

# Lecture 8 - PN Junction and MOS Electrostatics (V)

## ELECTROSTATICS OF METAL-OXIDE-SEMICONDUCTOR STRUCTURE (*cont.*)

October 4, 2005

### Contents:

1. Overview of MOS electrostatics under bias
2. Depletion regime
3. Flatband
4. Accumulation regime
5. Threshold
6. Inversion regime

### Reading assignment:

Howe and Sodini, Ch. 3, §§3.8-3.9

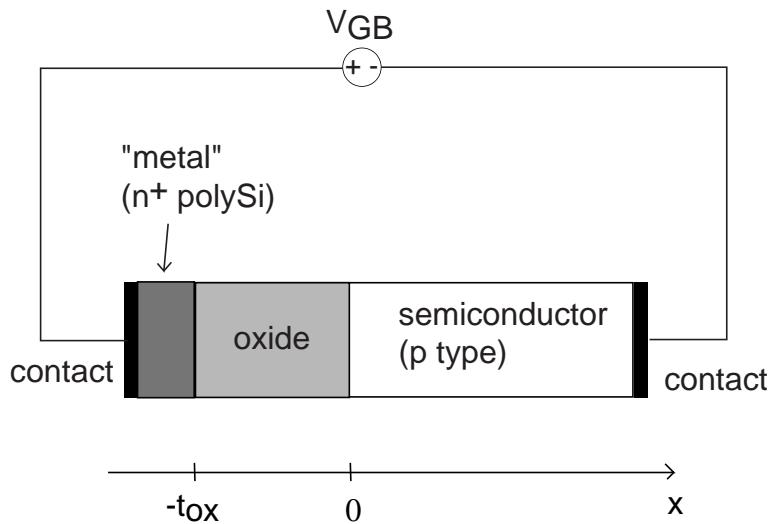
### Announcements:

Quiz 1: 10/13, 7:30-9:30 PM,  
(lectures #1-9); open book; must have calculator.

## Key questions

- Is there more than one regime of operation of the MOS structure under bias?
- What does "carrier inversion" mean and what is the big deal about it?
- How does the carrier inversion charge depend on the gate voltage?

# 1. Overview of MOS electrostatics under bias



Application of bias:

- built-in potential across MOS structure increases from  $\phi_B$  to  $\phi_B + V_{GB}$
- oxide forbids current flow  $\Rightarrow$ 
  - $J = 0$  everywhere in semiconductor
  - need *drift=-diffusion* in SCR
- must maintain boundary condition at Si/SiO<sub>2</sub> interface:  $E_{ox}/E_s \simeq 3$

How can this be accommodated simultaneously?  $\Rightarrow$   
**quasi-equilibrium situation** with potential build up across MOS equal to  $\phi_B + V_{GB}$

Important consequence of quasi-equilibrium:

⇒ Boltzmann relations apply in semiconductor

[they were derived starting from  $J_e = J_h = 0$ ]

