CHAPTER 12

Pulse Amplitude Modulation (PAM), Quadrature Amplitude Modulation (QAM)

12.1 PULSE AMPLITUDE MODULATION

In Chapter 2, we discussed the discrete-time processing of continuous-time signals, and in that context reviewed and discussed D/C conversion for reconstructing a continuous-time signal from a discrete-time sequence. Another common context in which it is useful and important to generate a continuous-time signal from a sequence is in communication systems, in which discrete data — for example, digital or quantized data — is to be transmitted over a channel in the form of a continuous-time signal. In this case, unlike in the case of DT processing of CT signals, the resulting continuous-time signal will be converted back to a discrete-time signal at the receiving end. Despite this difference in the two contexts, we will see that the same basic analysis applies to both.

As examples of the communication of DT information over CT channels, consider transmitting a binary sequence of 1's and 0's from one computer to another over a telephone line or cable, or from a digital cell phone to a base station over a high-frequency electromagnetic channel. These instances correspond to having analog channels that require the transmitted signal to be continuous in time, and to also be compatible with the bandwidth and other constraints of the channel. Such requirements impact the choice of continuous-time waveform that the discrete sequence is modulated onto.

The translation of a DT signal to a CT signal appropriate for transmission, and the translation back to a DT signal at the receiver, are both accomplished by devices referred to as modems (modulators/demodulators). Pulse Amplitude Modulation (PAM) underlies the operation of a wide variety of modems.

12.1.1 The Transmitted Signal

The basic idea in PAM for communication over a CT channel is to transmit a sequence of CT pulses of some pre-specified shape $p(t)$, with the sequence of pulse amplitudes carrying the information. The associated baseband signal at the transmitter (which is then usually modulated onto some carrier to form a bandpass signal before actual transmission — but we shall ignore this aspect for now) is given by

$$x(t) = \sum_n a[n]\, p(t - nT) \tag{12.1}$$

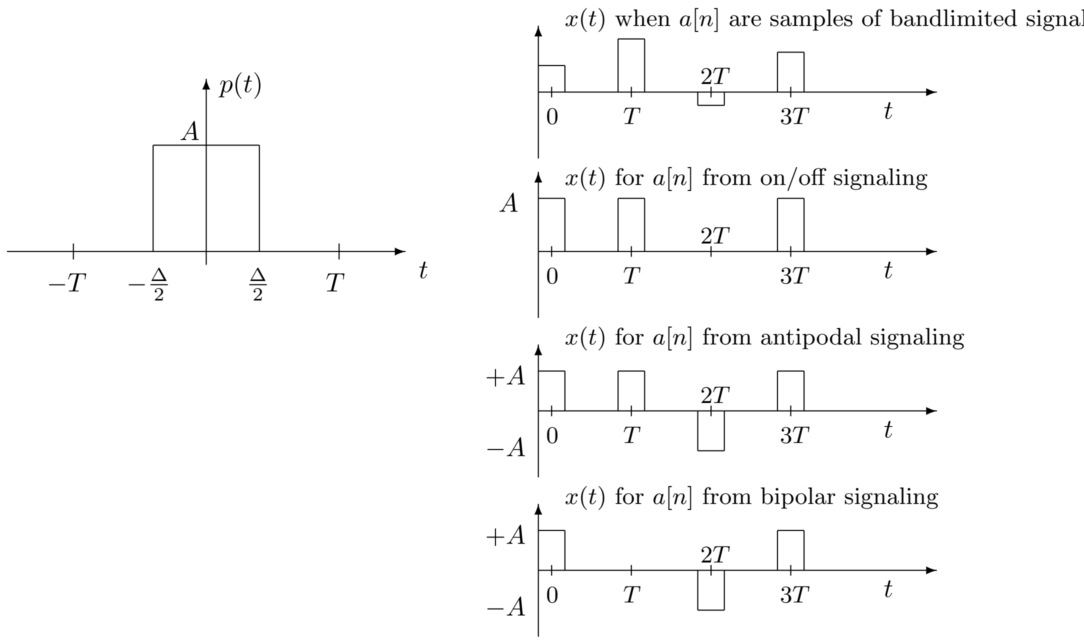

FIGURE 12.1 Baseband signal at the transmitter in Pulse Amplitude Modulation (PAM).

