_{DS}, v_{BS}) = \begin{cases} 0 & \text{for} \quad [v_{GS} - V_T(v_{BS})] < 0 < \alpha v_{DS} \\ \dfrac{1}{2}\dfrac{W}{\alpha L}\mu_e C_{ox}^* [v_{GS} - V_T(v_{BS})]^2 & \text{for} \quad 0 < [v_{GS} - V_T(v_{BS})] < \alpha v_{DS} \\ \dfrac{W}{\alpha L}\mu_e C_{ox}^* \left\{ v_{GS} - V_T(v_{BS}) - \alpha \dfrac{v_{DS}}{2} \right\} \alpha v_{DS} & \text{for} \quad 0 < \alpha v_{DS} < [v_{GS} - V_T(v_{BS})] \end{cases}$$

with $$V_T(v_{BS}) \equiv V_{FB} - 2\phi_{p-Si} + \frac{1}{C_{ox}^*}\left\{2\varepsilon_{Si} q N_A \left[\left|2\phi_{p-Si}\right| - v_{BS}\right]\right\}^{1/2}$$

$$\alpha \equiv 1 + \frac{1}{C_{ox}^*}\left\{\frac{\varepsilon_{Si} q N_A}{2\left[\left|2\phi_{p-Si}\right| - v_{BS}\right]}\right\}^{1/2} \qquad \mathrm{C}_{\mathrm{ox}}^* \equiv \frac{\varepsilon_{ox}}{t_{ox}}$$

Gradual Channel Approximation, cont.

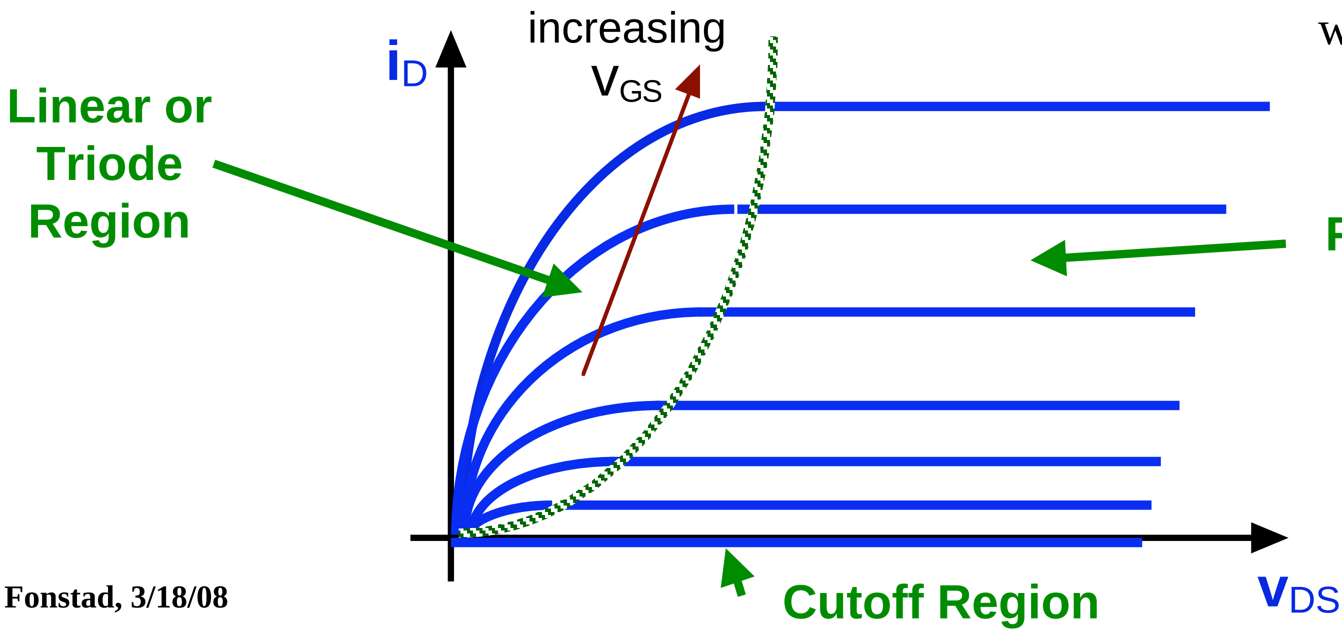


The full model, cont:

