Lecture 23 - Frequency Response of Amplifiers (I)

Common-Source Amplifier

December 1, 2005

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Reading assignment:

Howe and Sodini, Ch. 10, §§10.1-10.4

Key questions

- How does one assess the intrinsic frequency response of a transistor?
- What limits the frequency response of an amplifier?
- What is the "Miller effect"?

1. Introduction

Frequency domain is a major consideration in most analog circuits.

Data rates, bandwidths, carrier frequencies all pushing up.

Motivation:

- Processor speeds ↑
- Traffic volume ↑ ⇒ data rates ↑
- More bandwidth available at higher frequencies in the spectrum

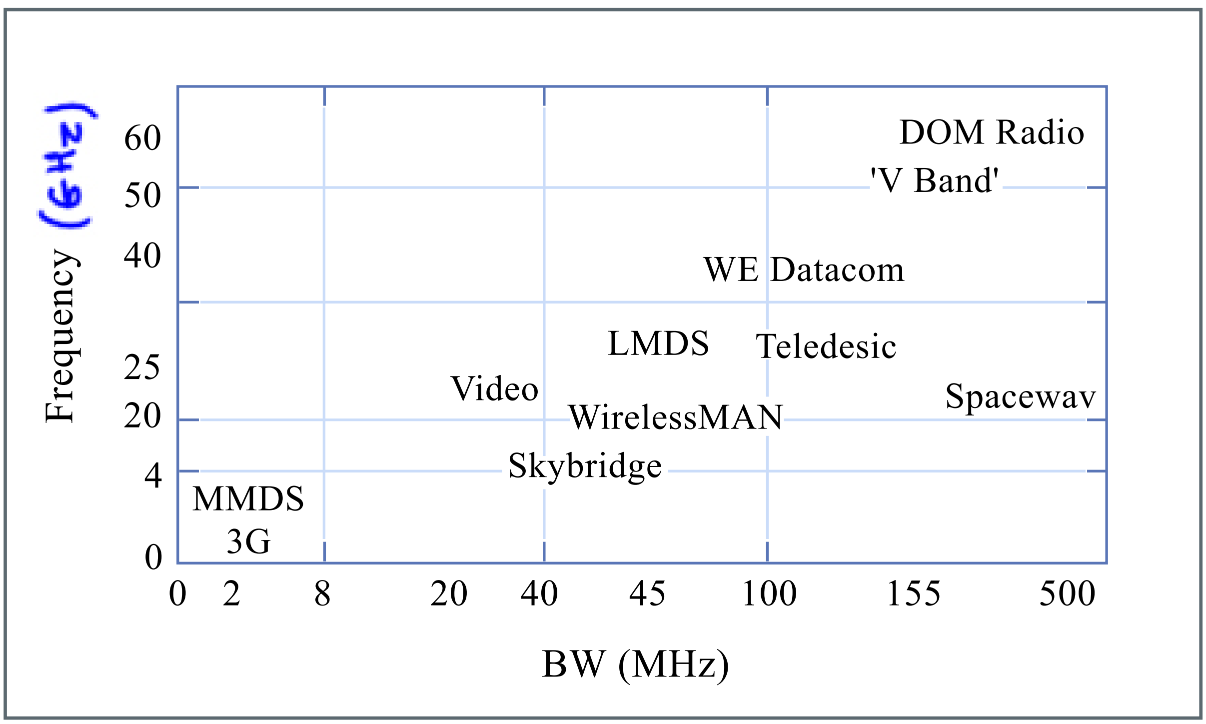


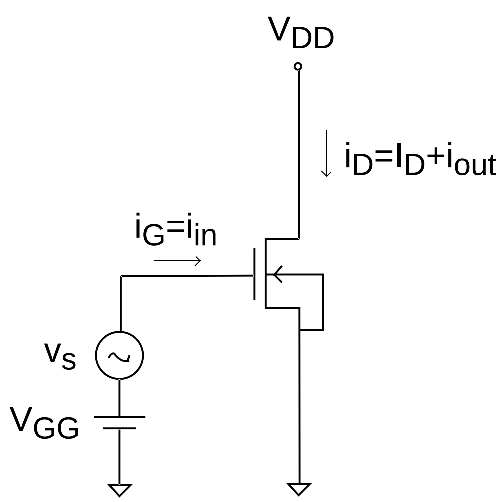
Figure by MIT OCW.