$$n(x) = n_i e^{q\phi(x)/kT}$$

$$p(x) = n_i e^{-q\phi(x)/kT}$$

and

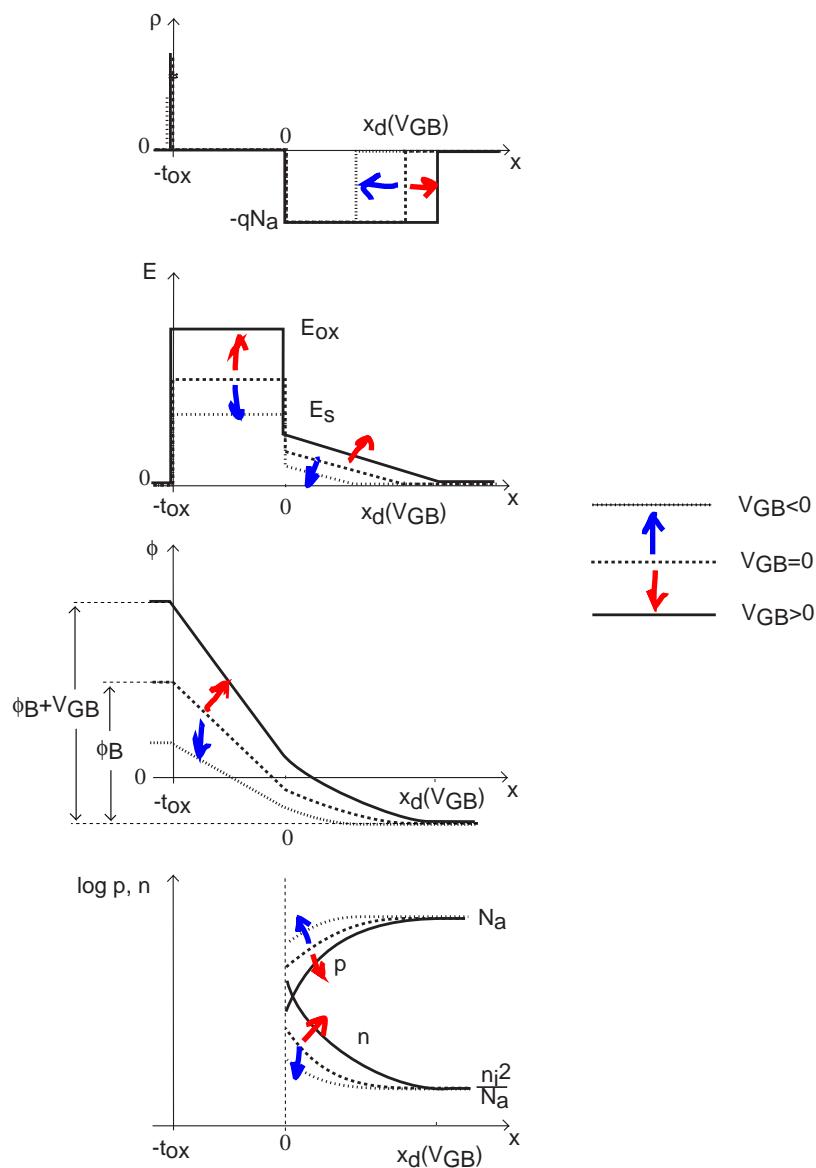
$$np = n_i^2 \text{ at every } x$$

[not the case in p-n junction or BJT under bias]

## 2. Depletion regime

For  $V_{GB} > 0$  gate "attracts" electrons, "repels" holes  
 $\Rightarrow$  depletion region widens

For  $V_{GB} < 0$  gate "repels" electrons, "attracts" holes  
 $\Rightarrow$  depletion region shrinks



In depletion regime, all results obtained for zero bias apply if  $\phi_B \rightarrow \phi_B + V_{GB}$ .

For example:

- depletion region thickness:

$$x_d(V_{GB}) = \frac{\epsilon_s}{C_{ox}} \left[ \sqrt{1 + \frac{4(\phi_B + V_{GB})}{\gamma^2}} - 1 \right]$$

- potential drop across semiconductor SCR:

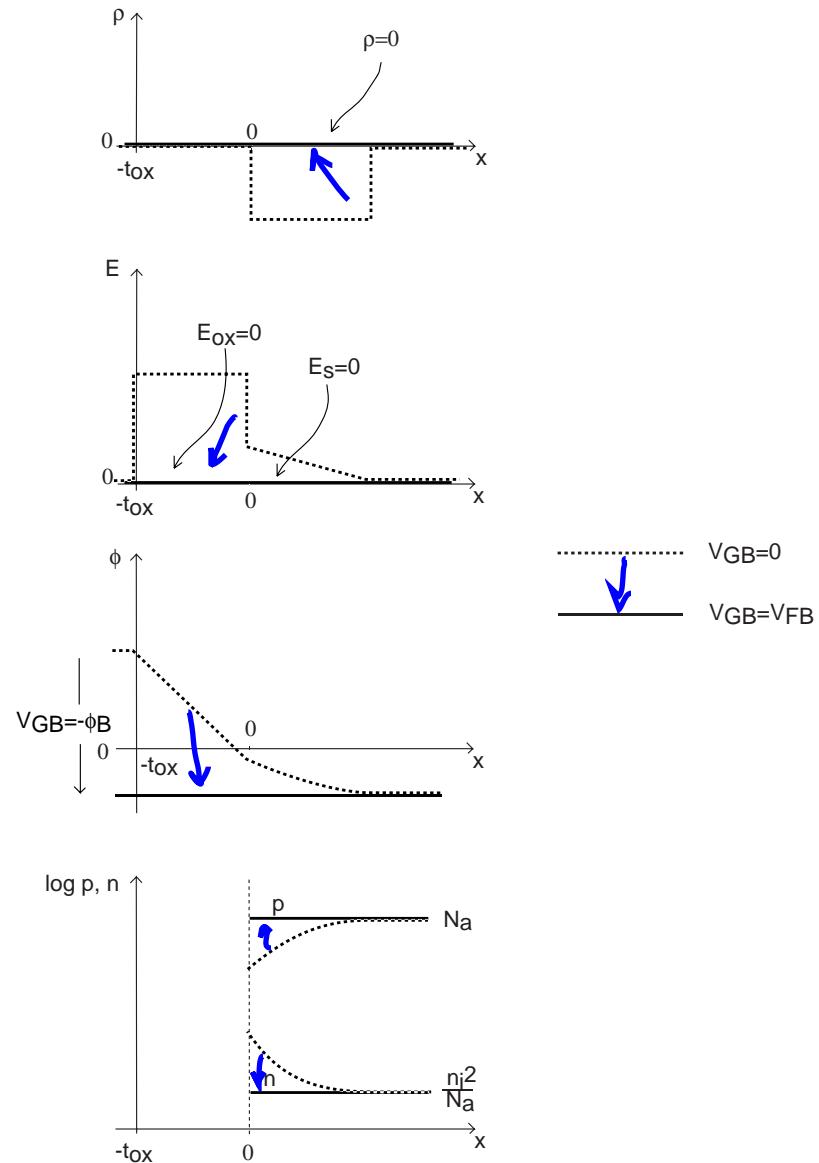
$$V_B(V_{GB}) = \frac{q N_a x_d^2(V_{GB})}{2 \epsilon_s}$$