$$i_G(v_{GS},v_{DS},v_{BS}) = 0 \qquad i_B(v_{GS},v_{DS},v_{BS}) = 0$$

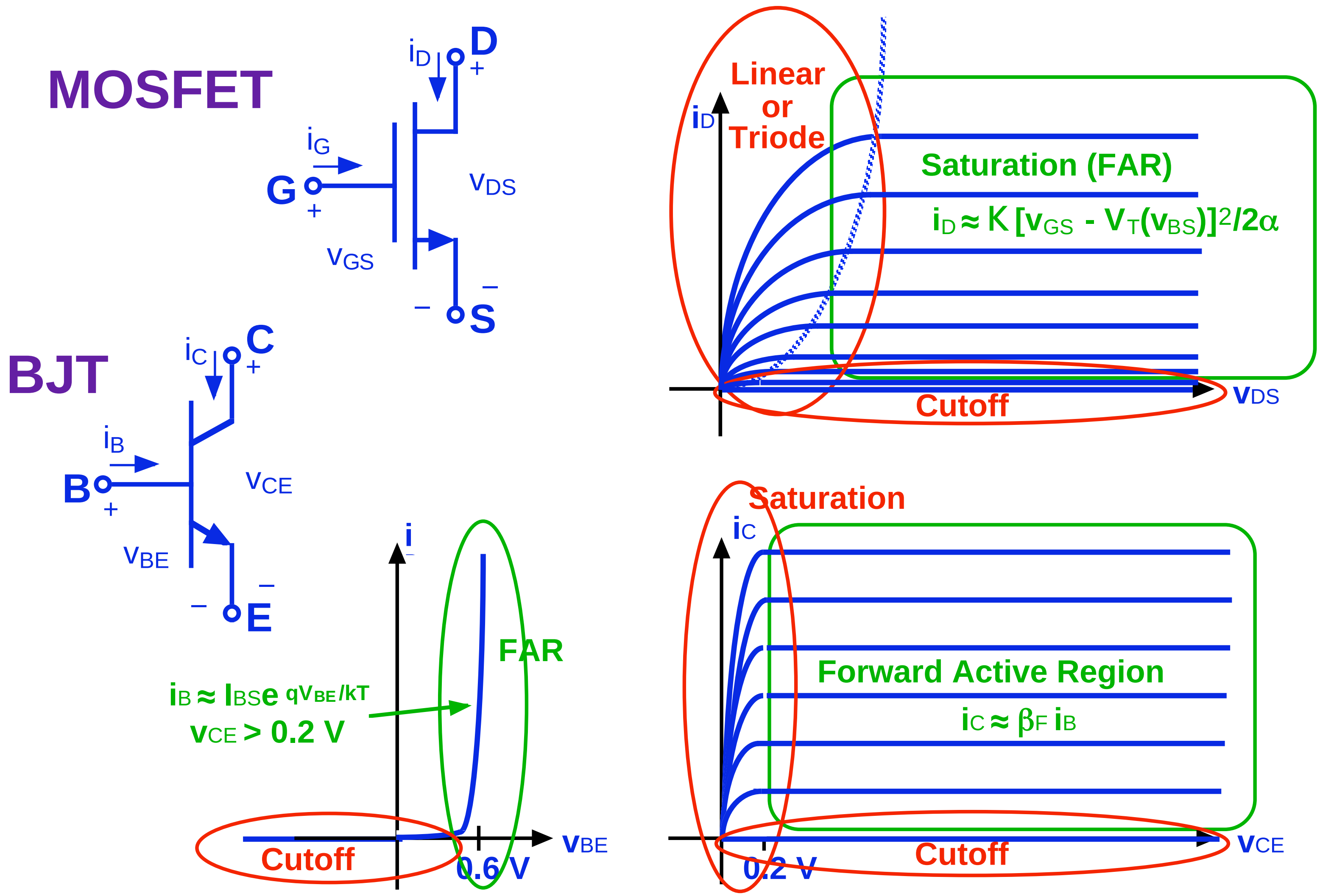
$$i_D(v_{GS},v_{DS},v_{BS}) = \begin{cases} 0 & \text{for } [v_{GS}-V_T(v_{BS})] < 0 < \alpha v_{DS} & \text{Cutoff} \\ \dfrac{K}{2}[v_{GS}-V_T(v_{BS})]^2 & \text{for } 0 < [v_{GS}-V_T(v_{BS})] < \alpha v_{DS} & \text{Saturation} \\ K\left\{v_{GS}-V_T(v_{BS})-\dfrac{\alpha v_{DS}}{2}\right\}\alpha v_{DS} & \text{for } 0 < \alpha v_{DS} < [v_{GS}-V_T(v_{BS})] & \text{Linear or Triode} \end{cases}$$

with $K \equiv \dfrac{W}{\alpha L}\mu_e C_{ox}^*$



The operating regions of MOSFETs and BJTs:

Comparing an n-channel MOSFET and an npn BJT



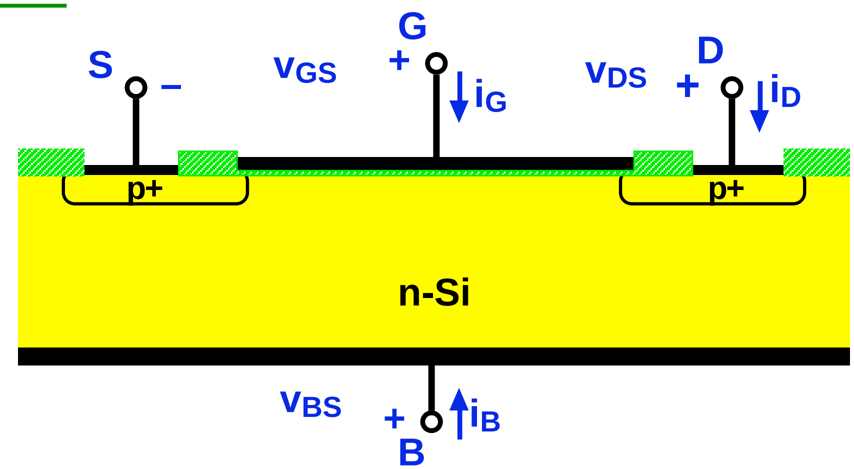
Input curve

Output family

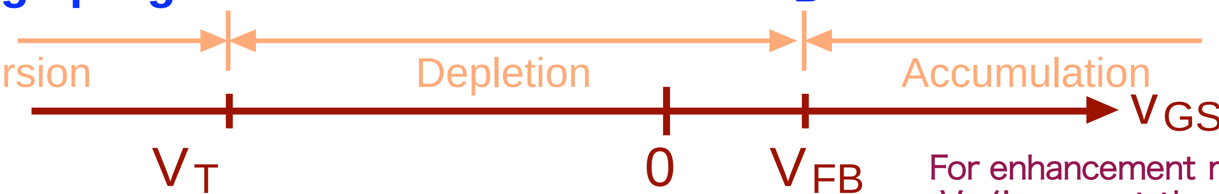
p-channel MOSFET's: The other "flavor" of MOSFET

p-channel

Structure:



The voltage progression:



For enhancement mode p-channel: V_T (i.e. v_{GS} at threshold) < 0, V_{FB} > 0..

Gradual channel model*:

Valid for $v_{SB} \le 0$, and $v_{SD} \ge 0$: $i_G(v_{SG}, v_{SD}, v_{SB}) = 0$ and $i_B(v_{SG}, v_{SD}, v_{SB}) = 0$

$$-i_D(v_{SG}, v_{SD}, v_{SB}) = \begin{cases} 0 & \text{for } \left[v_{SG} - |V_T(v_{SB})|\right] < 0 < \alpha v_{SD} \\ \dfrac{1}{2}\dfrac{W}{\alpha L}\mu_e C_{ox}^*\left[v_{SG} - |V_T(v_{SB})|\right]^2 & \text{for } 0 < \left[v_{SG} - |V_T(v_{SB})|\right] < \alpha v_{SD} \\ \dfrac{W}{\alpha L}\mu_e C_{ox}^*\left\{v_{SG} - |V_T(v_{SB})| - \alpha\dfrac{v_{SD}}{2}\right\}\alpha v_{SD} & \text{for } 0 < \alpha v_{SD} < \left[v_{SG} - |V_T(v_{SB})|\right] \end{cases}$$

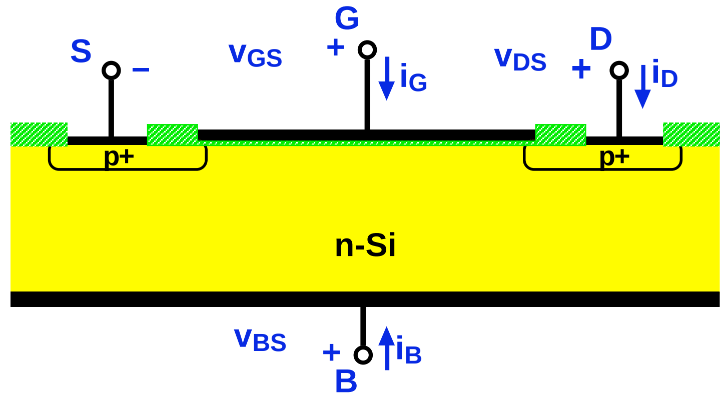
$$V_T(v_{SB}) = V_{FB} - 2\phi_{n-Si} - \gamma\left[2\phi_{n-Si} - v_{SB}\right]^{1/2} \quad \text{with } \gamma \equiv \frac{1}{C_{ox}^*}\left[2\varepsilon_{Si} q N_D\right]^{1/2}$$

