6.012 - Electronic Devices and Circuits

Lecture 2 - Uniform Excitation; Non-uniform conditions

- **Announcements**
- **Review**
  - **Carrier concentrations in TE given the doping level**
  - **What happens above and below room temperature?**
  - **Drift and mobility -** The full story.
- **Uniform excitation: optical generation**
  - **Generation/recombination in TE**
  - **Uniform optical generation - external excitation**
  - **Population excesses, p' and n', and their transients**
  - **Low level injection; minority carrier lifetime**
- **Uniform excitation: applied field and optical generation**
  - **Photoconductivity, photoconductors**
- **Non-uniform doping/excitation: diffusion, continuity**
  - **Fick's 1st law; diffusion**
  - **Diffusion current; total current (**drift plus diffusion**)**
  - **Fick's 2nd law; carrier continuity**

Extrinsic Silicon, cont.: solutions in Cases I and II

Case I - n-type: $N_d > N_a$:, $(N_d - N_a) >> n_i$ "n-type Si"

Define the net donor concentration, N_D: $$N_D \equiv (N_d - N_a)$$

We find:

$$n_o \approx N_D, \quad p_o = n_i^2(T)/n_o \approx n_i^2(T)/N_D$$

$$n_o >> n_i >> p_o$$

In Case I the concentration of electrons is much greater than that of holes. Silicon with net donors is called "n-type".

Case II - p-type: $N_a > N_d$:, $(N_a - N_d) >> n_i$ "p-type Si"

Define the net acceptor concentration, N_A: $$N_A \equiv (N_a - N_d)$$

We find:

$$p_o \approx N_A, \quad n_o = n_i^2(T)/p_o \approx n_i^2(T)/N_A$$

$$p_o >> n_i >> n_o$$

In Case II the concentration of holes is much greater than that of electrons. Silicon with net acceptors is called "p-type".

Variation of carrier concentration with temperature

(Note: for convenience we assume an n-type sample)

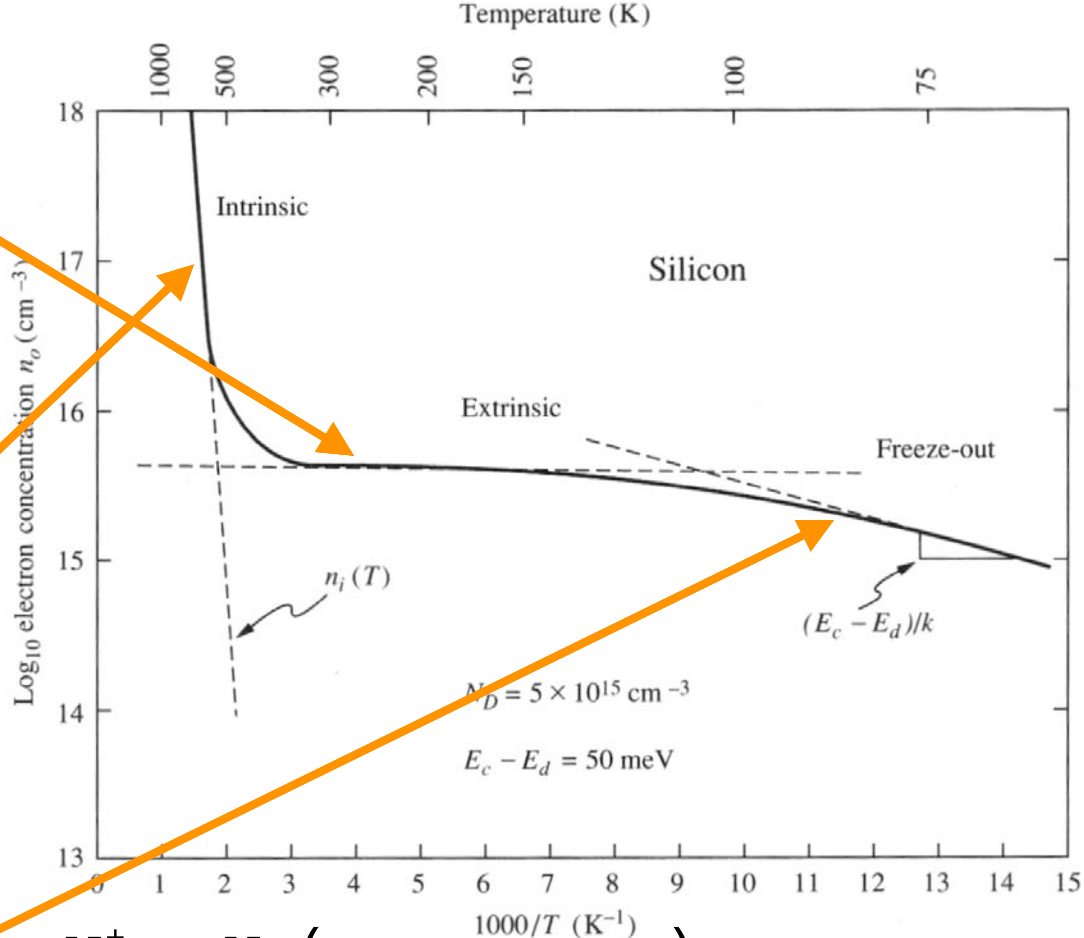
- **Around R.T.**
  Full ionization
  Extrinsic doping

$$N_d^+ \approx N_d,\ N_a^- \approx N_a$$
$$\left(N_d^+ - N_a^-\right) >> n_i$$
$$n_o \approx \left(N_d - N_a\right),\ p_o = n_i^2 / n_o$$