2. Intrinsic frequency response of MOSFET

□ How does one assess the intrinsic frequency response of a transistor?

$f_t \equiv$ *short-circuit current-gain cut-off frequency [GHz]*

Consider MOSFET biased in saturation regime with small-signal source applied to gate:



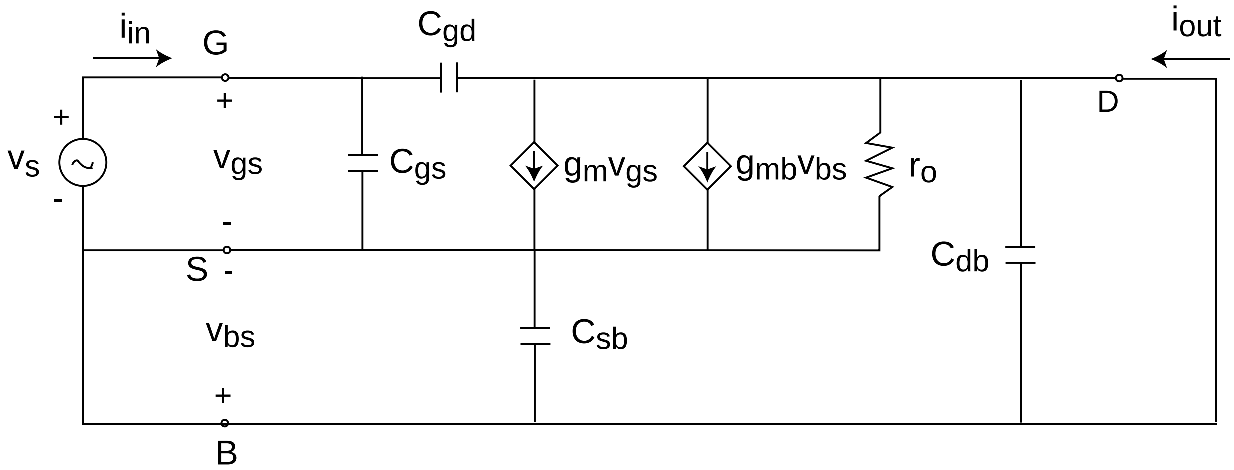
v_s at input $\Rightarrow i_{out}$: transistor effect

$\Rightarrow i_{in}$ due to gate capacitance

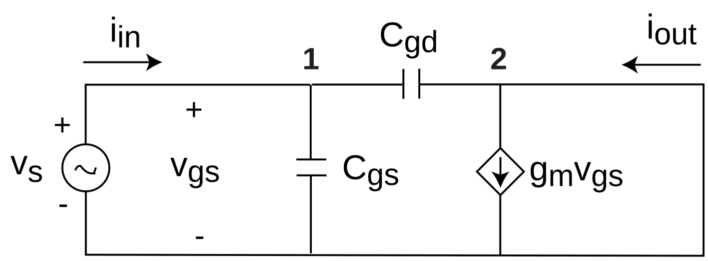
Frequency dependence: $f \uparrow \Rightarrow \; i_{in} \uparrow \Rightarrow |\frac{i_{out}}{i_{in}}| \downarrow$

$$f_t \equiv \text{ frequency at which } |\frac{i_{out}}{i_{in}}| = 1$$

Complete small-signal model in saturation:



vbs=0



node **1**: $$i_{in} - v_{gs} j\omega C_{gs} - v_{gs} j\omega C_{gd} = 0$$

$$\Rightarrow i_{in} = v_{gs} j\omega (C_{gs} + C_{gd})$$

node **2**: $$i_{out} - g_m v_{gs} + v_{gs} j\omega C_{gd} = 0$$

$$\Rightarrow i_{out} = v_{gs}(g_m - j\omega C_{gd})$$

Current gain:

$$h_{21} = \frac{i_{out}}{i_{in}} = \frac{g_m - j\omega C_{gd}}{j\omega(C_{gs} + C_{gd})}$$