* Enhancement mode only, V_T (i.e. v_{GS} at threshold) < 0.

p-channel MOSFET's: cont.

p-channel

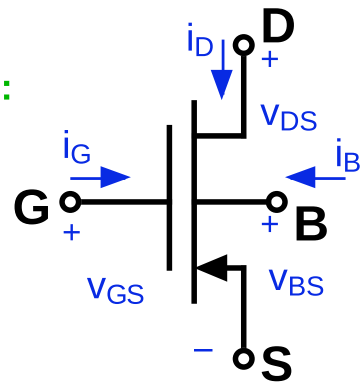
Structure:



Symbol and FAR model:

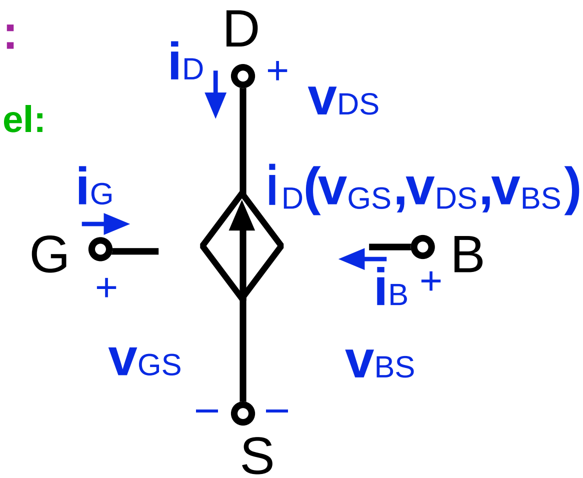
Oriented with source down like n-channel:

Symbol:



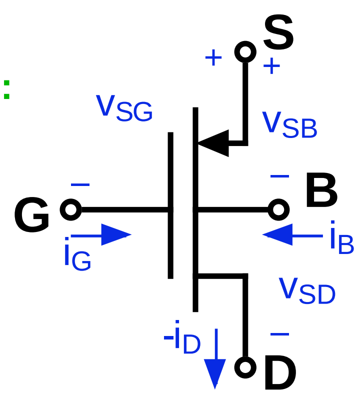
FAR model:

$v_{GS} < V_T$
$v_{BS} > 0$
$v_{DS} < 0$



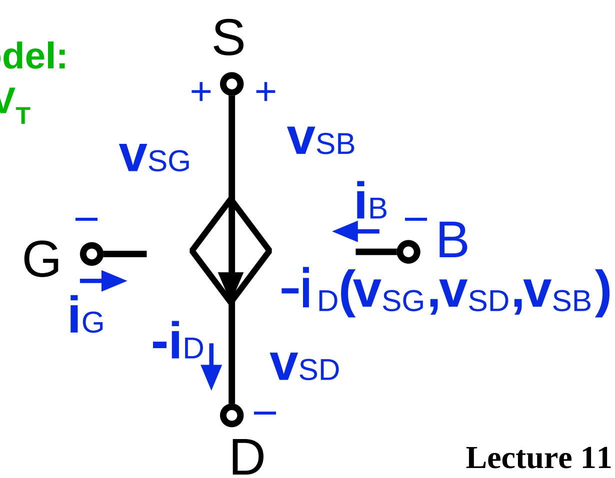
Oriented as found in circuits:

Symbol:



FAR model:

$v_{SG} > -V_T$
$v_{SB} < 0$
$v_{SD} > 0$

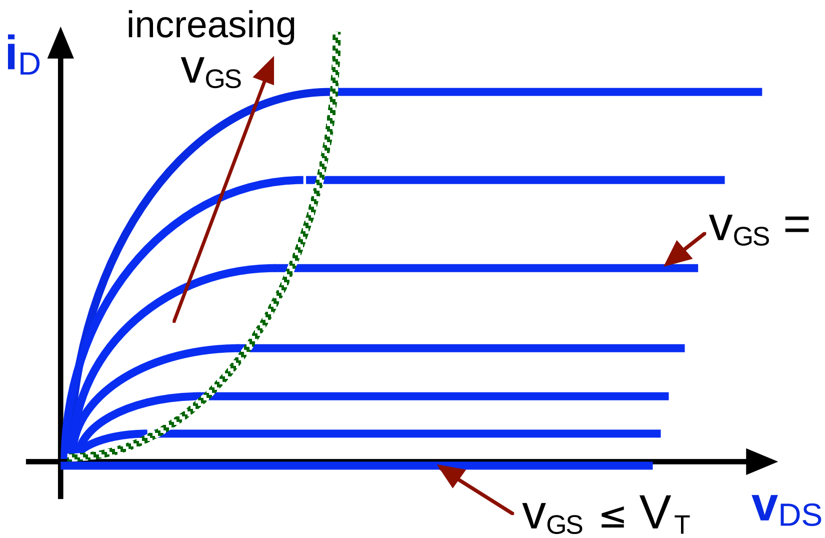


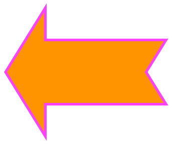
Depletion mode MOSFET's: The very last MOSFET variant

It is possible to have n-channel MOSFETs with $V_T < 0$.

In this situation the channel exists with $v_{GS} = 0$, and a negative bias must be applied to turn it off.

This type of device is called a "depletion mode" MOSFET. Devices with $V_T > 0$ are "enhancement mode."