- **At very high T**
  Full ionization
  Intrinsic behavior

$$N_d^+ \approx N_d,\ N_a^- \approx N_a$$
$$n_i >> \left|N_d^+ - N_a^-\right|$$
$$n_o \approx p_o \approx n_i$$

- **At very low T**
  Incomplete ionization
  Extrinsic doping, but with carrier freeze-out



$$N_d^+ << N_d\ \left(\text{assuming n – type}\right)$$
$$\left|N_d^+ - N_a^-\right| >> n_i$$
$$n_o \approx \left(N_d^+ - N_a^-\right) << \left(N_d - N_a\right),\ p_o = n_i^2 / n_o$$

Uniform material with uniform excitations

(pushing semiconductors out of thermal equilibrium)

A. Uniform Electric Field, E_x , cont.

Drift motion:

Holes and electrons acquire a constant net velocity, s_x, proportional to the electric field:

$$\overline{s_{ex}} = -\mu_e E_x, \quad \overline{s_{hx}} = \mu_h E_x$$

At low and moderate |E|, the mobility, μ, is constant.
At high |E| the velocity saturates and μ deceases.

Drift currents:

Net velocities imply net charge flows, which imply currents:

$$J_{ex}^{dr} = -q\, n_o\, \overline{s_{ex}} = q\mu_e n_o E_x \qquad J_{hx}^{dr} = q\, p_o\, \overline{s_{hx}} = q\mu_h p_o E_x$$

Note: Even though the semiconductor is no longer in thermal equilibrium the hole and electron populations still have their thermal equilibrium values.

Conductivity, σ_o:

Ohm's law on a microscale states that the drift current density is linearly proportional to the electric field:

$$J_x^{dr} = \sigma_o E_x$$

The total drift current is the sum of the hole and electron drift currents. Using our early expressions we find:

$$J_x^{dr} = J_{ex}^{dr} + J_{hx}^{dr} = q\mu_e n_o E_x + q\mu_h p_o E_x = q\left(\mu_e n_o + \mu_h p_o\right) E_x$$

From this we see obtain our expression for the conductivity:

$$\sigma_o = q\left(\mu_e n_o + \mu_h p_o\right) \quad \text{[S/cm]}$$

Majority vs. minority carriers:

Drift and conductivity are dominated by the most numerous, or "majority," carriers:

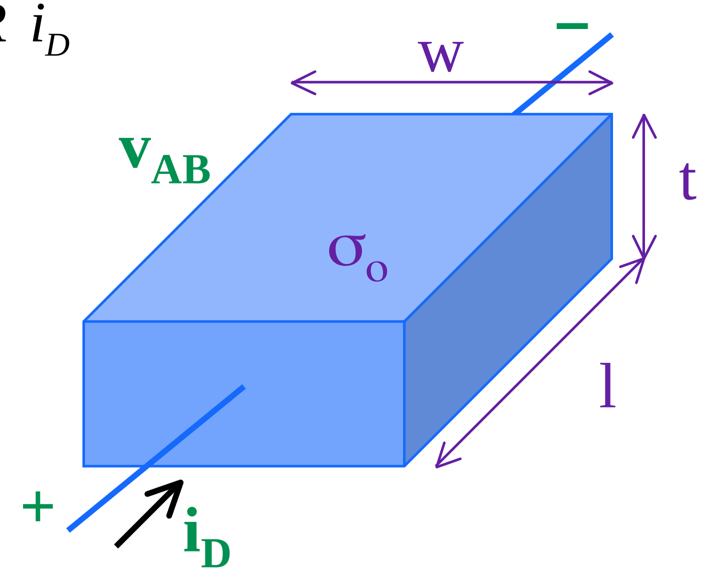
n-type $\quad n_o >> p_o \Rightarrow \sigma_o \approx q\mu_e n_o$

p-type $\quad p_o >> n_o \Rightarrow \sigma_o \approx q\mu_h p_o$

Resistance, R, and resistivity, ρ_o:

Ohm's law on a macroscopic scale says that the current and voltage are linearly related: $v_{ab} = R\ i_D$

The question is, "What is R?"



We have: $J_x^{dr} = \sigma_o E_x$

with $E_x = \dfrac{v_{AB}}{l}$ and $J_x^{dr} = \dfrac{i_D}{w \cdot t}$

Combining these we find:

$$\frac{i_D}{w \cdot t} = \sigma_o \frac{v_{AB}}{l}$$

which yields: $$v_{AB} = \frac{l}{w \cdot t}\frac{1}{\sigma_o} i_D = R\, i_D$$

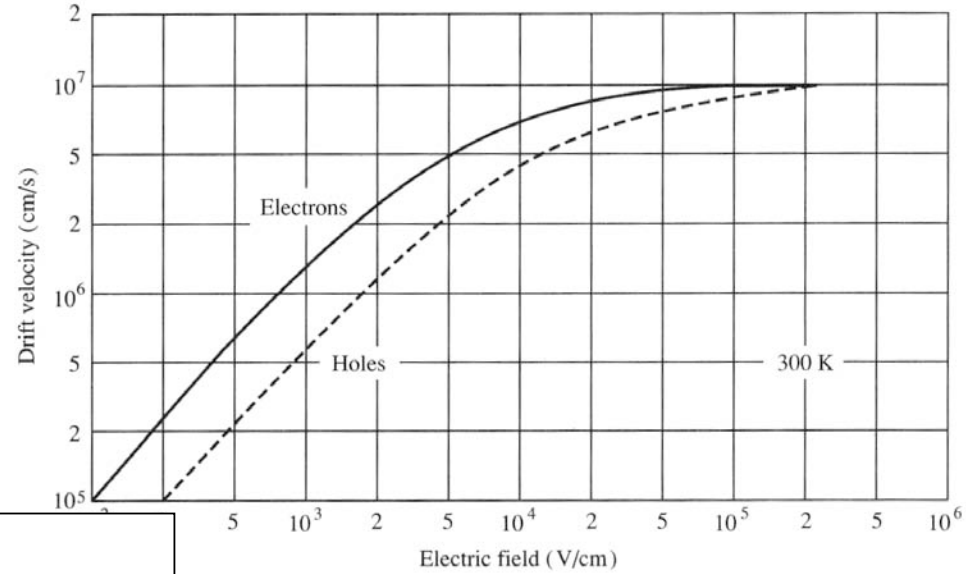
where $$R \equiv \frac{l}{w \cdot t}\frac{1}{\sigma_o} = \frac{l}{w \cdot t}\rho_o = \frac{l}{A}\rho_o$$