□ Magnitude of h_{21}:

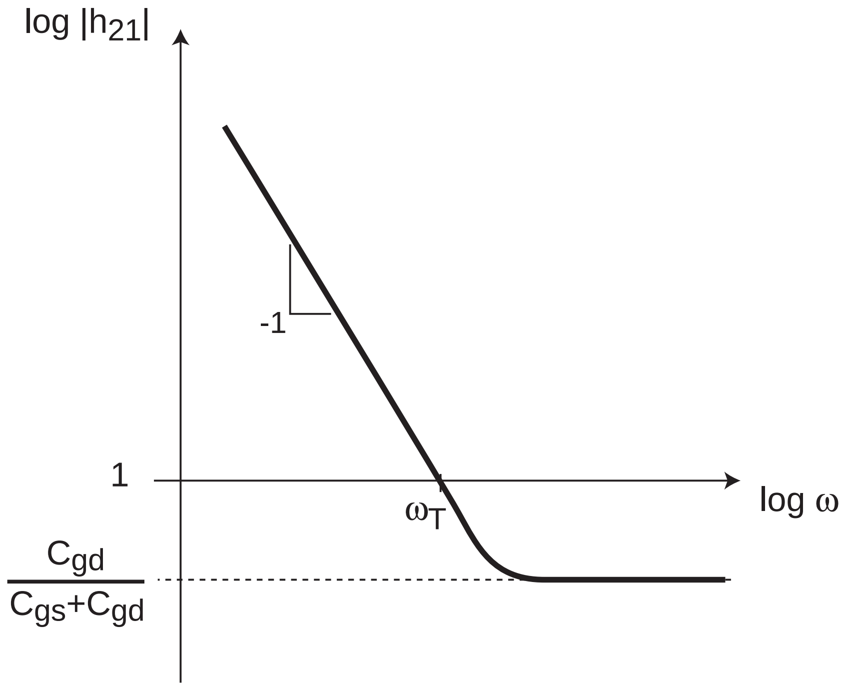
$$|h_{21}| = \frac{\sqrt{g_m^2 + \omega^2 C_{gd}^2}}{\omega(C_{gs} + C_{gd})}$$

- For low frequency, $\omega \ll \frac{g_m}{C_{gd}}$,

$$|h_{21}| \simeq \frac{g_m}{\omega(C_{gs} + C_{gd})}$$

- For high frequency, $\omega \gg \frac{g_m}{C_{gd}}$,

$$|h_{21}| \simeq \frac{C_{gd}}{C_{gs} + C_{gd}} < 1$$



$|h_{21}|$ becomes unity at:

$$\omega_T = 2\pi f_T = \frac{g_m}{C_{gs} + C_{gd}}$$

Then:

$$f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd})}$$

**

□ Physical interpretation of f_T:

Consider:

$$\frac{1}{2\pi f_T} = \frac{C_{gs} + C_{gd}}{g_m} \simeq \frac{C_{gs}}{g_m}$$

Plug in device physics expressions for C_{gs} and g_m:

$$\frac{1}{2\pi f_T} \simeq \frac{C_{gs}}{g_m} = \frac{\frac{2}{3}LWC_{ox}}{\frac{W}{L}\mu C_{ox}(V_{GS} - V_T)} = \frac{L}{\mu\frac{3}{2}\frac{V_{GS}-V_T}{L}}$$

or

$$\frac{1}{2\pi f_T} \simeq \frac{L}{\mu < E_{chan} >} = \frac{L}{< v_{chan} >} = \tau_t$$