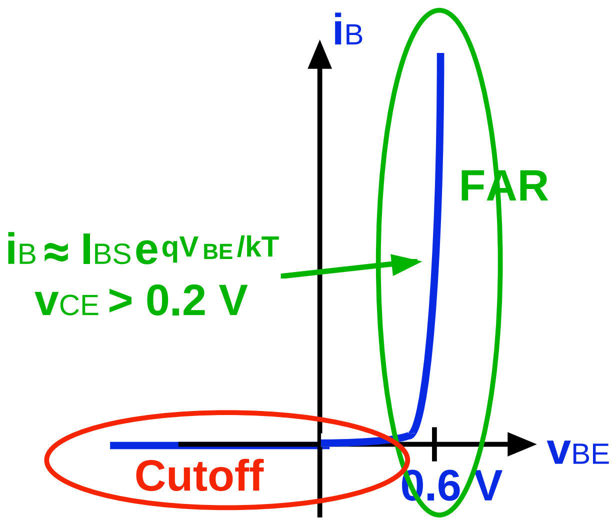




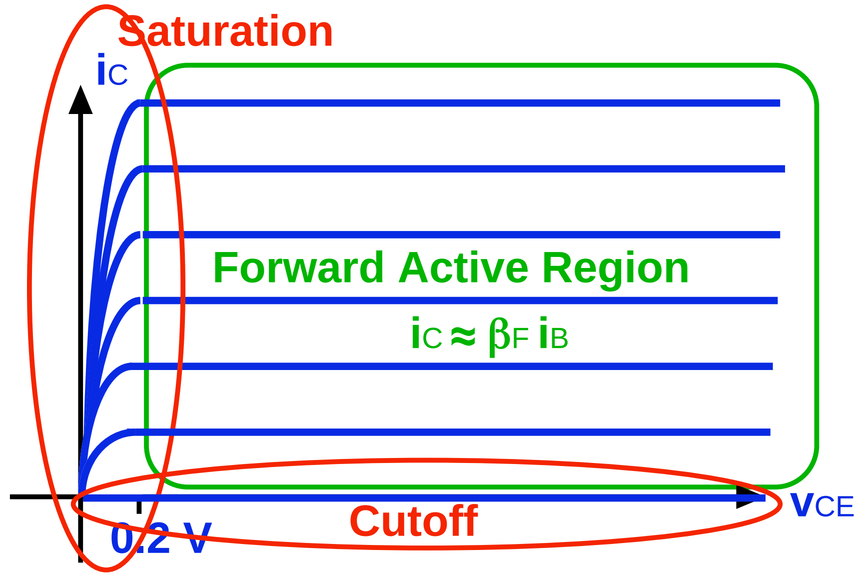
For a p-channel depletion mode MOSFET, $V_T > 0$.

The expressions for $i_D(v_{GS}, v_{DS}, v_{BS})$ are exactly the same for enhancement mode and depletion mode MOSFETs.

BJT Characteristics (npn)

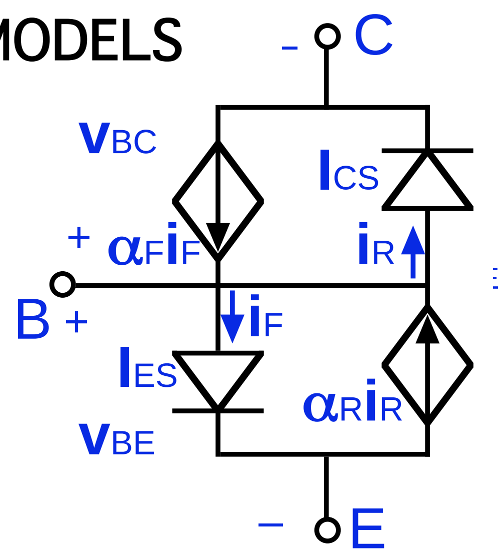


Input curve



Output family

BJT MODELS



Forward active region

$v_{BE} > 0.6$ V

$v_{CE} > 0.2$ V

(i.e. $v_{BC} < 0.4$V)

i_R is negligible

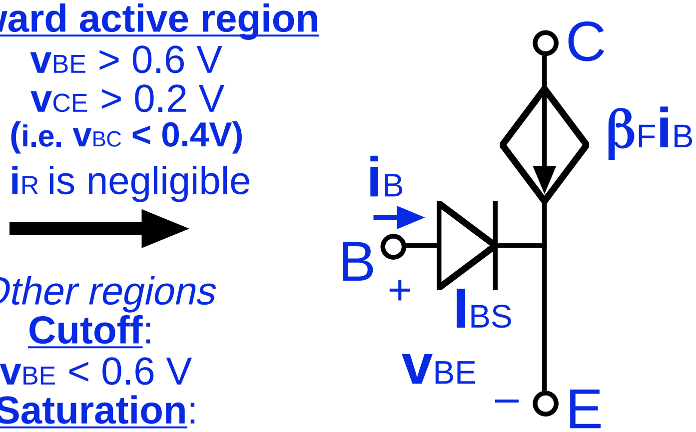
Other regions

Cutoff:

$v_{BE} < 0.6$ V

Saturation:

$v_{CE} < 0.2$ V



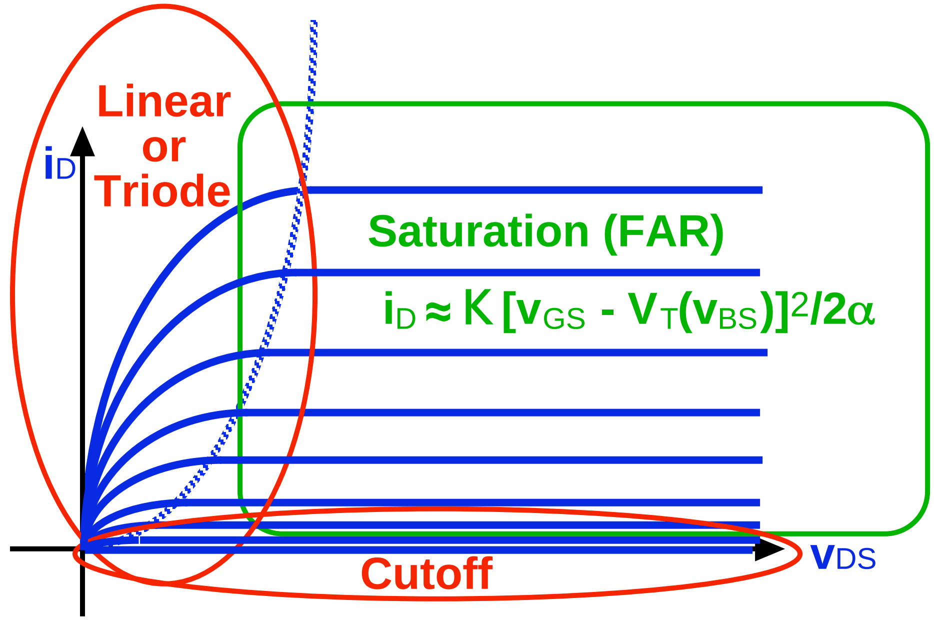
MOSFET Characteristics
(n-channel)

Also:

$i_G \approx 0$

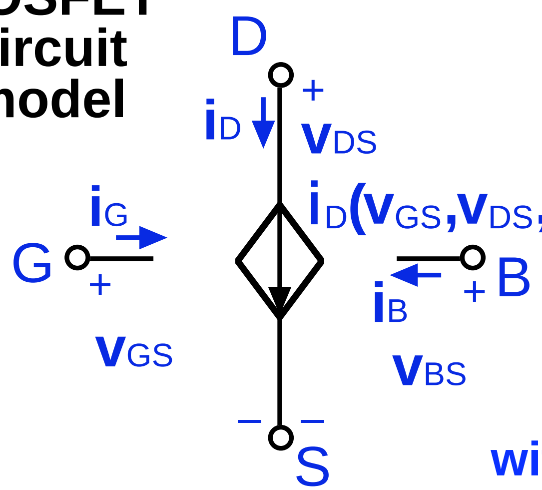
$i_B \approx 0$

$K = (W/L)\mu_e C_{ox}^*$



Output family

MOSFET circuit model



Model valid for $v_{BS} \leq 0$ and $v_{DS} \geq 0$, insuring

$i_G(v_{GS}, v_{DS}, v_{BS}) \approx 0,\ i_B(v_{GS}, v_{DS}, v_{BS}) \approx 0$

$$i_D(v_{GS}, v_{DS}, v_{BS}) \approx \begin{cases} 0 & \text{for } (v_{GS} - V_T) \leq 0 \leq \alpha v_{DS} \quad \text{(cutoff)} \\ (W/2\alpha L)\mu_e C_{ox}^*(v_{GS} - V_T)^2 & \text{for } 0 \leq (v_{GS} - V_T) \leq \alpha v_{DS} \quad \text{(saturation)} \\ (W/\alpha L)\mu_e C_{ox}^*(v_{GS} - V_T - \alpha v_{DS}/2)\alpha v_{DS} & \text{for } 0 \leq \alpha v_{DS} \leq (v_{GS} - V_T) \quad \text{(linear)} \end{cases}$$

with $V_T = V_{FB} - 2\phi_{p\text{-}Si} + [2\varepsilon_{Si}\, qN_A(|2\phi_{p\text{-}Si}| - v_{BS})]^{1/2}/C_{ox}^*$

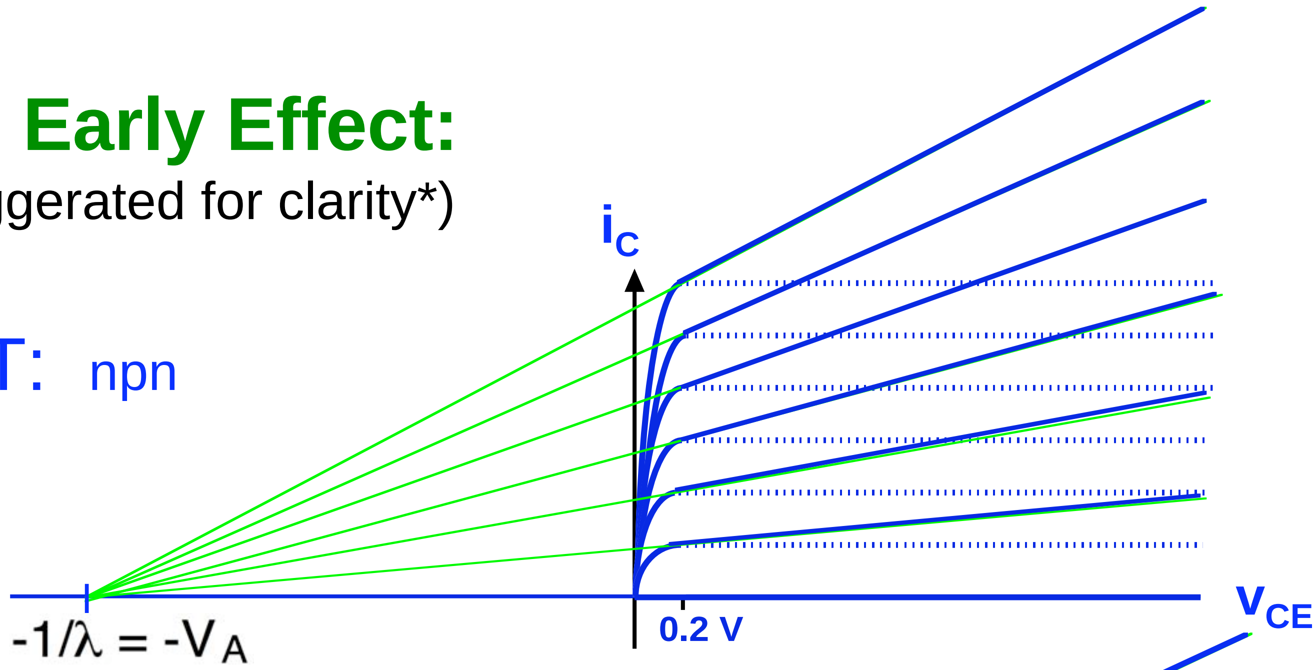
$\alpha = 1 + [(\varepsilon_{Si}\, qN_A/2(|2\phi_{p\text{-}Si}| - v_{BS})]^{1/2}/C_{ox}$

(frequently $\alpha \approx 1$)

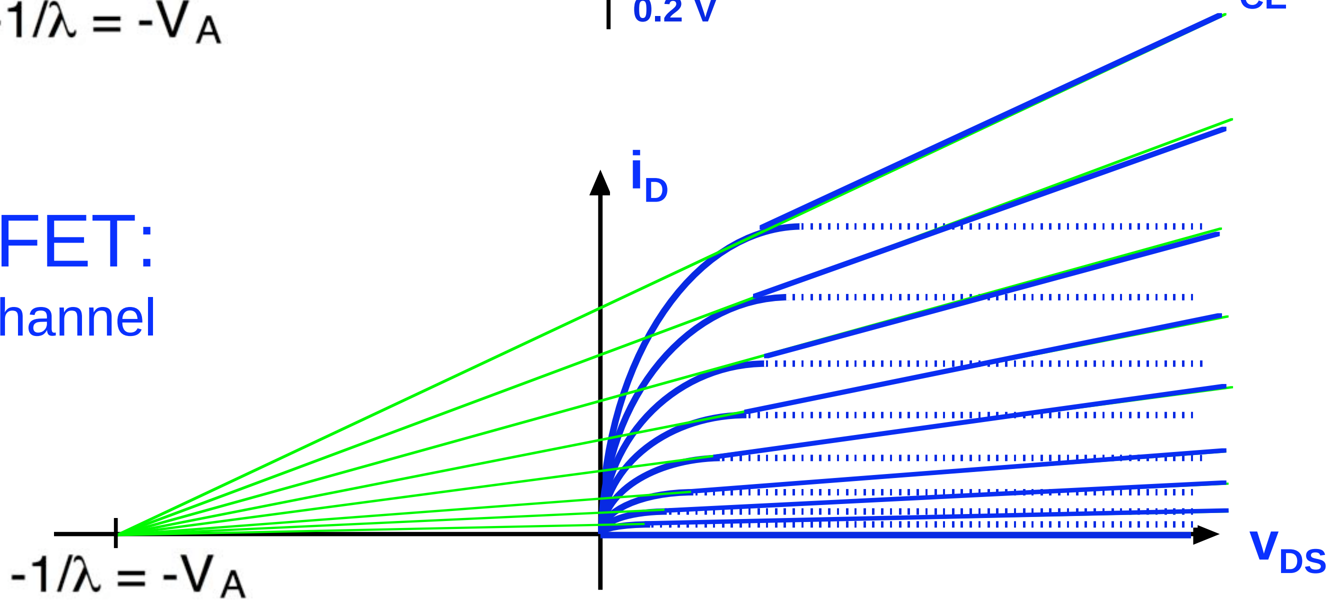
The Early Effect:

(exaggerated for clarity*)

BJT: npn



MOSFET:
n-channel



* Typically the Early effect is far more important in small-signal applications than large signal.

Active Length Modulation - the Early Effect: MOSFET

"Channel length modulation"

MOSFET:

We begin by recognizing that the channel length decreases with increasing v_{DS} and writing this dependence to first order in v_{DS}:

$$L \approx L_o\left[1-\lambda\left(v_{DS}-V_{DSat}\right)\right] \quad \text{and} \quad \frac{1}{L} \approx \frac{\left[1+\lambda\left(v_{DS}-V_{DSat}\right)\right]}{L_o}$$

$$K = \frac{W}{\alpha L}\mu_e C_{ox}^*$$

Inserting the channel length variation with v_{DS} into K we have:

$$K \approx K_o\left[1+\lambda\left(v_{DS}-V_{DSat}\right)\right] \quad \text{where} \quad K_o \equiv \frac{W}{\alpha L_o}\mu_e C_{ox}^*$$

Thus, in saturation:

$$i_D \approx \frac{K_o}{2}\left(v_{GS}-V_T\right)^2\left[1+\lambda\left(v_{DS}-V_{DSat}\right)\right]$$

Note: λ is the inverse of the Early Voltage, V_A (i.e., $\lambda = 1/V_A$).

Active Length Modulation - the Early Effect: BJT

"Base width modulation"

BJT:

We begin by recognizing that the base width decreases with increasing v_{CE} and writing this dependence to first order in v_{CE}:

$$w_B^* \approx w_{Bo}^*(1 - \lambda v_{CE}) \quad \text{and} \quad \frac{1}{w_B^*} \approx \frac{1}{w_{Bo}^*}(1 + \lambda v_{CE})$$

Then we recall that in a modern BJT the base defect, δ_B, is negligible and β_F depends primarily on the emitter defect, δ_E, and can be written:

$$\beta_F = \frac{(1 + \delta_B)}{(\delta_E + \delta_B)} \approx \frac{1}{\delta_E} = \frac{D_e}{D_h} \frac{N_{DE}}{N_{AB}} \frac{w_E^*}{w_B^*}$$

Inserting the base width variation with v_{CE} into β_F we have:

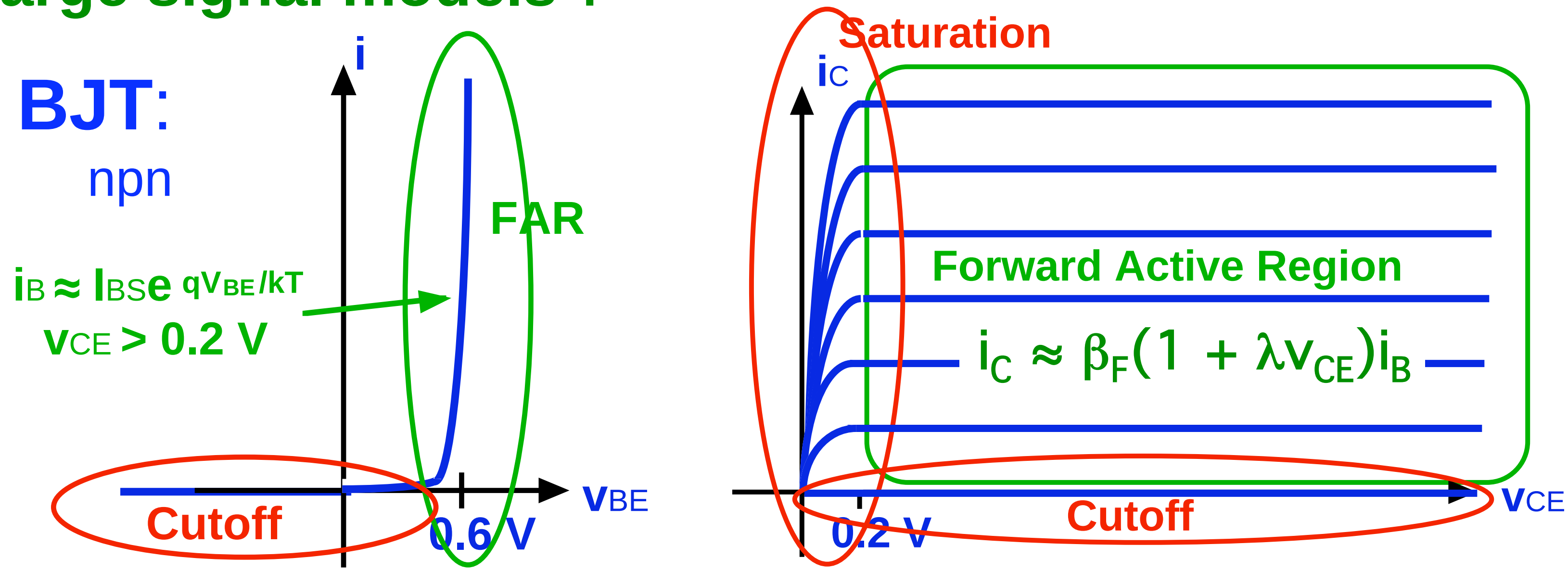
$$\beta_F \approx \beta_{Fo}(1 + \lambda v_{CE}) \quad \text{where} \quad \beta_{Fo} \equiv \frac{D_e}{D_h} \frac{N_{DE}}{N_{AB}} \frac{w_E^*}{w_{Bo}^*}$$