where the numbers $a[n]$ are the pulse amplitudes, and T is the pulse repetition interval or the inter-symbol spacing, so $1/T$ is the symbol rate (or "baud" rate). An individual pulse may be confined to an interval of length T, as shown in Figure 12.1, or it may extend over several intervals, as we will see in several examples shortly. The DT signal $a[n]$ may comprise samples of a bandlimited analog message (taken at the Nyquist rate or higher, and generally quantized to a specified set of levels, for instance 32 levels); or 1 and 0 for on/off or "unipolar" signaling; or 1 and -1 for antipodal or "polar" signaling; or 1, 0 and -1 for "bipolar" signaling; each of these possibilities is illustrated in Figure 12.1.

The particular pulse shape in Figure 12.1 is historically referred to as an RZ (return-to-zero) pulse when $\Delta < T$ and an NRZ (non-return-to-zero) pulse when $\Delta = T$. These pulses would require substantial channel bandwidth (of the order of $1/\Delta$) in order to be transmitted without significant distortion, so we may wish to find alternative choices that use less bandwidth, to accommodate the constraints of the channel. Such considerations are important in designing appropriate pulse shapes, and we shall elaborate on them shortly.

If $p(t)$ is chosen such that $p(0) = 1$ and $p(nT) = 0$ for $n \neq 0$, then we could recover the amplitudes $a[n]$ from the PAM waveform $x(t)$ by just sampling $x(t)$ at times nT, since $x(nT) = a[n]$ in this case. However, our interest is in recovering the amplitudes from the signal at the *receiver*, rather than directly from the transmitted signal, so we need to consider how the communication channel affects $x(t)$. Our objective will be to recover the DT signal in as simple a fashion as possible, while compensating for distortion and noise in the channel.

12.1.2 The Received Signal

When we transmit a PAM signal through a channel, the characteristics of the channel will affect our ability to accurately recover the pulse amplitudes $a[n]$ from the received signal $r(t)$. We might model $r(t)$ as

$$r(t) = h(t) * x(t) + \eta(t) \tag{12.2}$$

corresponding to the channel being modeled as LTI with impulse response $h(t)$, and channel noise being represented through the additive noise signal $\eta(t)$. We would still typically try to recover the pulse amplitudes $a[n]$ from samples of $r(t)$ — or from samples of an appropriately filtered version of $r(t)$ — with the samples taken at intervals of T.

The overall model is shown in Figure 12.2, with $f(t)$ representing the impulse response of an LTI filter at the receiver. This receiver filter will play a key role in filtering out the part of the noise that lies outside the frequency bands in which the signal information is concentrated. Here, we first focus on the noise-free case (for which one would normally set $f(t) = \delta(t)$, corresponding to no filtering before sampling at the receiver end), but for generality we shall take account of the effect of the filter $f(t)$ as well.

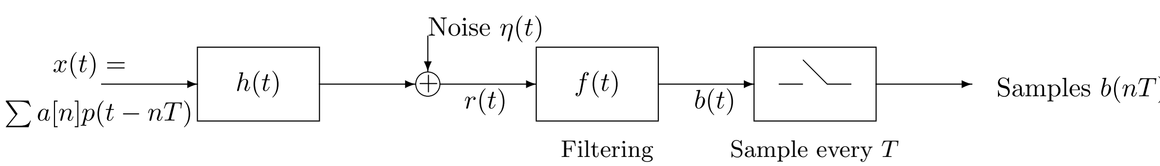

FIGURE 12.2 Transmitter, channel and receiver model for a PAM system.

12.1.3 Frequency-Domain Characterizations

Denote the CTFT of the pulse $p(t)$ by $P(j\omega)$, and similarly for the other CT signals in Figure 12.2. If the frequency response $H(j\omega)$ of the channel is unity over the frequency range where $P(j\omega)$ is significant, then a single pulse $p(t)$ is transmitted essentially without distortion. In this case, we might invoke the linearity and time invariance of our channel model to conclude that $x(t)$ in (12.1) is itself transmitted essentially without distortion, in which case $r(t) \approx x(t)$ in the noise-free case that we are considering. However, this conclusion leaves the possiblity that distortions which are insignificant when a single pulse is transmitted accumulate in a non-negligible way when a succession of pulses is transmitted. We should therefore directly examine $x(t)$, $r(t)$, and their corresponding Fourier transforms. The understanding we obtain from this is a prerequisite for designing $P(j\omega)$ and picking the inter-symbol time T for a given channel, and also allows us to determine the influence of the DT signal $a[n]$ on the CT signals $x(t)$ and $r(t)$.