$\tau_t \equiv$ *transit time* from source to drain $[s]$

Then:

$$\boxed{f_T \simeq \frac{1}{2\pi\tau_t}}$$ ***

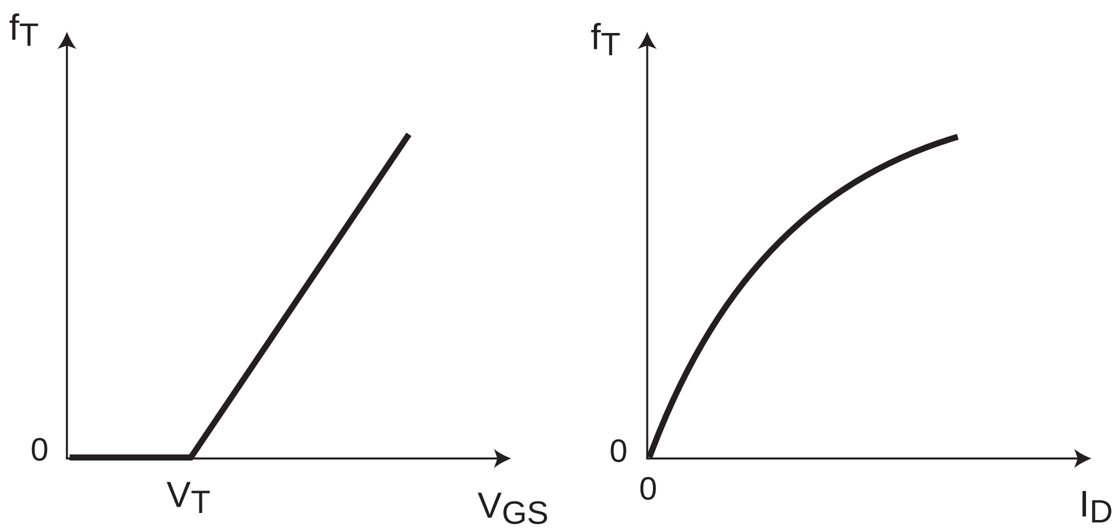
f_T gives an idea of the *intrinsic delay* of the transistor: good first-order figure of merit for frequency response.

To reduce τ_t and increase f_T:

- $L \downarrow$: trade-off is cost
- $(V_{GS} - V_T) \uparrow \Rightarrow I_D \uparrow$: trade-off is power
- $\mu \uparrow$: hard to do
- note: f_T independent of W (for constant $V_{GS} - V_T$)

Impact of bias point on f_T:

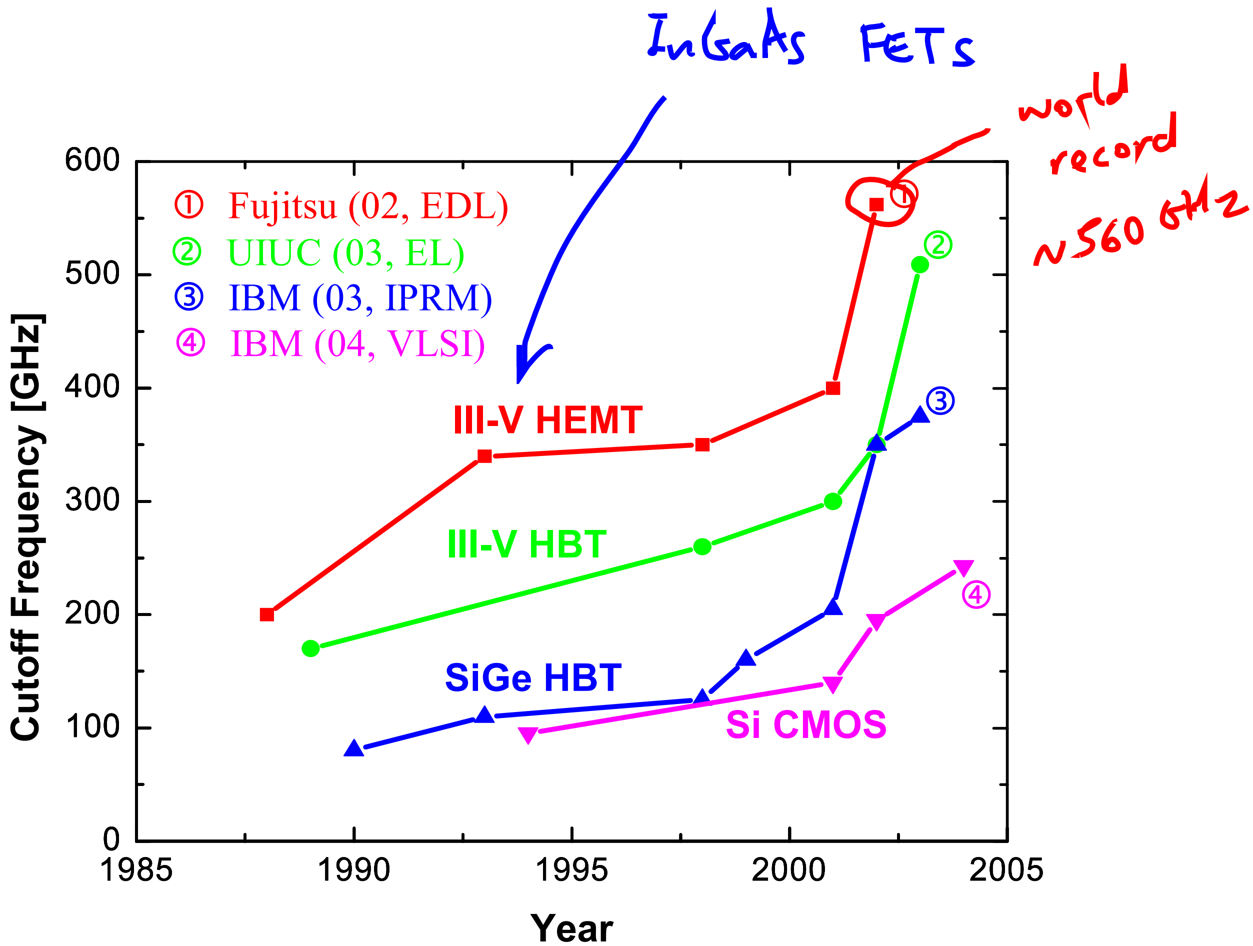
$$f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd})} = \frac{\frac{W}{L}\mu C_{ox}(V_{GS} - V_T)}{2\pi(C_{gs} + C_{gd})} = \frac{\sqrt{2\frac{W}{L}\mu C_{ox} I_D}}{2\pi(C_{gs} + C_{gd})}$$



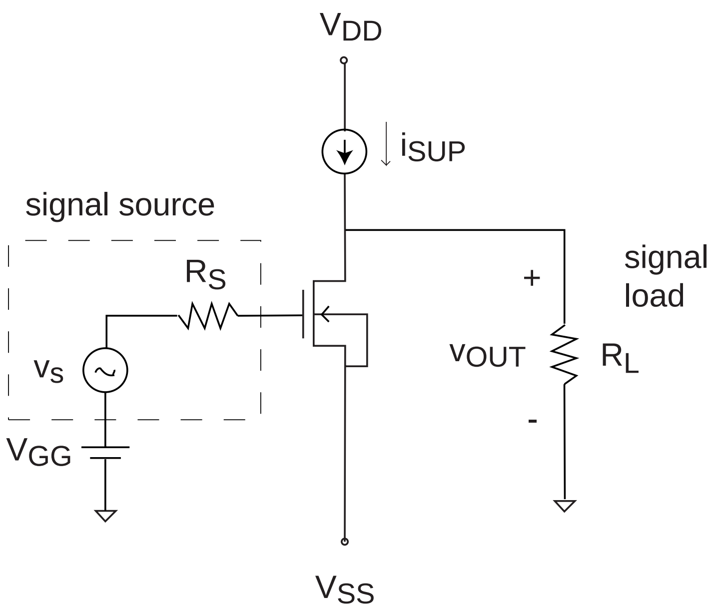
In typical MOSFET at typical bias points:

$$f_T \sim 5 - 50\ GHz$$

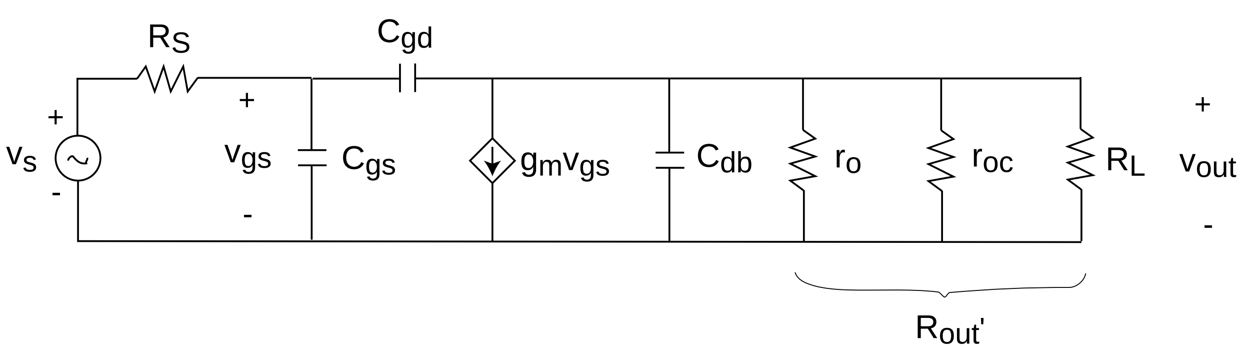
f_t of different device technologies



3. Frequency response of common-source amp



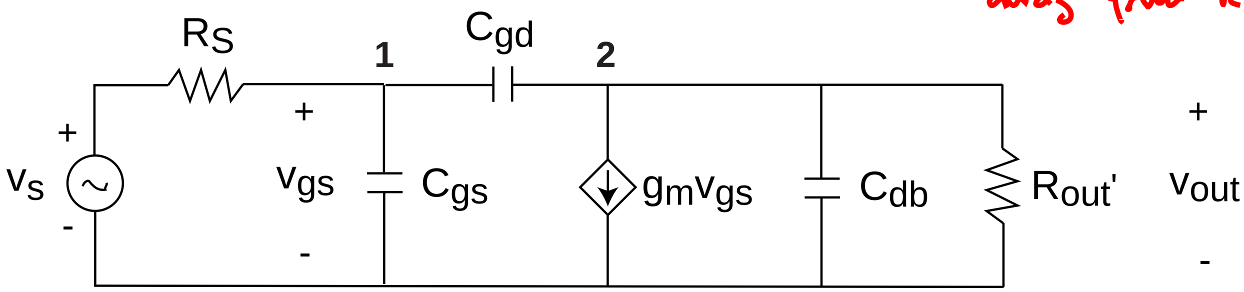
Small-signal equivalent circuit model (assuming current source has no parasitic capacitance):



Low-frequency voltage gain:

$$A_{v,LF} = \frac{v_{out}}{v_s} = -g_m(r_o//r_{oc}//R_L) = -g_m R'_{out}$$

Capacitors hurt bandwidth because:
- current into Cgs, Cgd makes $v_{gs} < v_s$
- Cdb diverts current away from Rout'



node **1**: $\frac{v_s - v_{gs}}{R_S} - v_{gs} j\omega C_{gs} - (v_{gs} - v_{out}) j\omega C_{gd} = 0$

node **2**: $(v_{gs} - v_{out}) j\omega C_{gd} - g_m v_{gs} - v_{out} j\omega C_{db} - \frac{v_{out}}{R'_{out}} = 0$

Solve for v_{gs} in **2**:

$$v_{gs} = v_{out} \frac{j\omega(C_{gd} + C_{db}) + \frac{1}{R'_{out}}}{j\omega C_{gd} - g_m}$$