Note: Resistivity, ρ_o, is defined as the inverse of the conductivity:

$$\rho_o \equiv 1/\sigma_o \qquad [\text{Ohm-cm}]$$

Velocity saturation

The breakdown of Ohm's law at large electric fields.



Silicon

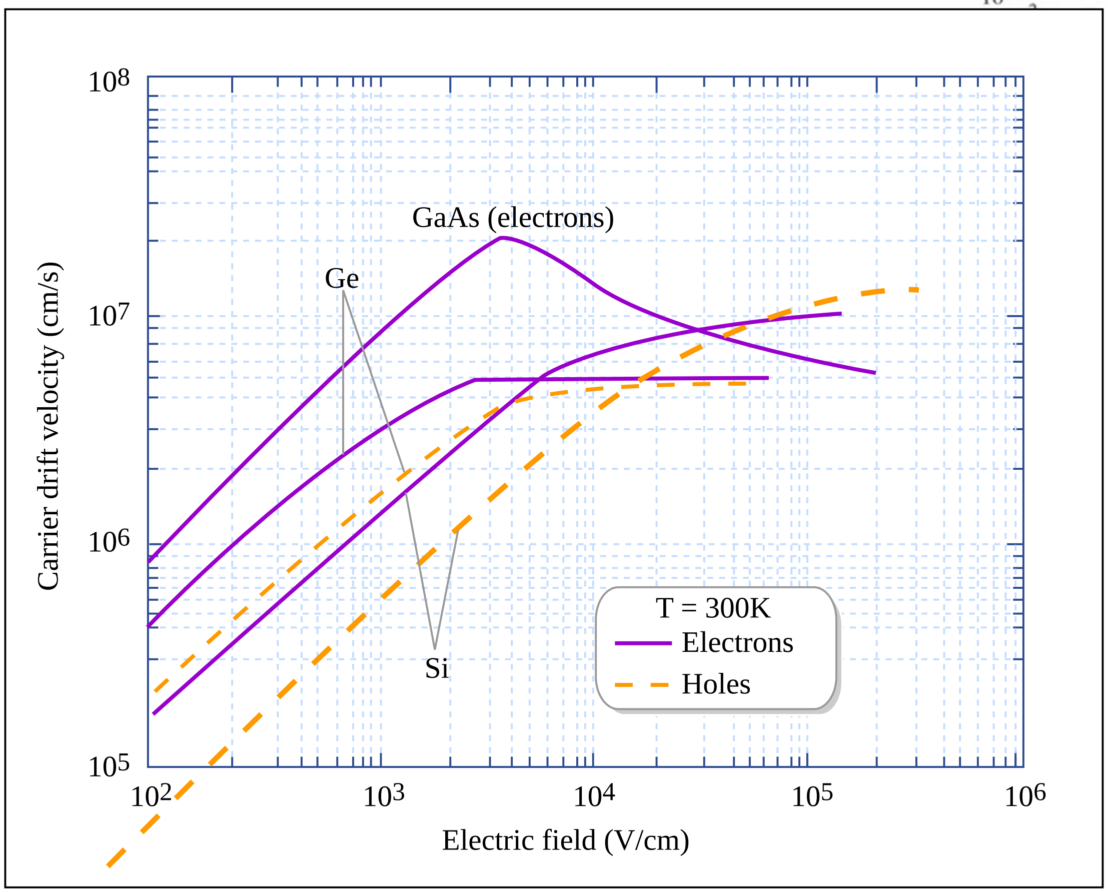


Figure by MIT OpenCourseWare.

Above: Velocity vs. field plot at R.T. for holes and electrons in Si (log-log plot). (Fonstad, Fig. 3.2)

Left: Velocity-field curves for Si, Ge, and GaAs at R.T. (log-log plot). (Neaman, Fig. 5.7)