To compute $X(j\omega)$, we take the transform of both sides of (12.1):

$$\begin{aligned} X(j\omega) &= \Big(\sum_n a[n]\, e^{-j\omega nT}\Big)\, P(j\omega) \\ &= A(e^{j\Omega})|_{\Omega=\omega T}\; P(j\omega) \end{aligned} \tag{12.3}$$

where $A(e^{j\Omega})$ denotes the DTFT of the sequence $a[n]$. The quantity $A(e^{j\Omega})|_{\Omega=\omega T}$ that appears in the above expression is simply a uniform re-scaling of the frequency axis of the DTFT; in particular, the point $\Omega = \pi$ in the DTFT is mapped to the point $\omega = \pi/T$ in the expression $A(e^{j\Omega})|_{\Omega=\omega T}$.

The expression in (12.3) therefore describes $X(j\omega)$ for us, assuming the DTFT of the sequence $a[n]$ is well defined. For example, if $a[n] = 1$ for all n, corresponding to periodic repetition of the basic pulse waveform $p(t)$, then $A(e^{j\Omega}) = 2\pi\delta(\Omega)$ for $|\Omega| \le \pi$, and repeats with period 2π outside this range. Hence $X(j\omega)$ comprises a train of impulses spaced apart by $2\pi/T$; the strength of each impulse is $2\pi/T$ times the value of $P(j\omega)$ at the location of the impulse (note that the scaling property of impulses yields $\delta(\Omega) = \delta(\omega T) = (1/T)\delta(\omega)$ for positive T).

In the absence of noise, the received signal $r(t)$ and the signal $b(t)$ that results from filtering at the receiver are both easily characterized in the frequency domain:

$$R(j\omega) = H(j\omega)X(j\omega)\;, \qquad B(j\omega) = F(j\omega)H(j\omega)X(j\omega)\;. \tag{12.4}$$

Some important constraints emerge from (12.3) and (12.4). Note first that for a general DT signal $a[n]$, necessary information about the signal will be distributed in its DTFT $A(e^{j\Omega})$ at frequencies Ω throughout the interval $|\Omega| \le \pi$; knowing $A(e^{j\Omega})$ only in a smaller range $|\Omega| \le \Omega_a < \pi$ will in general be *insufficient* to allow reconstruction of the DT signal. Now, setting $\Omega = \omega T$ as specified in (12.3), we see that $A(e^{j\omega T})$ will contain necessary information about the DT signal at frequencies ω that extend throughout the interval $|\omega| \le \pi/T$. Thus, if $P(j\omega) \neq 0$ for $|\omega| \le \pi/T$ then $X(j\omega)$ preserves the information in the DT signal; and if $H(j\omega)P(j\omega) \neq 0$ for $|\omega| \le \pi/T$ then $R(j\omega)$ preserves the information in the DT signal; and if $F(j\omega)H(j\omega)P(j\omega) \neq 0$ for $|\omega| \le \pi/T$ then $B(j\omega)$ preserves the information in the DT signal.

The above constraints have some design implications. A pulse for which $P(j\omega)$ was nonzero only in a strictly smaller interval $|\omega| \le \omega_p < \pi/T$ would cause loss of information in going from the DT signal to the PAM signal $x(t)$, and would not be a suitable pulse for the chosen symbol rate $1/T$ (but could become a suitable pulse if the symbol rate was reduced appropriately, to ω_p/π or less).

Similarly, even if the pulse was appropriately designed so that $x(t)$ preserved the information in the DT signal, if we had a lowpass channel for which $H(j\omega)$ was nonzero only in a strictly smaller interval $|\omega| \leq \omega_c < \pi/T$ (so ω_c is the cutoff frequency of the channel), then we would lose information about the DT signal in going from $x(t)$ to $r(t)$; the chosen symbol rate $1/T$ would be inappropriate for this channel, and would need to be reduced to ω_c/π in order to preserve the information in the DT signal.

