MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.012 MICROELECTRONIC DEVICES AND CIRCUITS

Answers to Exam 1 - Fall 2009

Problem 1: Graded by Prof. Fonstad

a) i) The resistivity is given, and is the inverse of the conductivity: $\sigma = 1/\rho = q\mu_e n_o$. Thus, $n_o = 1/\rho q\mu_e = 1/(10^2 x1.6x10^{-19} x1.6x10^3) = 3.9x10^{13}$ cm$^{-3} \approx 4 \text{ x } 10^{13}$ cm^{-3}.

ii) $p_o = n_i^2/n_o = 10^{20}/(3.9x10^{13}) = 2.56 \text{ x } 10^6$ cm^{-3}.

b) i) $i_h = A\ j_h = -\ A\ q\ D_h\ dp'/dx$. $D_h = \mu_h(kT/q) = 640 \text{ x } 0.025 = 640/40 = 16$ cm^2/s. Thus, $i_h = -10^{-4}x1.6x10^{-19}x1.6x10^1x(-5x10^{14}/2x10^{-4}) = 6.4 \text{ x } 10^{-4}$ A.

ii) The total number is the area under the profile times the cross-sectional area: Excess holes = $(5x10^{14}x2x10^{-4}/2)x10^{-4} = 5 \text{ x } 10^6$

iii) The total recombination is the total excess divided by the minority carrier lifetime: $5x10^6/\tau_h = 5x10^6/10^{-4} = 5 \text{ x } 10^{10}$ holes/s

The hole recombination current, i.e. the current supplying the holes is the total hole recombination rate times q: $5 \text{ x } 10^{10} \text{ x } 1.6 \text{ x } 10^{-19} = 8 \text{ x } 10^{-9}$ A. This is much smaller than the hole diffusion current found in 1-b-i, i.e. negligible.

c) i) Diode B, because the saturation current varies inversely with the effective width of the diode. Both diodes have the same physical length, but since $L_h < w_N$ in Diode B, it has the smaller effective width, and thus the larger I_S.

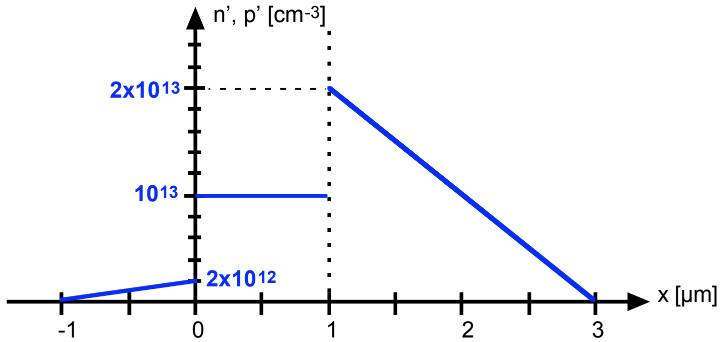
ii) Diode A, because the excess hole population in it falls off linearly to zero at $x = w_N$, whereas in Diode B the excess population falls off more quickly, i.e. exponentially. As a result there is more area under the excess carrier profile in Diode A, and thus more excess carriers in the n-side quasi-neutral region.

iii) They are similar, because the space charge layer width does not depend on τ_{min}.

d) i) The thermal equilibrium minority carrier populations in the emitter, base, and collector are: $p_{oE} = 2 \text{ x } 10^2$ cm^{-3}, $n_{oB} = 10^3$ cm^{-3}, $p_{oC} = 2 \text{ x } 10^3$ cm^{-3}. When a 0.6 V bias is applied to the junctions, the excess minority carrier concentrations at the edges of the space charge layer increase by 0.6/0.06 = 10 orders of magnitude, i.e. a factor of 10^{10}, to: $p'(0^-) = 2 \text{ x } 10^{12}$ cm^{-3}, $n'(0^+) = 10^{13}$ cm^{-3}, $n'(1^-) = 10^{13}$ cm^{-3}, and $p'(1^+) = 2 \text{ x } 10^{13}$ cm^{-3}. The plot of the corresponding carrier profiles is shown in the figure at the top of the next page

Note 1: Some students did not use the 60 mV rule, but calculated $\exp(qV_{AB}/kT)$ with kT/q = 25 mV. In this case the increase is by a factor of $e^{0.6/0.025} = e^{24} \approx 2.65 \text{ x } 10^{10}$, instead of 10^{10}; either answer was accepted.