Variation of mobility with temperature and doping

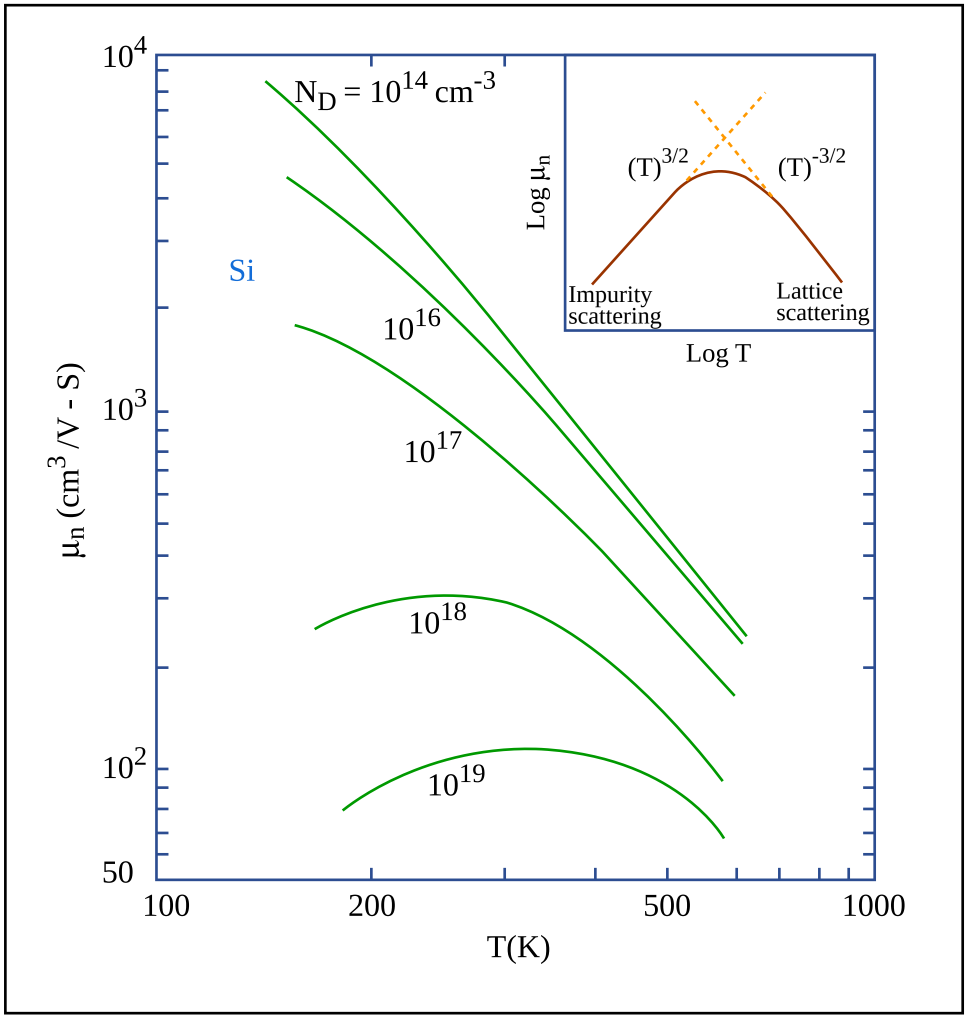


Figure by MIT OpenCourseWare.

μ_e vs T in Si at several doping levels

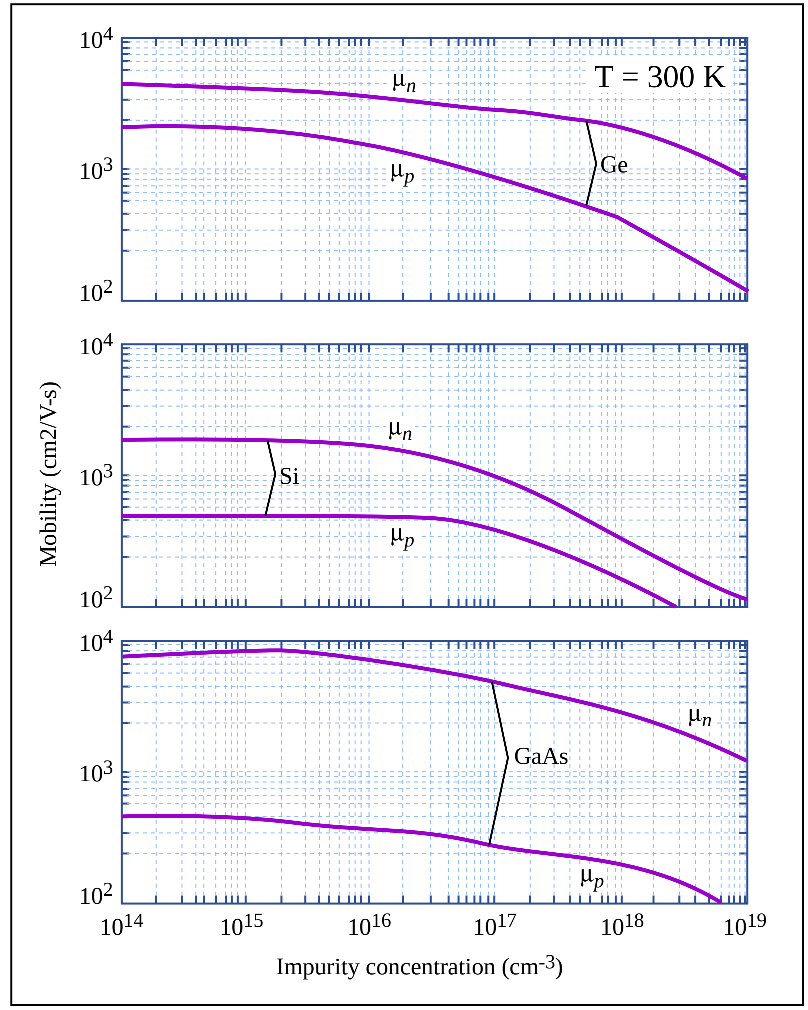


Figure by MIT OpenCourseWare.

μ vs doping for Si, Ge, and GaAs at R.T.
(Neaman, Fig. 5.3)

Having said all of this,…

…it is good to be aware that the mobilities vary with doping and temperature, but in 6.012 we will

1. use only one value for the hole mobility in Si, and one for the electron mobility in Si, and will not consider the variation with doping. Typically for bulk silicon we use

$$\mu_e = 1600\ cm^2/V\text{-}s \qquad \text{and} \qquad \mu_h = 600\ cm^2/V\text{-}s$$

2. assume uniform temperature (isothermal) conditions and room temperature operation, and

3. only consider velocity saturation when we talk about MOSFET scaling near the end of the term.