- potential drop across oxide:

$$V_{ox}(V_{GB}) = \frac{q N_a x_d(V_{GB}) t_{ox}}{\epsilon_{ox}}$$

### 3. Flatband

At a certain negative  $V_{GB}$ , depletion region is wiped out  
 $\Rightarrow$  *Flatband*

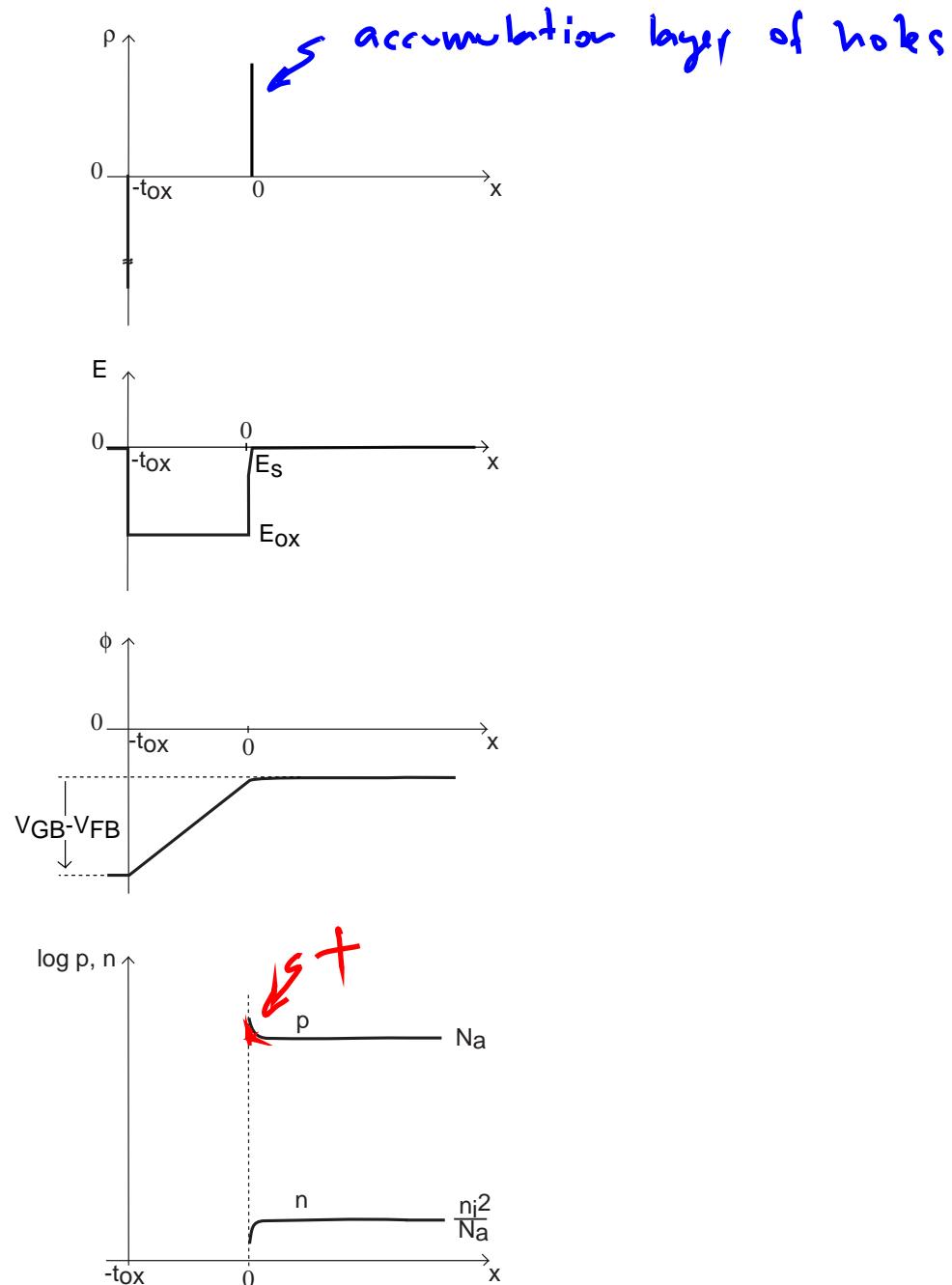


*Flatband voltage:*

$$V_{FB} = -\phi_B$$

## 4. Accumulation regime

If  $V_{GB} < V_{FB}$  accumulation of holes at Si/SiO<sub>2</sub> interface

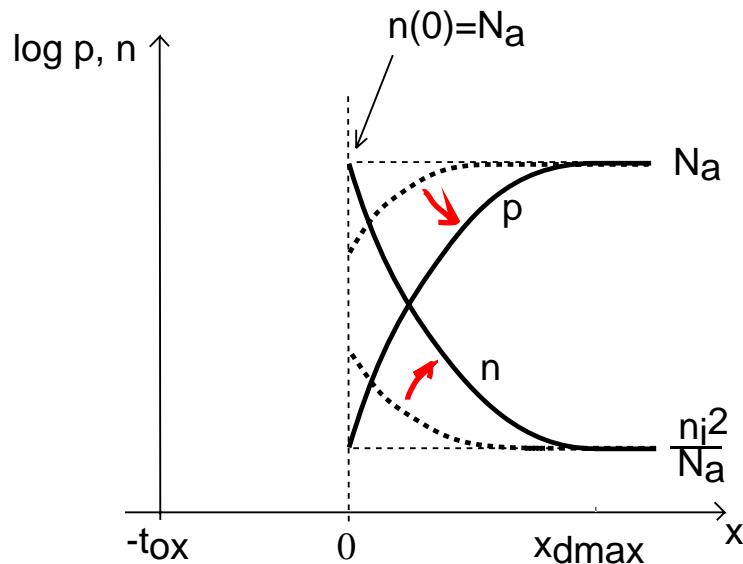


## 5. Threshold

Back to  $V_{GB} > 0$ .