12.1.4 Inter-Symbol Interference at the Receiver

In the absence of any channel impairments, the signal values can be recovered from the transmitted pulse trains shown in Figure 12.1 by re-sampling at the times which are integer multiples of T. However, these pulses, while nicely time localized, have infinite bandwidth. Since any realistic channel will have a limited bandwidth, one effect of a communication channel on a PAM waveform is to "de-localize" or disperse the energy of each pulse through low-pass filtering. As a consequence, pulses that may not have overlapped (or that overlapped only benignly) at the transmitter may overlap at the receiver in a way that impedes the recovery of the pulse amplitudes from samples of $r(t)$, i.e. in a way that leads to *inter-symbol interference* (ISI). We now make explicit what condition is required in order for ISI to be eliminated

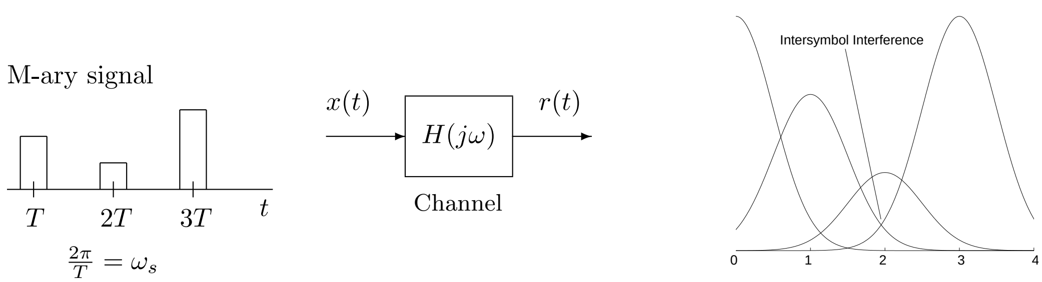

FIGURE 12.3 Illustration of Inter-symbol Interference (ISI).

from the filtered signal $b(t)$ at the receiver. When this no-ISI condition is met, we will again be able to recover the DT signal by simply sampling $b(t)$. Based on this condition, we can identify the additional constraints that must be satisfied by the pulse shape $p(t)$ and the impulse response $f(t)$ of the filter (or channel compensator or *equalizer*) at the receiver so as to eliminate or minimize ISI.

With $x(t)$ as given in (12.1), and noting that $b(t) = f(t) * h(t) * x(t)$ in the noise-free case, we can write

$$b(t) = \sum_n a[n]\, g(t - nT) \tag{12.5}$$

where

$$g(t) = f(t) * h(t) * p(t) \tag{12.6}$$

We assume that $g(t)$ is continuous (i.e., has no discontinuity) at the sampling times

nT. Our requirement for no ISI is then that

$$g(0) = c\ , \quad \text{and} \quad g(nT) = 0 \ \text{ for nonzero integers } n, \tag{12.7}$$

where c is some nonzero constant. If this condition is satisfied, then if follows from (12.5) that $b(nT) = c.a[n]$, and consequently the DT signal is exactly recovered (to within the known scale factor c).

As an example, suppose that $g(t)$ in (12.6) is

$$g(t) = \frac{\sin \omega_c t}{\omega_c t}\ , \tag{12.8}$$

with corresponding $G(j\omega)$ given by

$$\begin{aligned} G(j\omega) &= \frac{\pi}{\omega_c} \quad \text{for} \quad |\omega| < \omega_c \\ &= 0 \quad \text{otherwise}\ . \end{aligned} \tag{12.9}$$

Then choosing the inter-symbol spacing to be $T = \dfrac{\pi}{\omega_c}$, we can avoid ISI in the received samples, since $g(t) = 1$ at $t = 0$ and is zero at other integer multiples of T, as illustrated in Figure 12.4.

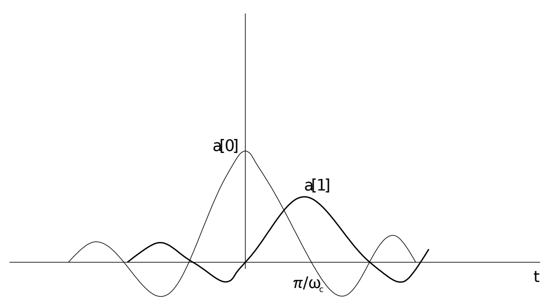

FIGURE 12.4 Illustration of the no-ISI property for PAM when $g(0) = 1$ and $g(t) = 0$ at other integer multiples of the inter-symbol time T.