Uniform material with uniform excitations

(pushing semiconductors out of thermal equilibrium)

B. Uniform Optical Generation, g_L (t)

The carrier populations, n and p:

The light supplies energy to "break" bonds creating excess holes, p', and electrons, n'. These excess carriers are generated in pairs.

Thus:

$$\left.\begin{array}{r}\text{Electron concentration: } n_o \Rightarrow n_o + n'(t) \\ \text{Hole concentration: } p_o \Rightarrow p_o + p'(t)\end{array}\right\} \text{ with } \quad n'(t) = p'(t)$$

Generation, G, and recombination, R:

In general:

$$\frac{dn}{dt} = \frac{dp}{dt} = G - R \quad \begin{cases} G > R \Rightarrow \dfrac{dn}{dt} = \dfrac{dp}{dt} > 0 \\ G < R \Rightarrow \dfrac{dn}{dt} = \dfrac{dp}{dt} < 0 \end{cases}$$

In thermal equilibrium: G = R

$$\left.\begin{array}{r} G = g_o \\ R = n_o p_o r \end{array}\right\} \quad G = R \Rightarrow g_o = n_o p_o r = n_i^2 r$$

B. Uniform Optical Generation, g_L (t), cont.

With uniform optical generation, $g_L(t)$:

$$G = g_o + g_L(t)$$

$$R = n\,p\,r = (n_o + n')(p_o + p')\,r$$

thus

$$\frac{dn}{dt} = \frac{dp}{dt} = G - R = g_o + g_L(t) - (n_o + n')(p_o + p')r$$

The question: *Given N_d, N_a, and $g_L(t)$, what are n(t) and p(t)?*

To answer: Using

(1) $$\frac{dn}{dt} = \frac{dp}{dt} = \frac{dn'}{dt} = \frac{dp'}{dt}$$

(2) $$g_o = n_o\,p_o\,r$$

(3) $$n' = p'$$

gives one equation in one unknown*:

$$\frac{dn'}{dt} = g_L(t) - (p_o + n_o + n')n'\,r$$

* Remember: n_o and p_o are known given N_d, N_a

B. Uniform Optical Generation, g_L (t), cont.

This equation is non-linear: It is in general hard to solve

$$\frac{dn'}{dt} = g_L(t) - (p_o + n_o + n')n'r$$

Special Case - Low Level Injection: assume p-type, $p_o \gg n_o$

LLI: $n' \ll p_o$

When LLI holds our equation becomes linear, and solvable:

$$\frac{dn'}{dt} \approx g_L(t) - p_o n' r$$

$$= g_L(t) - \frac{n'}{\tau_{\min}} \quad \text{with} \quad \tau_{\min} \equiv 1/p_o r$$

This first order differential equation is very familiar to us. The homogeneous solution is:

$$n'(t) = Ae^{-t/\tau_{\min}}$$

Important facts about τ_{min} and recombination:

The minority carrier lifetime is a gauge of how quickly excess carriers recombine in the bulk of a semiconductor sample.

Recombination also occurs at surfaces and contacts.
(Problem x in P.S. #2 deals with estimating the relative importance of recombination in the bulk relative to that at surfaces and contacts.)

In **silicon**:

- the minority carrier lifetime is relatively very long,
- the surface recombination can be made negligible, and
- the only significant recombination occurs at ohmic contacts
  (Furthermore, the lifetime is zero at a well built ohmic contact, and any excess carrier reaching a contact immediately recombines, so the excess population at an ohmic contact is identically zero.)

In **most other semiconductors**:

- both bulk and surface recombination are likely to be important, but it is hard to make any further generalizations

Uniform material with uniform excitations

(pushing semiconductors out of thermal equilibrium)

C. Photoconductivity - drift and optical generation

When the carrier populations change because of optical generation… $g_L(t) \Rightarrow n(t) = n_o + n'(t), \quad p(t) = p_o + n'(t)$

Used: $p'(t) = n'(t)$

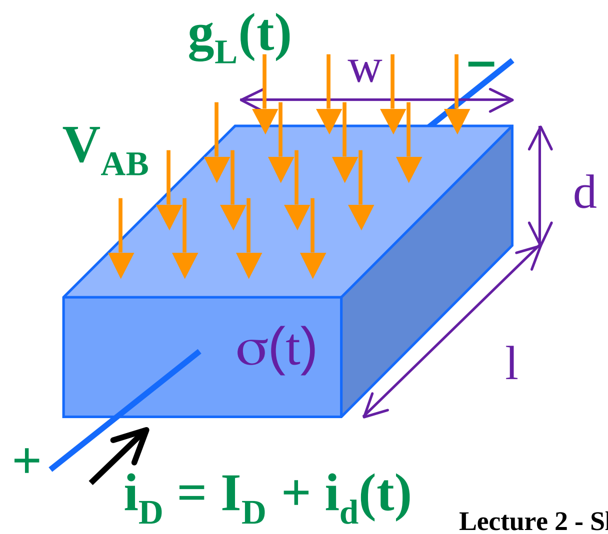
...the conductivity changes:

$$\sigma(t) = q[\mu_e n(t) + \mu_h p(t)]$$
$$= q[\mu_e n_o + \mu_h p_o] + q[\mu_e + \mu_h]n'(t)$$
$$= \sigma_o + \sigma'(\mathrm{t})$$