Plug in **1** and solve for v_{out}/v_s:

$$A_v = \frac{-(g_m - j\omega C_{gd}) R'_{out}}{DEN}$$

with

$$\begin{aligned} DEN &= 1 + j\omega\{R_S C_{gs} + R_S C_{gd}[1 + R'_{out}(\frac{1}{R_S} + g_m)] + R'_{out} C_{db}\} \\ &\quad -\omega^2 R_S R'_{out} C_{gs}(C_{gd} + C_{db}) \end{aligned}$$

[check that for $\omega = 0$, $A_{v,LF} = -g_m R'_{out}$]

Simplify:

1. Operate at $\omega \ll \omega_T = \frac{g_m}{C_{gs}+C_{gd}} \Rightarrow$

$$g_m \gg \omega(C_{gs} + C_{gd}) > \omega C_{gs},\ \omega C_{gd}$$

2. Assume g_m high enough so that

$$\frac{1}{R_S} + g_m \simeq g_m$$

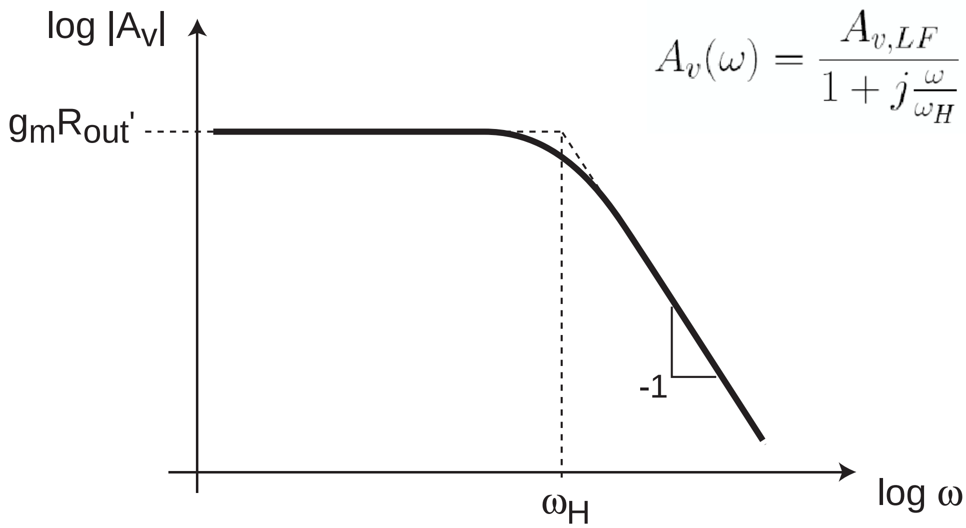
3. Eliminate ω^2 term in denominator of A_v
   $\Rightarrow$ worst-case estimation of bandwidth

Then:

$$A_v \simeq \frac{-g_m R'_{out}}{1 + j\omega[R_S C_{gs} + R_S C_{gd}(1 + g_m R'_{out}) + R'_{out} C_{db}]}$$

This has the form:

$$\boxed{A_v(\omega) = \frac{A_{v,LF}}{1 + j\frac{\omega}{\omega_H}}}$$



At $\omega = \omega_H$:

$$|A_v(\omega_H)| = \frac{1}{\sqrt{2}}|A_{v,LF}|$$

ω_H gives idea of frequency beyond which $|A_v|$ starts rolling off quickly $\Rightarrow$ *bandwidth*

For common-source amplifier:

$$\omega_H = \frac{1}{R_S C_{gs} + R_S C_{gd}(1 + g_m R'_{out}) + R'_{out} C_{db}}$$

Frequency response of common-source amplifier limited by C_{gs} and C_{gd} shorting out the input, and C_{db} shorting out the output.