We are thereby able to transmit at a symbol rate that is twice the cutoff frequency of the channel. From what was said earlier, in the discussion following (12.3) on constraints involving the symbol rate and the channel cutoff frequency, we cannot expect to do better in general.

More generally, in the next section we translate the no-ISI time-domain condition in (12.7) to one that is useful in designing $p(t)$ and $f(t)$ for a given channel. The approach is based on the frequency-domain translation of the no-ISI condition, leading to a result that was first articulated by Nyquist.

12.2 NYQUIST PULSES

The frequency domain interpretation of the no-ISI condition of (12.7) was explored by Nyquist in 1924 (and extended by him in 1928 to a statement of the sampling theorem — this theorem then waited almost 20 years to be brought to prominence by Gabor and Shannon).

Consider sampling $g(t)$ with a periodic impulse train:

$$\widehat{g}(t) = g(t) \sum_{n=-\infty}^{+\infty} \delta(t - nT) \ . \tag{12.10}$$

Then our requirements on $g(t)$ in (12.7) imply that $\widehat{g}(t) = c\,\delta(t)$, an impulse of strength c, whose transform is $\widehat{G}(j\omega) = c$. Taking transforms of both sides of (12.10), and utilizing the fact that multiplication in the time domain corresponds to convolution in the frequency domain, we obtain

$$\widehat{G}(j\omega) = c = \frac{1}{T} \sum_{m=-\infty}^{+\infty} G(j\omega - jm\frac{2\pi}{T}) \ . \tag{12.11}$$

The expression on the right hand side of (12.11) represents a replication of $G(j\omega)$ (scaled by $1/T$) at every integer multiple of $2\pi/T$ along the frequency axis. The Nyquist requirement is thus that $G(j\omega)$ and its replications, spaced $2\pi m/T$ apart for all integer m, add up to a constant. Some examples of $G(j\omega) = F(j\omega)H(j\omega)P(j\omega)$ that satisfy this condition are given below.

The particular case of the sinc function of (12.8) and (12.9) certainly satisfies the Nyquist condition of (12.11).

If we had an ideal lowpass channel $H(j\omega)$ with bandwidth ω_c or greater, then choosing $p(t)$ to be the sinc pulse of (12.8) and not doing any filtering at the receiver — so $F(j\omega) = 1$ — would result in no ISI. However, there are two problems with the sinc characteristic. First, the signal extends indefinitely in time in both directions. Second, the sinc has a very slow roll-off in time (as $1/t$). This slow roll-off in time is coupled to the sharp cut-off of the transform of the sinc in the frequency domain. This is a familiar manifestation of time-frequency duality: quick transition in one domain means slow transition in the other.

It is highly desirable in practice to have pulses that taper off more quickly in time than a sinc. One reason is that, given the inevitable inaccuracies in sampling times due to timing jitter, there will be some unavoidable ISI, and this ISI will propagate for unacceptably long times if the underlying pulse shape decays too slowly. Also, a faster roll-off allows better approximation of a two-sided signal by a one-sided signal, as would be required for a causal implementation. The penalty for more rapid pulse roll-off in time is that the transition in the frequency domain has to be more gradual, necessitating a larger bandwidth for a given symbol rate (or a reduced symbol rate for a given bandwidth).

The two examples in Figure 12.5 have smoother transitions than the previous case, and correspond to pulses that fall off as $1/t^2$. It is evident that both can be made to satisfy the Nyquist condition by appropriate choice of T.

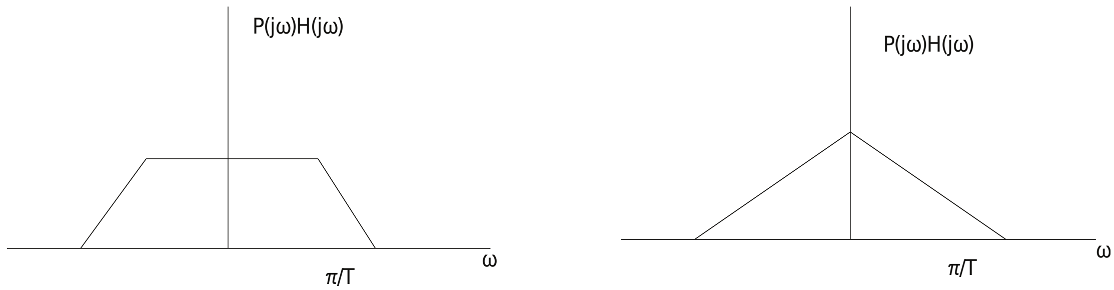

FIGURE 12.5 Two possible choices for the Fourier transform of pulses that decay in time as $1/t^2$ and satisfy the Nyquist zero-ISI condition for appropriate choice of T.