This change is used in photoconductive detectors to sense light:

$$i_D(t) = [\sigma_o + \sigma'(t)]\frac{w \cdot d}{l}V_{AB} = I_D + i_d(\mathrm{t})$$

with $\quad i_d(t) = \sigma'(t)\frac{w \cdot d}{l}V_{AB}$



The current varies in response to the light $\quad g_L(t) \Rightarrow i_\mathrm{d}(t)$

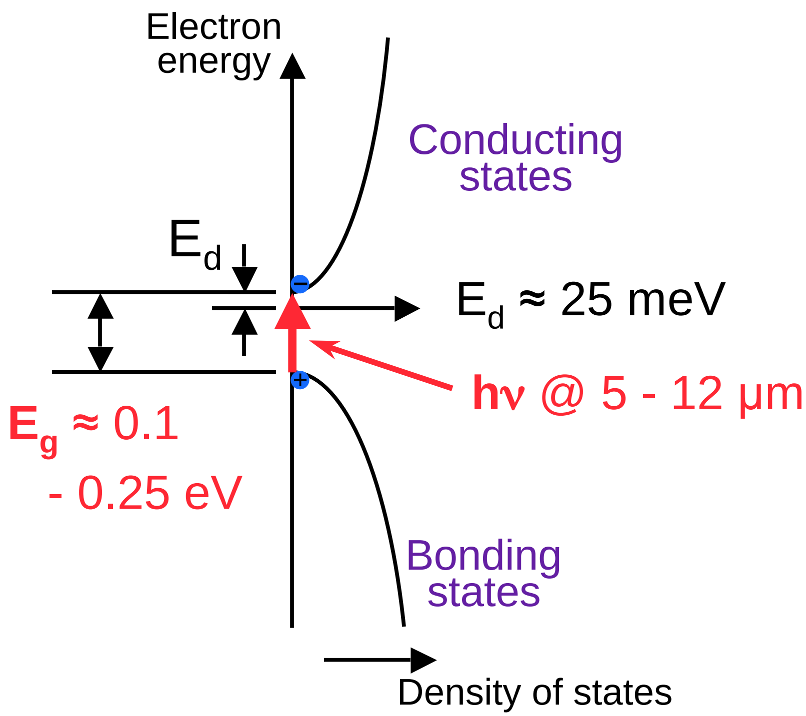
An antique photoconductor at MIT:

A Stanley Magic Door with a lensed photoconductor-based sensor unit.

Do you know where it is on campus?

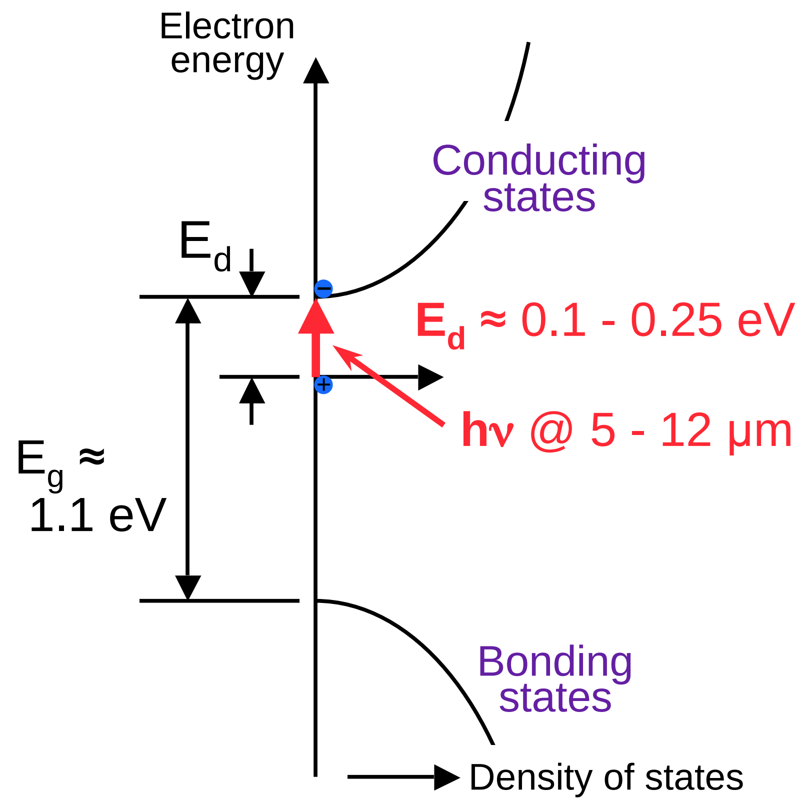
Modern photoconductors - mid-infrared sensors, imagers*

mid-infrared: λ = 5 to 12 µm, hν = 0.1 to 0.25 eV



Intrinsic, band to band option:

- small energy gap; n_i very large
- n' (= p') small relative to n_o
- signal very weak



Extrinsic, donor to band option:

- large energy gap; n_i very small
- n' (= N_D^+) large relative to n_o
- signal much stronger