Can rewrite as:

$$f_H = \frac{1}{2\pi\{R_S[C_{gs} + C_{gd}(1 + \underbrace{|A_{v,LF}|}_{g_m R_{out}'})] + R'_{out}C_{db}\}}$$

Compare with:

$$f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd})}$$

□ In general: $f_H \ll f_T$ due to

high f_T good, but not enough to get high bandwidth

- typically: $g_m \gg \frac{1}{R_S}$
- C_{db} enters f_H but not f_T
- presence of $|A_{v,LF}|$ in denominator

□ To improve bandwidth,

- $C_{gs}, C_{gd}, C_{db} \downarrow \Rightarrow$ small transistor with low parasitics
- $|A_{v,LF}| \downarrow \Rightarrow$ don't want more gain than really needed

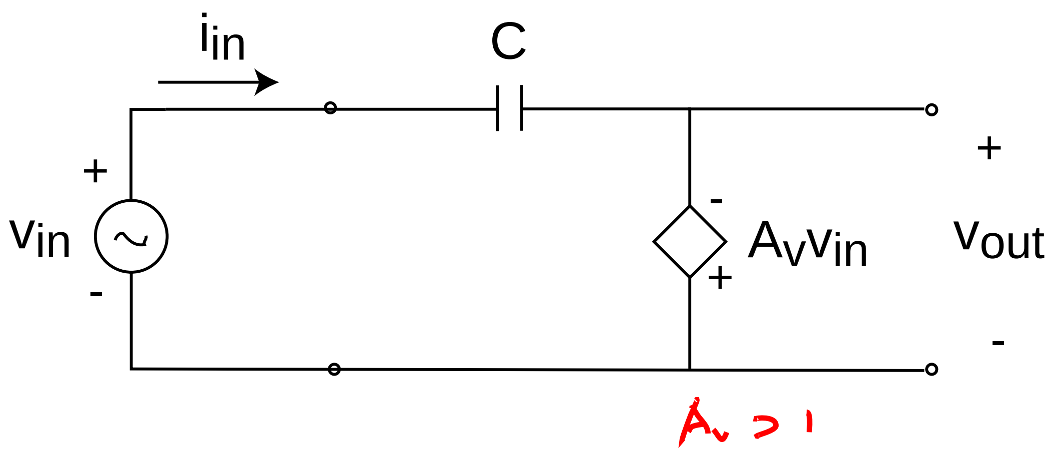
but...

why is it that effect of C_{gd} on f_H appears to being amplified by $1 + |A_{v,LF}|$??!!

4. Miller effect

In common-source amplifier, C_{gd} looks much bigger than it really is.

Consider simple voltage-gain stage:



What is the input impedance?

$$i_{in} = (v_{in} - v_{out}) j\omega C$$

But

$$v_{out} = -A_v v_{in}$$

Then:

$$i_{in} = v_{in}(1 + A_v)C$$

or

$$Z_{in} = \frac{v_{in}}{i_{in}} = \frac{1}{j\omega\underbrace{(1 + A_v)C}_{C_{Miller}}}$$

From input, C, looks much bigger than it really is. This is called the *Miller effect*.

When a capacitor is located across nodes where there is voltage gain, its effect on bandwidth is amplified by the voltage gain ⇒ *Miller c pacitance*:

$$C_{Miller} = C(1 + A_v)$$

Why?

$$v_{in} \uparrow \Rightarrow \; v_{out} = -A_v v_{in} \downarrow\downarrow \Rightarrow (v_{in} - v_{out}) \uparrow\uparrow \Rightarrow i_{in} \uparrow\uparrow$$

In amplifier stages with voltage gain, it is critical to have small capacitance across voltage gain nodes.

As a result of the Miller effect, there is a fundamental *gain-bandwidth tradeoff* in amplifiers.

Key conclusions

- f_T (*short-circuit current-gain cut-off frequency*): figure of merit to assess intrinsic frequency response of transistors.
- In MOSFET, to first order,

$$f_t = \frac{1}{2\pi\tau_t}$$

  where τ_t is *transit time* of electrons through channel.
- In common-source amplifier, voltage gain rolls off at high frequency because C_{gs} and C_{gd} short out input and C_{db} shorts out output.
- In common-source amplifier, effect of C_{gd} on bandwidth is magnified by amplifier voltage gain.
- *Miller effect*: effect of capacitance across voltage gain nodes is magnified by voltage gain

  $\Rightarrow$ *trade-off between gain and bandwidth.*