For sufficiently large  $V_{GB} > 0$ , electrostatics change when  $n(0) = N_a \Rightarrow \text{threshold}$ .

Beyond *threshold*, cannot neglect contributions of electrons towards electrostatics.



Let's compute the voltage (*threshold voltage*) that leads to  $n(0) = N_a$ .

Key assumption: use electrostatics of depletion (neglect electron concentration at threshold).

- Computation of threshold voltage.

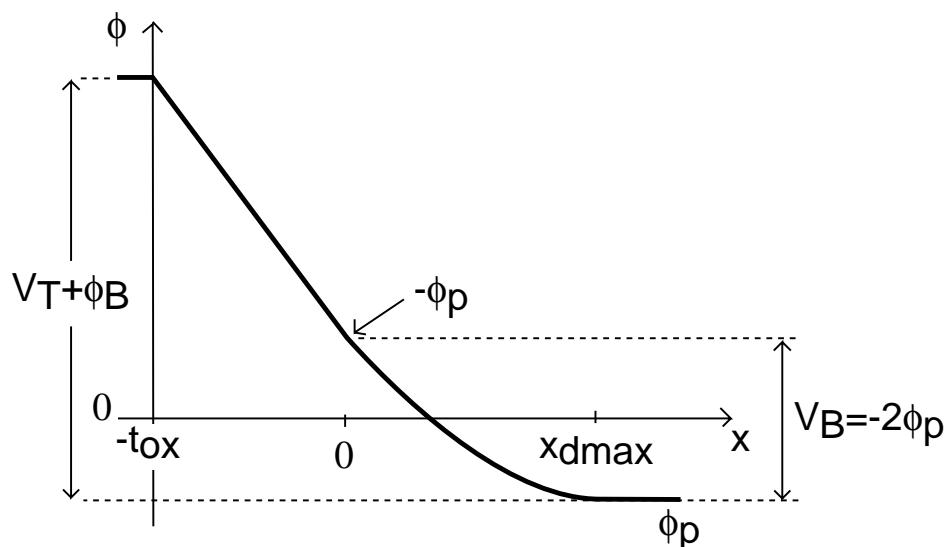
Three-step process:

- First, compute potential drop in semiconductor at threshold. Start from:

$$n(0) = n_i e^{q\phi(0)/kT}$$

Solve for  $\phi(0)$  at  $V_{GB} = V_T$ :

$$\phi(0)|_{V_T} = \frac{kT}{q} \ln \frac{n(0)}{n_i}|_{V_T} = \frac{kT}{q} \ln \frac{N_a}{n_i} = -\phi_p$$



Hence:

$$V_B(V_T) = -2\phi_p$$

- Second, compute potential drop in oxide at threshold.

Obtain  $x_d(V_T)$  using relationship between  $V_B$  and  $x_d$  in depletion:

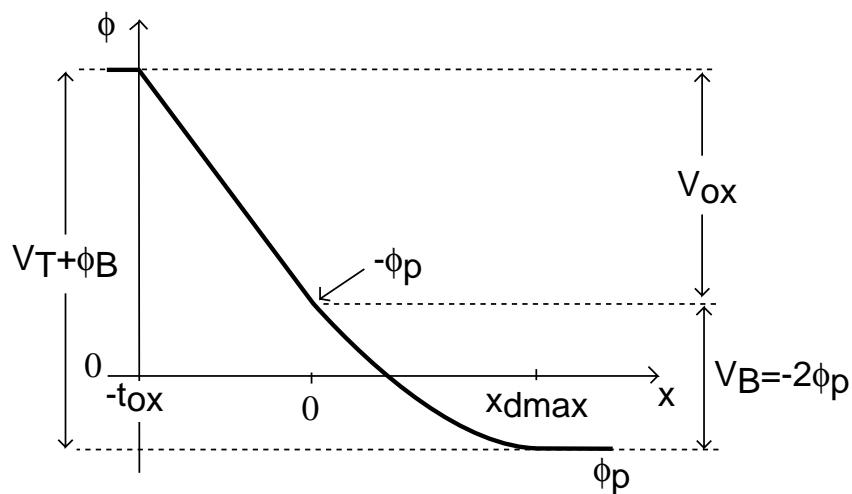
$$V_B(V_T) = \frac{qN_a x_d^2(V_T)}{2\epsilon_s} = -2\phi_p$$

Solve for  $x_d(V_T)$ :

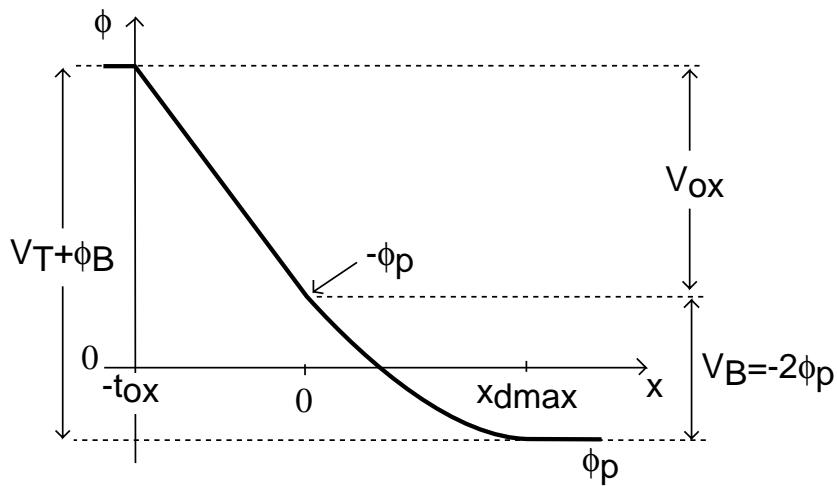
$$x_d(V_T) = x_{dmax} = \sqrt{\frac{2\epsilon_s(-2\phi_p)}{qN_a}}$$

Then:

$$\boxed{V_{ox}(V_T) = E_{ox}(V_T)t_{ox} = \frac{qN_a x_d(V_T)}{\epsilon_{ox}}t_{ox} = \gamma\sqrt{-2\phi_p}}$$



- Finally, sum potential drops across structure.



$$V_T + \underbrace{\phi_B}_{-V_{FB}} = V_B(V_T) + V_{ox}(V_T) = -2\phi_p + \gamma\sqrt{-2\phi_p}$$