Thus, in the F.A.R.: $$i_C \approx \beta_{Fo}(1 + \lambda v_{CE})\, i_B$$

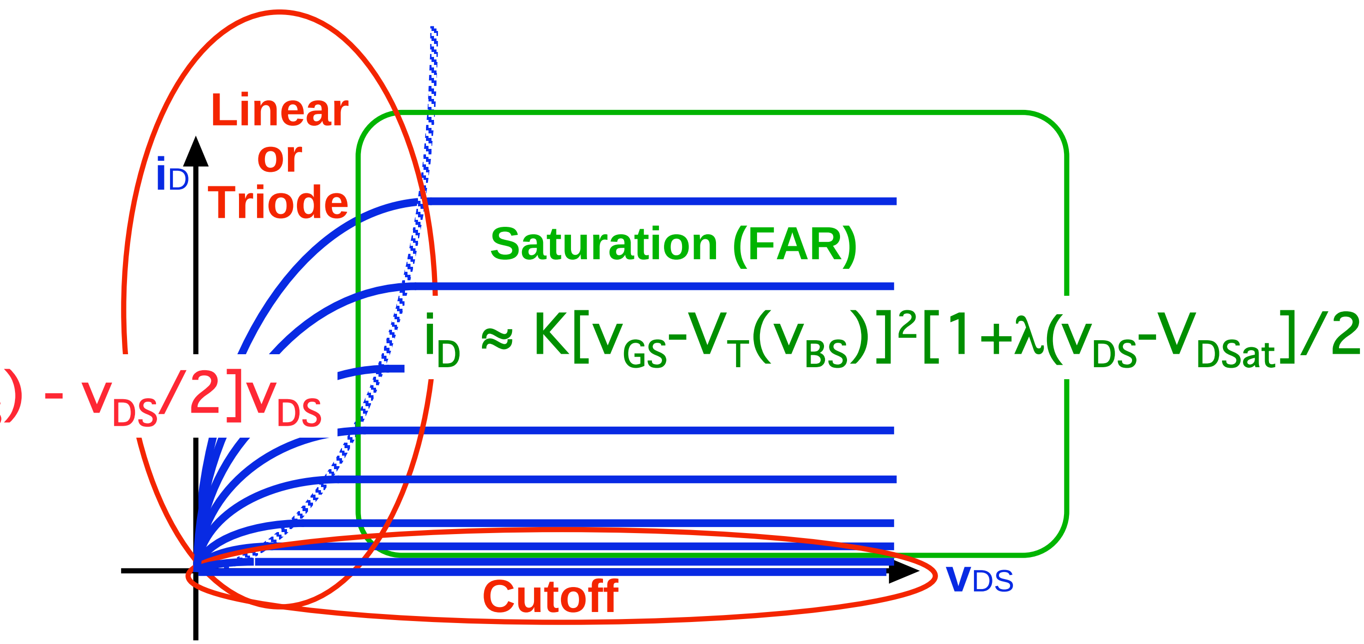
Note: λ is the inverse of the Early Voltage, V_A (i.e., $\lambda = 1/V_A$).

Large signal models*:

BJT:
npn



MOSFET:
n-channel



Clif Fonstad, 3/18/08

* The Early effect is included, but barely visible.

Large signal models: when will we use them?

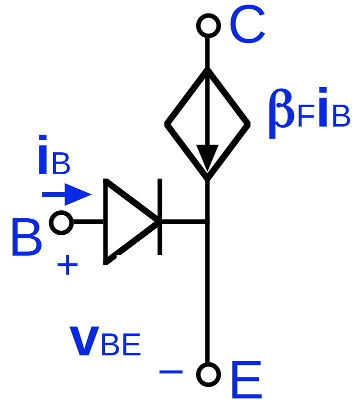
Digital circuit analysis/design:

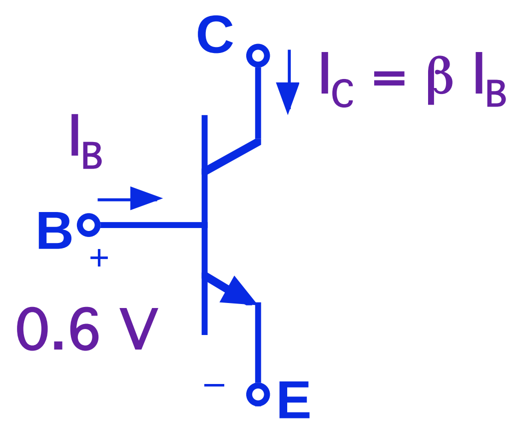
This requires use of the entire circuit, and will be the topic of Lectures 14, 15, and 16.

Bias point analysis/design:

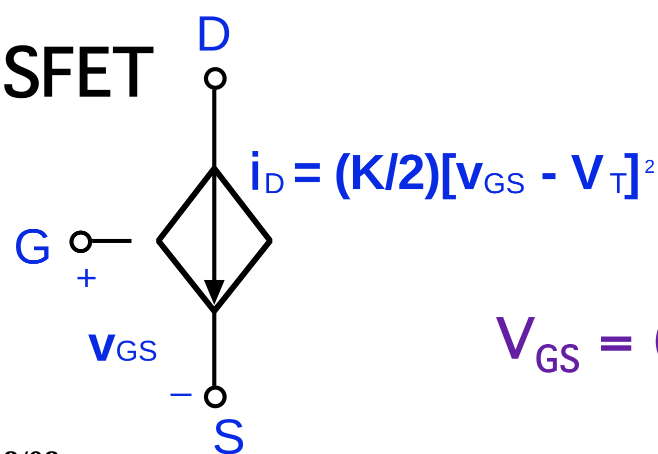
This uses the FAR models (below and Lec. 17ff).