Note 2: $10 \approx e^{2.3}$ so so using kT/q = 26 mV (i.e., the 60 mV rule) says the increase is by a factor of e^{23}, whereas using kT/q = 25 mV yields e^{24}. The minority carrier population, and the diode current as well, are clearly very sensitive functions of temperature. A good design doesn't rely on either being having a fixed, precisely known value.



ii) $\beta \approx 1/\delta_E = (N_{DE}/N_{AB})(D_h/D_e)(W_B/W_E) = (5x10^{17}/10^{17})(40/15)(1/1) = 13.3$

iii) The choice is between emitter and base, and from the expression above we see the correct choice to increase the doping of the emitter, which increases β. We want to make β 100/13.3 = 7.5 times bigger, so we should increase N_{DE} by a factor of 7.5 to 3.75 x 10^{18} cm^{-3}.

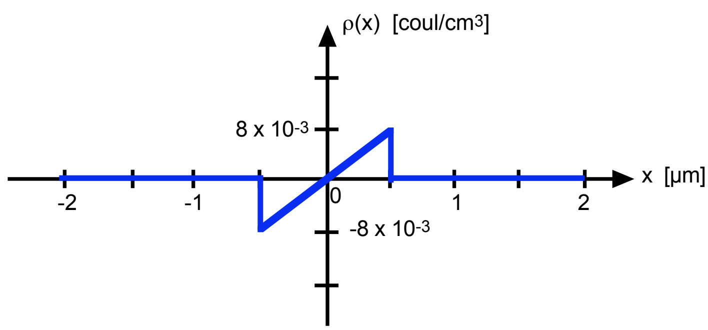
Problem 2: Graded by Prof. Palacios

a) $\phi(-1.5\ \mu m) = -(kT/q)\ \ln(10^{17}/10^{10}) \approx\ -7 \times 0.06 = -0.42$ V

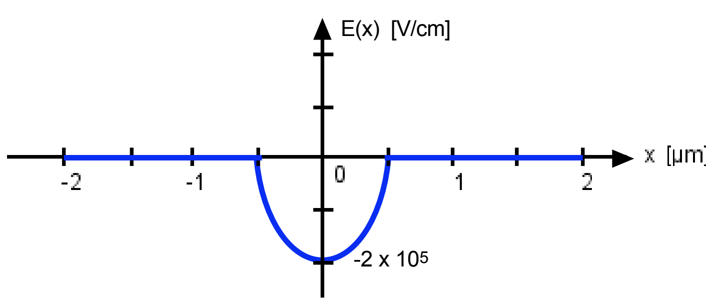
$\phi(1.5\ \mu m) = (kT/q)\ \ln(10^{17}/10^{10}) \approx\ 7 \times 0.06 =\ 0.42$ V

$\Delta\phi = \phi(1.5\ \mu m) - \phi(-1.5\ \mu m) = 0.42 - (-0.42) = 0.84$ V

b) $\rho(x) =\ 160$ x for $-5x10^{-5}$ cm < x < $5x10^{-5}$ cm (i.e.- 0.5 μm < x < 0.5 μm), and ≈ 0 elsewhere.



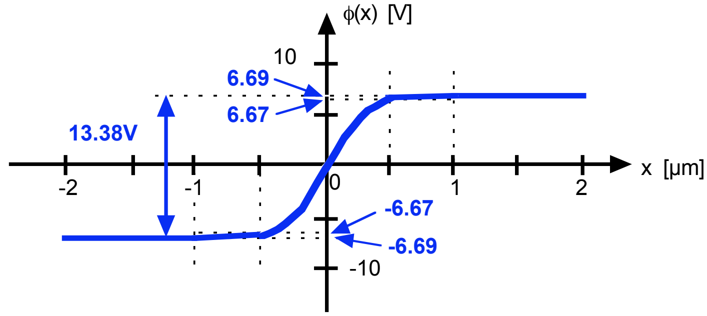
c) $E(x) = \int\rho(x)\ dx/\varepsilon_{Si} = 8x10^{13}\ (x^2 - 2.5x10^{-9})$ for $-\ 5x10^{-5}$ cm < x < $5x10^{-5}$ cm, and ≈ 0 elsewhere. The peak value is $-2x10^5$ V/cm at x = 0.