Solve for  $V_T$ :

$$V_T = V_{FB} - 2\phi_p + \gamma\sqrt{-2\phi_p}$$

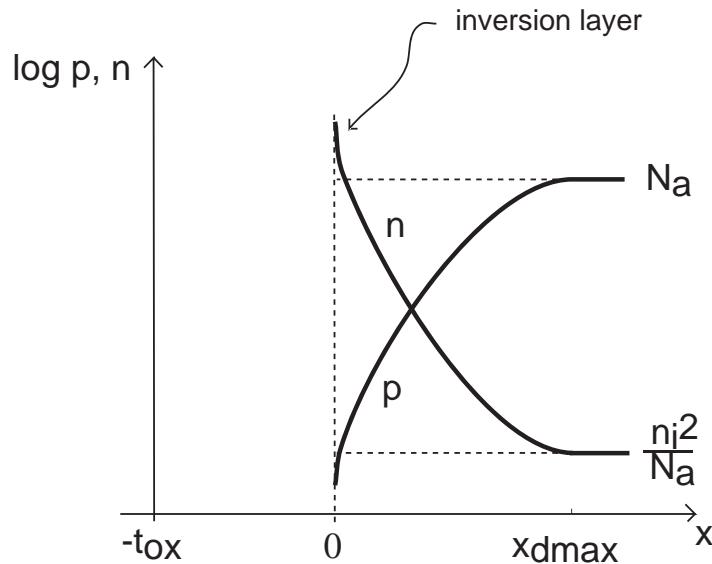
Key dependencies:

- If  $N_a \uparrow \rightarrow V_T \uparrow$ . The higher the doping level, the more voltage required to produce  $n(0) = N_a$ .
- If  $C_{ox} \uparrow$  ( $t_{ox} \downarrow$ )  $\rightarrow V_T \downarrow$ . The thinner the oxide, the less voltage dropped across it.

## 6. Inversion

What happens for  $V_{GB} > V_T$ ?

More electrons at Si/SiO<sub>2</sub> interface than acceptors  
 $\Rightarrow$  *inversion*.



Electron concentration at Si/SiO<sub>2</sub> interface modulated by  $V_{GB}$   $\Rightarrow V_{GB} \uparrow \rightarrow n(0) \uparrow \rightarrow |Q_n| \uparrow$

**field-effect control of mobile charge!**

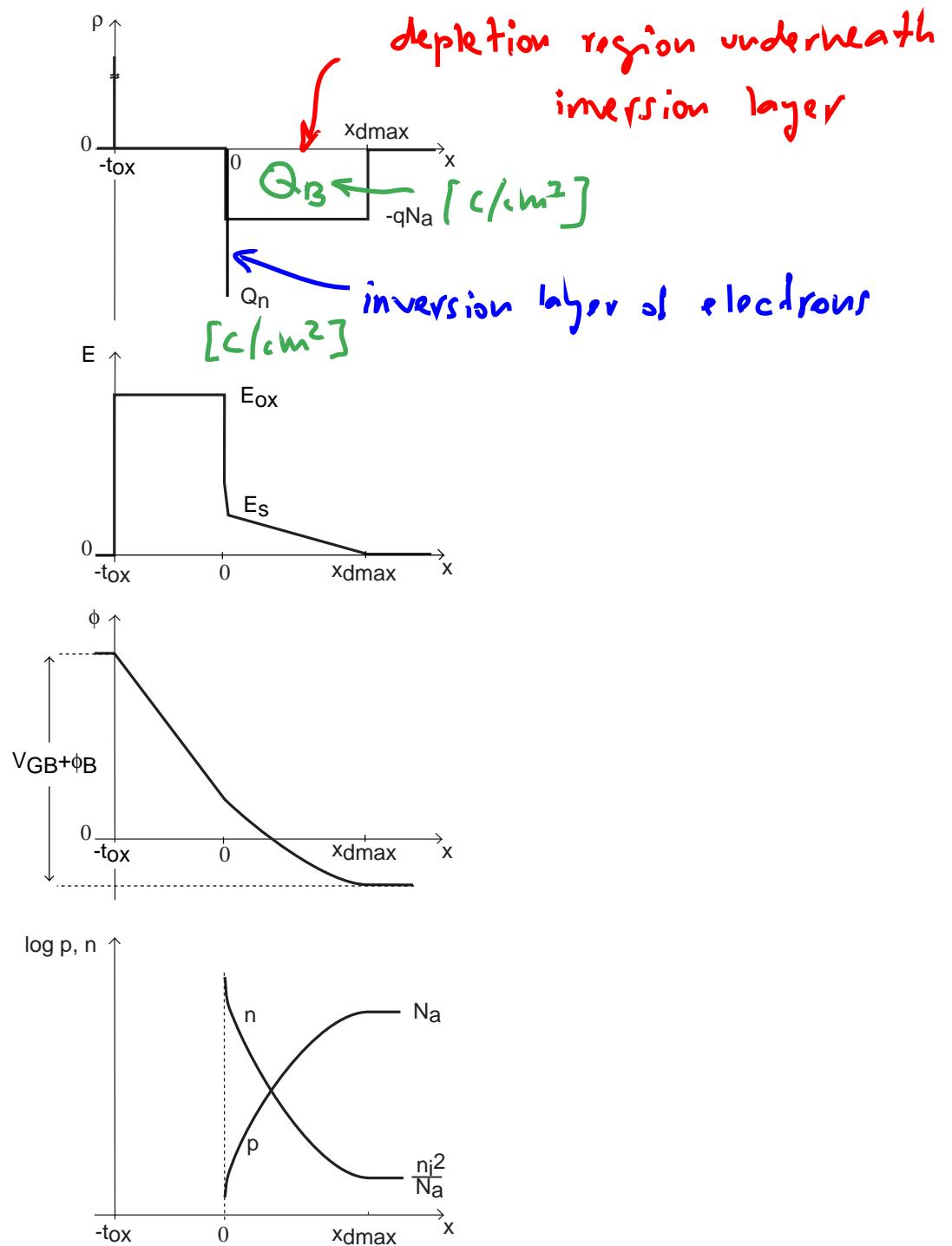
[essence of MOSFET]