Still smoother transitions can be obtained with a family of frequency-domain characteristics in which there is a *cosine* transition from 1 to 0 over the frequency range $\omega = \frac{\pi}{T}(1-\beta)$ to $\omega = \frac{\pi}{T}(1+\beta)$, where β is termed the roll-off parameter. The corresponding formula for the received and filtered pulse is

$$f(t) * h(t) * p(t) \;=\; \frac{\sin \frac{\pi}{T}t}{\frac{\pi}{T}t} \, \frac{\cos \beta\frac{\pi}{T}t}{1-(2\beta t/T)^2} \tag{12.12}$$

which falls off as $1/t^3$ for large t.

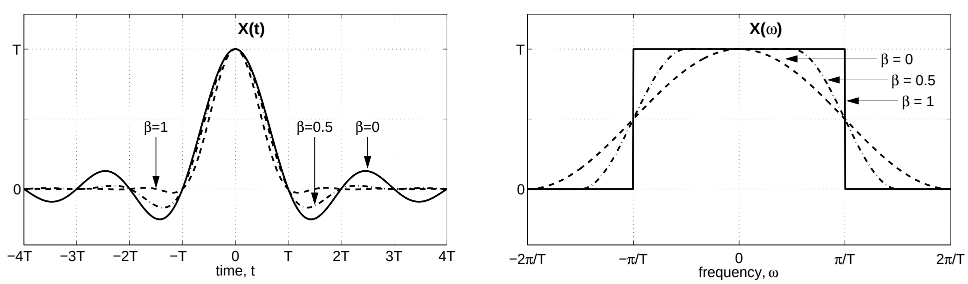

FIGURE 12.6 Time and frequency characteristics of the family of pulses in Eq. (12.12)

Once $G(j\omega)$ is specified, knowledge of the channel characteristic $H(j\omega)$ allows us to determine the corresponding pulse transform $P(j\omega)$, if we fix $F(j\omega) = 1$. In the presence of channel noise that corrupts the received signal $r(t)$, it turns out that it is best to only do part of the pulse shaping at the transmitter, with the rest done at the receiver prior to sampling. For instance, if the channel has no distortion in the passband (i.e., if $H(j\omega) = 1$ in the passband) and if the noise intensity is

TABLE 5.4: Selected CCITT International Telephone Line Modem Standards

Bit Rate	Symbol Rate	Modulation	CCITT Standard
330	300	2FSK	V.21
1,200	600	QPSK	V.22
2,400	600	16QAM	V.22bis
1,200	1,200	2FSK	V.23
2,400	1,200	QPSK	V.26
4,800	1,600	8PSK	V.27
9,600	2,400	Fig. 3.15(a)	V.29
4,800	2,400	QPSK	V.32
9,600	2,400	16QAM	V.32ALT
14,400	2,400	128QAM,TCM	V.32bis
28,800	3,429	1024QAM,TCM	V.fast(V.34)

FIGURE 12.7 From *Digital Transmission Engineering* by J.B.Anderson, IEEE Press 1999. The reference to Fig. 3.15 a is a particular QAM constellation.

uniform in this passband, then the optimal choice of pulse is $P(j\omega) = \sqrt{G(j\omega)}$, assuming that $G(j\omega)$ is purely real, and this is also the optimal choice of receiver filter $F(j\omega)$. We shall say a little more about this sort of issue when we deal with matched filtering in a later chapter.

12.3 CARRIER TRANSMISSION

The previous discussion centered around the design of baseband pulses. For transmission over phone lines, wireless links, satellites, etc. the baseband signal needs to be modulated onto a carrier, i.e. converted to a passband signal. This also opens opportunities for augmentation of PAM. The table in Figure 12.7 shows the evolution of telephone line digital modem standards. FSK refers to frequency-shift-keying, PSK to phase-shift-keying, and QAM to quadrature amplitude modulation, each of which we describe in more detail below. The indicated increase in symbol rate (or *baud* rate) and bit rates over the years corresponds to improvements in signal processing, to better modulation schemes, to the use of better conditioned channels, and to more elaborate coding (and correspondingly complex decoding, but now well within real-time computational capabilities of digital receivers).