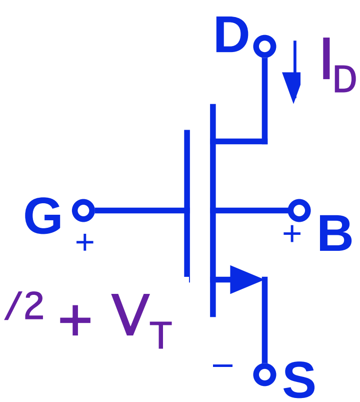
BJT





MOSFET

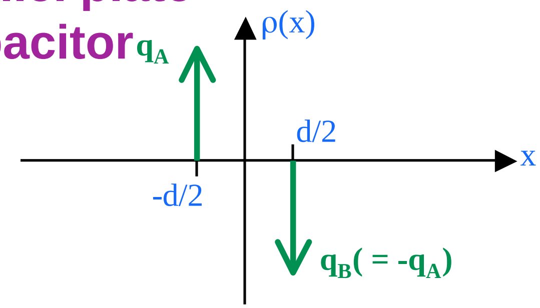




$$V_{GS} = (2I_{DS}/K)^{1/2} + V_T$$

Charge stores in devices: we must add them to our device models

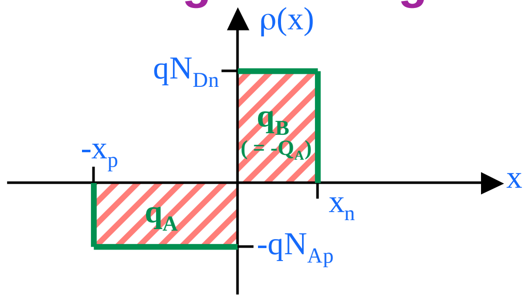
Parallel plate capacitor



$$q_{A,PP} = A\frac{\varepsilon}{d}v_{AB}$$

$$C_{pp}(V_{AB}) \equiv \left.\frac{\partial q_{A,PP}}{\partial v_{AB}}\right|_{v_{AB}=V_{AB}} = \frac{A\varepsilon}{d}$$

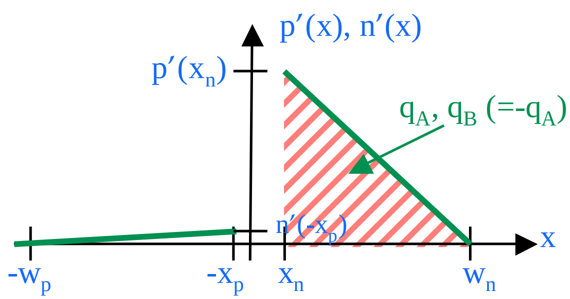
Depletion region charge store



$$q_{A,DP}(v_{AB}) = -A\sqrt{2q\varepsilon_{Si}\left[\phi_b - v_{AB}\right]\frac{N_{Ap}N_{Dn}}{\left[N_{Ap}+N_{Dn}\right]}}$$

$$C_{dp}(V_{AB}) = A\sqrt{\frac{q\varepsilon_{Si}}{2\left[\phi_b - V_{AB}\right]}\frac{N_{Ap}N_{Dn}}{\left[N_{Ap}+N_{Dn}\right]}} = \frac{A\,\varepsilon_{Si}}{w(V_{AB})}$$

QNR region diffusion charge store



$$q_{AB,DF}(v_{AB}) \approx Aqn_i^2\frac{D_h}{N_{Dn}w_{n,eff}}\left[e^{qV_{AB}/kT} - 1\right]$$

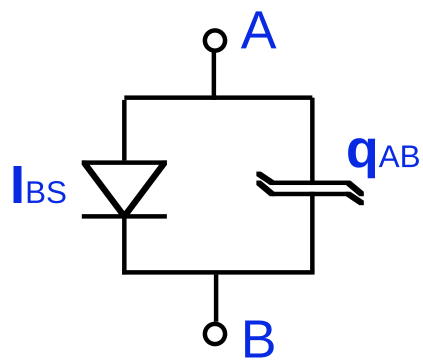
Note: Approximate because we are only accounting for the charge store on the lightly doped side.

$$= \frac{w_{n,eff}^2}{2D_h}i_D(v_{AB})$$

$$C_{df}(V_{AB}) \approx \frac{w_{n,eff}^2}{2D_h}\frac{q\,I_D(V_{AB})}{kT}$$

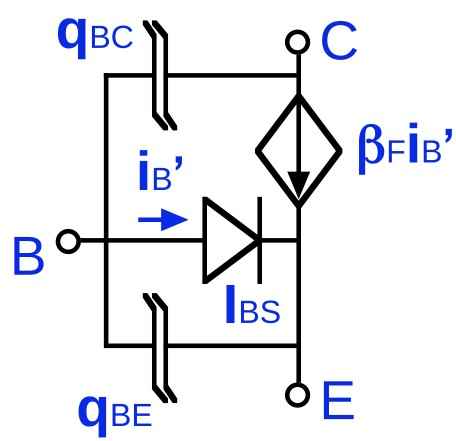
Adding charge stores to the large signal models:

p-n diode:



q_{AB}: Excess carriers on p-side plus excess carriers on n-side plus junction depletion charge.

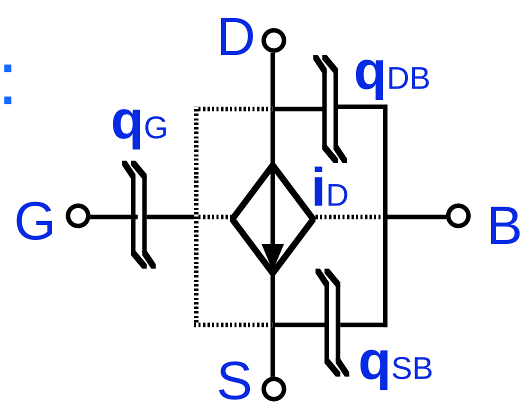
BJT: npn
(in F.A.R.)



q_{BE}: Excess carriers in base plus E-B junction depletion charge

q_{BC}: C-B junction depletion charge

MOSFET:
n-channel



q_G: Gate charge; a function of v_{GS}, v_{DS}, and v_{BS}.

q_{DB}: D-B junction depletion charge

q_{SB}: S-B junction depletion charge

Lecture 11 - MOSFETS II; Large-Signal Models - Summary

- **Gradual channel approximation for FETs**

  General approach

  MOSFETS in strong inversion

  1. **Ignore variation of V_T along channel**
  2. **Linearize variation of V_T along channel:** introduces α factor

- **Additional device model issues**

  The Early Effect:

  1. **Base-width modulation in BJTs:** $w_B(v_{CE})$

     In the F.A.R.: $i_C \approx \beta_{Fo}(1 + l v_{CE}) i_B$

  2. **Channel-length modulation in MOSFETs:** $L(v_{DS})$

     In saturation: $i_D \approx K_o (v_{GS} - V_T)^2 [1 + \lambda(v_{DS} - V_{DSat})]/2\alpha$

  Charge stores:

  1. **Junction diodes - depletion and diffusion charge**
  2. **BJTs - at EB junction:** depletion and diffusion charge

     at CB junction: depletion charge **(focus on FAR)**

  3. **MOSFETs - between B and S, D: depletion charge of n^+-p junctions**

     between G and S, D, B: gate charge **(the dominant store)**

     in cut-off: $C_{gs} \approx C_{gd} \approx 0$; all is C_{gb}

     linear region: $C_{gs} = C_{gd} = W\,L\,C_{ox}^*/2$

     in saturation region: $C_{gs} = (2/3)\,W\,L\,C_{ox}^*$

     $C_{gd} = 0$ (only parasitic overlap)

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