d) We can obtain the potential step between - 0.5 μm and 0.5 μm by integrating the electric field over this range. Doing this yields: $\phi(x) = -8x10^{13}[\ (x^3/3) - 2.5x10^{-9}\ x]$. Using this: $\phi(-0.5\mu m) = -6.67$ V, and $\phi(0.5\mu m) = 6.67$ V, so $\Delta\phi_{Depl.\ Reg.} = 13.34$ V.

e) Between $x = 0.5\ \mu m$ and $x = 1.0\ \mu m$, the doping changes by a factor of 2. Thus, using the 60 mV rule, the electrostatic potential must change by 0.06 x log 2 Volts, or 0.06 x 0.3 = 0.018 V. $\Delta\phi = 0.018$ V between $x = -1.0\ \mu m$ and $x = -0.5\ \mu m$, also.

f)



g) In thermal equilibrium, $\Delta\phi$ crossing the junction, ϕ_b, from Part a, is 0.84 V. Now the total change, $(\phi_b - V_{AB})$, is 0.02 + 13.34 + 0.02 = 13.38 V. Subtracting, we find the applied voltage, V_{AB}: $V_{AB} = -12.54$ V

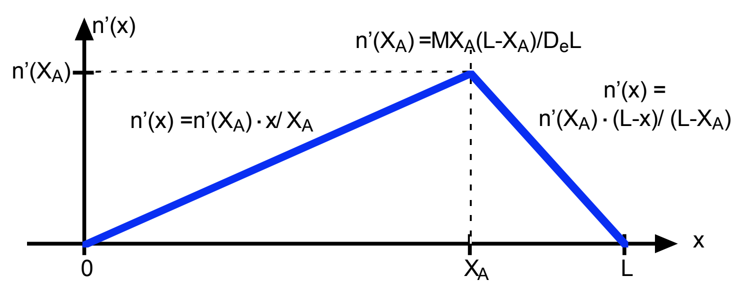
Problem 3: Graded by Prof. Weinstein

a) B.C. @ $x = 0$: $n'(0) = 0$. B.C. @ $x = L$: $n'(L) = 0$.

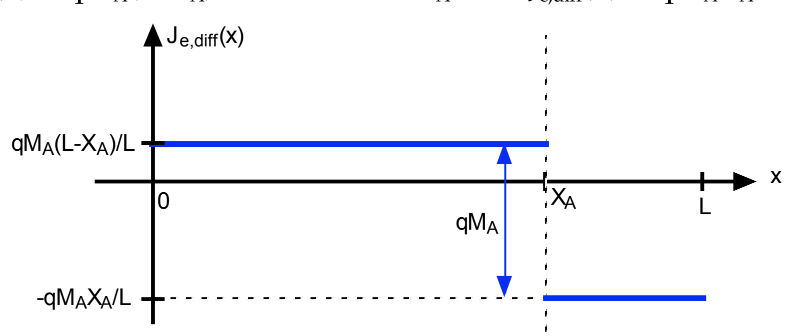
b) We require $L_e >> L$, and since $L_e = (D_e\tau_e)^{1/2}$, we must have $\tau_e >> L^2/D_e$

c) Crossing the plane of injection at X_A, n' must be continuous, and the sum of the carrier fluxes away from X_A must be M. So, the constraint on n' is $n'(X_{A-}) = n'(X_{A+})$, and the constraint on dn'/dx is $D_e(dn'/dx|_{X_{A-}} - dn'/dx|_{X_{A+}}) = M_A$.

d) For $0 < x < X_A$: $n'(x) = n'(X_A)\, x/X_A$, and for $X_A < x < L$, $n'(x) = n'(X_A)\,(L-x)/(L-X_A)$. Note: The value of $n'(X_A)$ is most easily found after first solving Part e, see below.