* Night vision, deep space imaging, thermal analysis

Photoconductors - quantum well infrared photodetectors QWIPs

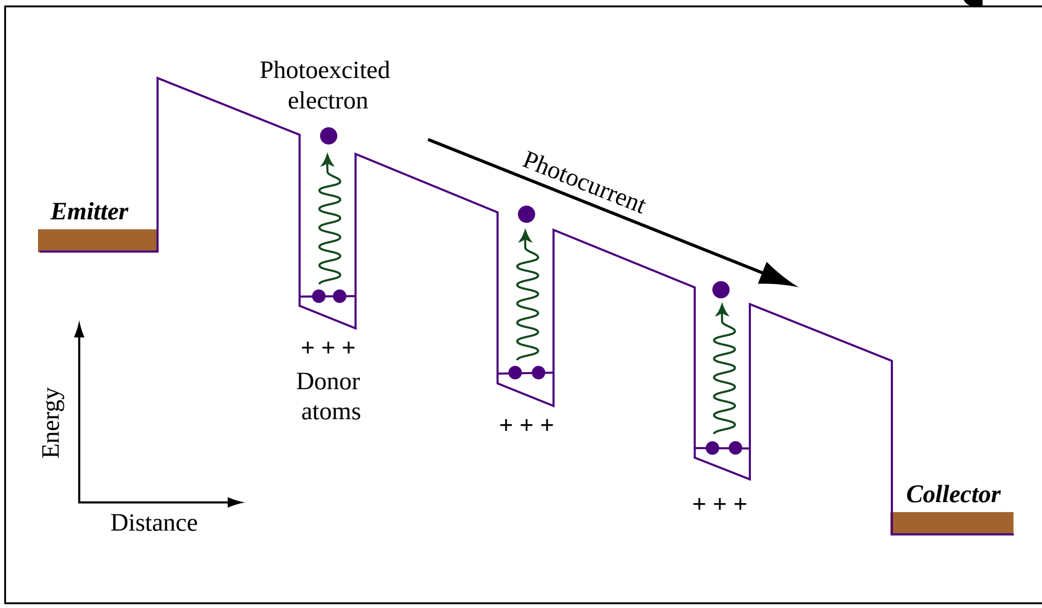


Figure by MIT OpenCourseWare.

Above: Schematic illustration of QWIP structure and function.

Upper right: A QWIP imager photograph

Lower right: Spectral response of a typical IR QWIPs

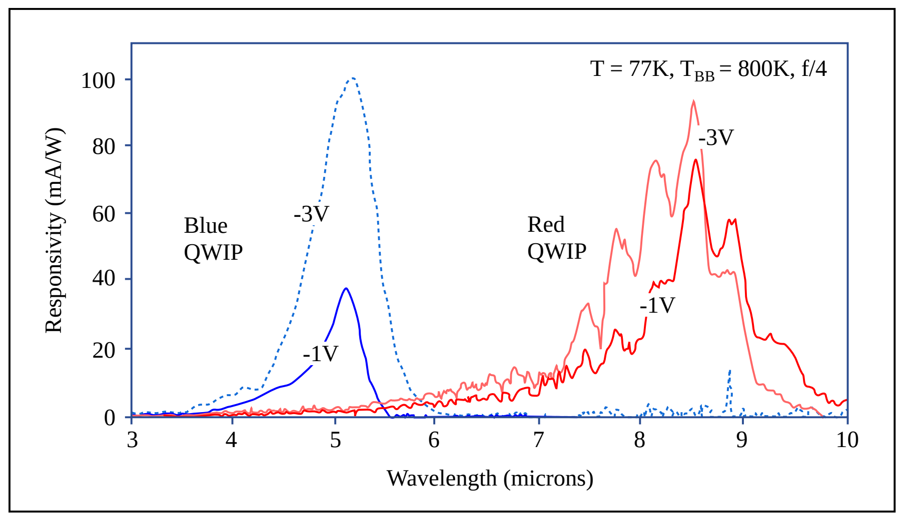


Figure by MIT OpenCourseWare.

Ref: Lockheed-Martin (now BAE Systems), Nashua, N.H.

Note: 5 μm ≈ 0.25 eV   8 μm ≈ 0.15 eV

Non-uniform doping/excitation: Diffusion

When the hole and electron populations are not uniform we have to add diffusion currents to the drift currents we discussed before.

Diffusion flux (Fick's First Law):

Consider particles m with a concentration distribution, $C_m(x)$. Their random thermal motion leads to a diffusion flux density:

$$F_m(x,t) = -D_m \frac{\partial C_m(x,t)}{\partial x} \quad \left[\text{particles/cm}^2\text{ - s}\right]$$

where D_m is the diffusion constant of the particles.

Note that the diffusion flux is <u>down the gradient</u>.

Diffusion current:

Diffusion depends only on the random thermal motion of the particles and has nothing to do with fact that they may be charged. However, if the particles carry a charge q_m, the particle flux is also an electric current density:

$$J_m(x,t) = -q_m D_m \frac{\partial C_m(x,t)}{\partial x} \quad \left[\text{A/cm}^2\right]$$

Non-uniform doping/excitation, cont.: Diffusion

Hole Diffusion Fluxes and Currents:

The hole concentration is p(x,t), each hole carries a charge +q, and the hole diffusion constant is D_h. The hole diffusion flux and current densities are :

$$F_h(x,t) = -D_h \frac{\partial p(x,t)}{\partial x} \qquad J_h(x,t) = -qD_h \frac{\partial p(x,t)}{\partial x}$$

Electron Diffusion Fluxes and Currents:

Similarly for electrons using n(x,t), -q, and D_e :

$$F_e(x,t) = -D_e \frac{\partial n(x,t)}{\partial x} \qquad J_e(x,t) = qD_e \frac{\partial n(x,t)}{\partial x}$$

Total Current Fluxes:

Adding the diffusion and drift current densities yield the total currents:

Holes:

$$J_h(x,t) = q\mu_h p(x,t)E(x,t) - qD_h \frac{\partial p(x,t)}{\partial x}$$

Electrons:

$$J_e(x,t) = q\mu_e n(x,t)E(x,t) + qD_e \frac{\partial n(x,t)}{\partial x}$$

<u>Non-uniform doping/excitation, cont.</u>: Diffusion Continuity

Total Current Fluxes, cont.:

An important different between the drift and diffusion currents:

Holes: $J_h(x,t) = q\mu_h p(x,t)E(x,t) - qD_h \dfrac{\partial p(x,t)}{\partial x}$

Electrons: $J_e(x,t) = q\mu_e n(x,t)E(x,t) + qD_e \dfrac{\partial n(x,t)}{\partial x}$

Drift depends on <u>total</u> carrier concentration

Diffusion depends on the concentration <u>gradient</u>

Continuity Relationships (Fick's Second Law):

Another consequence of non-uniform doping/excitations is that fluxes can vary in space, leading to concentration increases or decreases with time:

$$\frac{\partial F_m(x,t)}{\partial x} = -\frac{\partial C_m(x,t)}{\partial t}$$

This effect must be added to generation and recombination when counting carriers.