Want to compute  $Q_n$  vs.  $V_{GB}$  [*charge-control relation*]

Make sheet charge approximation: electron layer at semiconductor surface is much thinner than any other dimension in problem ( $t_{ox}$ ,  $x_d$ ).

□ Charge-control relation

Let us look at overall electrostatics:



Key realization:

$$|Q_n| \propto n(0) \propto e^{q\phi(0)/kT}$$

*charge in depletion region*

$$|Q_B| \propto \sqrt{\phi(0)}$$

Hence, as  $V_{GB} \uparrow$  and  $\phi(0) \uparrow$ ,  $|Q_n|$  will change a lot, but  $|Q_B|$  will change very little.

Several consequences:

- little change in  $\phi(0)$  beyond threshold
- $V_B$  does not increase much beyond  $V_B(V_T) \equiv -2\phi_p$   
(*a thin sheet of electrons does not contribute much to  $V_B$* ):

$$V_B(\text{inv.}) \simeq V_B(V_T) = -2\phi_p$$

- little change in  $Q_B$  beyond threshold
- $x_d$  does not increase much beyond threshold:

$$x_d(\text{inv.}) \simeq x_d(V_T) = \sqrt{\frac{2\epsilon_s(-2\phi_p)}{qN_a}} = \boxed{x_{dmax}}$$

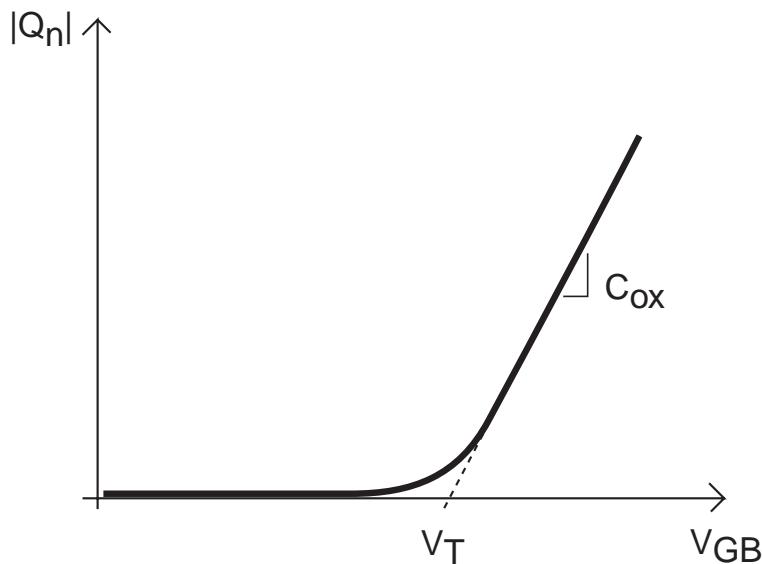
- All extra voltage beyond  $V_T$  used to increase inversion charge  $Q_n$ . Think of it as capacitor:
  - top plate: metal gate
  - bottom plate: inversion layer