e) For $0 < x < X_A$: $J_{e,diff}(x) = qD_e n'(X_A)/X_A$; for $X_A < x < L$, $J_{e,diff}(x) = -\,qD_e n'(X_A)/(L-X_A)$. The current step at $x = X_A$ must be qM_A. Using this gives $n'(X_A) = MX_A(L-X_A)/D_eL$; thus: $J_{e,diff}(x) = qM_A(L-X_A)/L$ for $0 < x < X_A$, and $J_{e,diff}(x) = qM_AX_A/L$ for $X_A < x < L$.



f) Minority carriers flow primarily by diffusion because any minority carrier drift current must be negligible compared to the majority carrier drift current because in low level injection the majority carriers far outnumber the minority carriers.

g) Superposition is valid, so the excess minority carrier populations and the minority carrier drift currents due to each excitation, $M_A\delta(X_A)$ and $M_B\delta(X_B)$, individually can be superimposed, i.e. added:

$$n'(x, g_L = M_A\delta(X_A) + M_B\delta(X_B)) = n'(x, g_L = M_A\delta(X_A)) + n'(x, g_L = M_B\delta(X_B))$$

$$J_{e,diff}(x, g_L = M_A\delta(X_A) + M_B\delta(X_B)) = J_{e,diff}(x, g_L = M_A\delta(X_A)) + J_{e,diff}(x, g_L = M_B\delta(X_B))$$

Exam Statistics

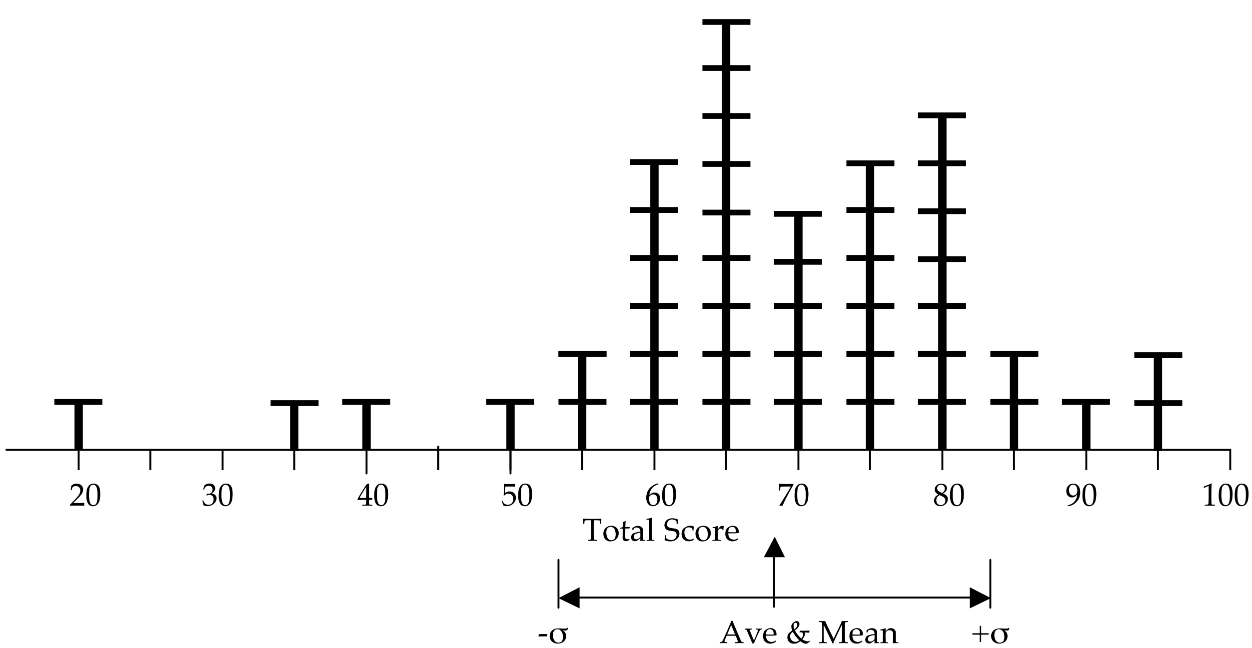
Average/Standard deviation:

Problem 1	26.4	4.9
Problem 2	23.0	6.2
Problem 3	18.8	6.4
Total	68.3	14.3

Class median: 68.5

Distribution to nearest 5:

Find your face in this picture



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