Non-uniform doping/excitation, cont.: Continuitity

Continuity, cont.:

For holes and electrons, Fick's Second Law translates to:

Holes: $$\frac{\partial F_h(x,t)}{\partial x} = \frac{1}{q}\frac{\partial J_h(x,t)}{\partial x} = -\frac{\partial p(x,t)}{\partial t}$$

Electrons: $$\frac{\partial F_e(x,t)}{\partial x} = \frac{1}{-q}\frac{\partial J_e(x,t)}{\partial x} = -\frac{\partial n(x,t)}{\partial t}$$

With these factors the total expressions for the dp/dt and dn/dt are:

Holes: $$\frac{\partial p(x,t)}{\partial t} = G - R - \frac{1}{q}\frac{\partial J_h(x,t)}{\partial x}$$

Electrons: $$\frac{\partial n(x,t)}{\partial t} = G - R + \frac{1}{q}\frac{\partial J_e(x,t)}{\partial x}$$

These can also be written as:

$$\frac{\partial p(x,t)}{\partial t} + \frac{1}{q}\frac{\partial J_h(x,t)}{\partial x} = \frac{\partial n(x,t)}{\partial t} - \frac{1}{q}\frac{\partial J_e(x,t)}{\partial x} = G - R$$

We'll come back to this in Lectures 3 and 6.

Non-uniform doping/excitation, cont.: Summary

What we have so far:

Five things we care about (i.e. want to know):

Hole and electron concentrations: $p(x,t)$ and $n(x,t)$

Hole and electron currents: $J_{hx}(x,t)$ and $J_{ex}(x,t)$

Electric field: $E_x(x,t)$

And, amazingly, we already have five equations relating them:

Hole continuity: $$\frac{\partial p(x,t)}{\partial t} + \frac{1}{q}\frac{\partial J_h(x,t)}{\partial x} = G - R \approx G_{ext}(x,t) - \left[n(x,t)p(x,t) - n_i^2\right]r(t)$$

Electron continuity: $$\frac{\partial n(x,t)}{\partial t} - \frac{1}{q}\frac{\partial J_e(x,t)}{\partial x} = G - R \approx G_{ext}(x,t) - \left[n(x,t)p(x,t) - n_i^2\right]r(t)$$

Hole current density: $$J_h(x,t) = q\mu_h p(x,t)E(x,t) - qD_h\frac{\partial p(x,t)}{\partial x}$$

Electron current density: $$J_e(x,t) = q\mu_e n(x,t)E(x,t) + qD_e\frac{\partial n(x,t)}{\partial x}$$

Charge conservation: $$\rho(x,t) = \frac{\partial\left[\varepsilon(x)E_x(x,t)\right]}{\partial x} \approx q\left[p(x,t) - n(x,t) + N_d(x) - N_a(x)\right]$$

So...we're all set, right? No, and yes.....

We'll see next time that it isn't easy to get a general solution, but we can prevail.

Lect 2 - Excitation; Non-Uniform Profiles - Summary

- **Uniform excitation:** optical generation

  In TE, $g_o(T) = n_o p_o\, r(T)$

  Uniform illumination adds uniform generation term, $g_L(t)$

  Populations increase: $n_o \rightarrow n_o + n'$, $p_o \rightarrow p_o + p'$, and $n' = p'$

  $dn'/dt = dp'/dt = g_o(T) + g_L(t) - np\, r(T) = g_L(t) - [np - n_o p_o] r(T)$

  focus is on minority $\approx g_L(t) - n'/\tau_{min}$ with $\tau_{min} \equiv [p_o\, r(T)]^{-1}$ if LLI holds

- **Uniform excitation:** both optical and electrical

  Photoconductivity: $\sigma_o \rightarrow \sigma_o + \sigma' = q\,[\mu_e (n_o + n') + \mu_h (p_o + p')]$

  $= \sigma_o + q\,(\mu_e + \mu_h)\, p'$

  Photoconductors: an important class of light detectors

- **Non-uniform doping/excitation:** diffusion added

  Fick's first law: $F_{mx} = -D_m dC_m/dx$ $\quad [J_{mx} = -q_m D_m dC_m/dx]$

  Diffusion currents: $J_{ex,df} = qD_e dn/dx$, $J_{hx,df} = -qD_h dp/dx$,

  Total currents: $J_{ex} = J_{ex,dr} + J_{ex,df} = qn\mu_e E_x + qD_e dn/dx$

  $J_{hx} = J_{hx,dr} + J_{hx,df} = qp\mu_h E_x - qD_h dp/dx$

  Fick's second law: $dC_m/dt = -dF_{mx}/dx$ $\quad [dC_m/dt = -(1/q_m) dJ_{mx}/dx]$

  Continuity: $dn/dt - (1/q) dJ_{ex}/dx = dp/dt + (1/q) dJ_{hx}/dx = G - R$