$$Q = CV$$

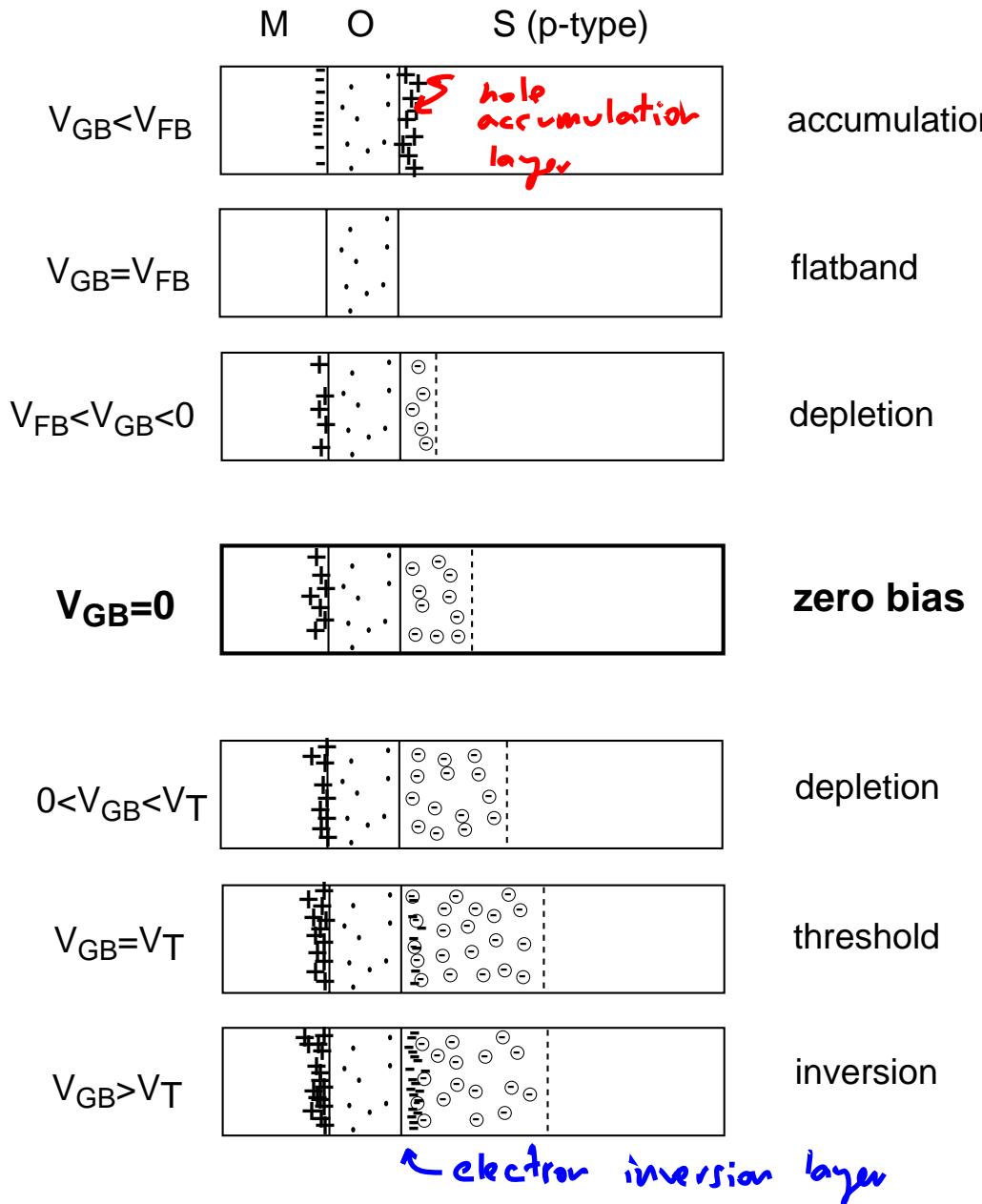
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$$\Rightarrow Q_n = -C_{ox}(V_{GB} - V_T) \quad \text{for } V_{GB} > V_T$$

Existence of  $Q_n$  and control over  $Q_n$  by  $V_{GB}$   $\Rightarrow$  key to MOS electronics



# Key conclusions



In inversion:

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$$|Q_n| = C_{ox}(V_{GB} - V_T) \quad \text{for } V_{GB} > V_T$$