For baseband PAM, the transmitted signal is of the form of equation (12.1) i.e.

$$x(t) = \sum_n a[n]\, p(t - nT) \tag{12.13}$$

where $p(t)$ is a lowpass pulse. When this is amplitude-modulated onto a carrier, the transmitted signal takes the form

$$s(t) = \sum_n a[n]\, p(t - nT) \cos(\omega_c t + \theta_c) \tag{12.14}$$

where ω_c and θ_c are the carrier frequency and phase.

In the simplest form of equation (12.14), specifically with ω_c and θ_c fixed, equation (12.14) corresponds to using amplitude modulation to shift the frequency content from baseband to a band centered at the carrier frequency ω_c. However, since two additional parameters have been introduced (i.e. ω_c and θ_c) this opens additional possibilities for embedding data in $s(t)$. Specifically, in addition to changing the amplitude in each symbol interval, we can consider changing the carrier frequency and/or the phase in each symbol interval. These alternatives lead to frequency-shift-keying (FSK) and phase-shift-keying (PSK).

12.3.1 FSK

With frequency shift keying (12.14) takes the form

$$s(t) = \sum_n a[n]\, p(t - nT) \cos((\omega_0 + \Delta_n)t + \theta_c) \tag{12.15}$$

where ω_0 is the nominal carrier frequency and Δ_n is the shift in the carrier frequency in symbol interval n. In principle in FSK both $a[n]$ and Δ_n can incorporate data although it is typically the case that in FSK the amplitude does not change.

12.3.2 PSK

In phase shift keying (12.14) takes the form

$$s(t) = \sum_n a[n]\, p(t - nT) \cos(\omega_c t + \theta_n) \tag{12.16}$$

In each symbol interval, information can then be incorporated in both the pulse amplitude $a[n]$ and the carrier phase θ_n. In what is typically referred to as PSK, information is only incorporated in the phase, i.e. $a[n] = a = constant$.

For example, with

$$\theta_n = \frac{2\pi b_n}{M} \ ; \ b_n \text{ a non-negative integer} \tag{12.17}$$

one of M symbols can be encoded in the phase in each symbol interval. For $M = 2$, $\theta_n = 0$ or π, commonly referred to as binary PSK (BPSK). With $M = 4$, θ_n takes on one of the four values 0, $\frac{\pi}{2}$, π, or $\frac{3\pi}{2}$.

To interpret PSK somewhat differently and as a prelude to expanding the discussion to a further generalization (quadrature amplitude modulation or QAM) it is convenient to express equation (12.16) in some alternate forms. For example,

$$s(t) = \sum_n Re\{a e^{j\theta_n} p(t - nT) e^{j\omega_c t}\} \tag{12.18}$$

and equivalently

$$s(t) = I(t)\cos(\omega_c t) - Q(t)\sin(\omega_c t) \tag{12.19}$$

with

$$I(t) = \sum_n a_i[n]\, p(t - nT) \tag{12.20}$$

$$Q(t) = \sum_n a_q[n]\, p(t - nT) \tag{12.21}$$

and

$$a_i[n] = a\cos(\theta_n) \tag{12.22}$$

$$a_q[n] = a\sin(\theta_n) \tag{12.23}$$

Equation 12.19 is referred to as the quadrature form of equation 12.16 and $I(t)$ and $Q(t)$ are referred to as the in-phase and quadrature components. For BPSK, $a_i[n] = \pm a$ and $a_q[n] = 0$.

For PSK with θ_n in the form of equation 12.17 and $M = 4$, θ_n can take on any of the four values 0, $\frac{\pi}{2}$, π, or $\frac{3\pi}{2}$. In the form of equations 12.22 and 12.23 $a_i[n]$ will then be either $+a, -a$, or zero and $a_q[n]$ will be either $+a, -a$, or zero. However, clearly QPSK can only encode four symbols in the phase not nine, i.e. the various possibilities for $a_i[n]$ and $a_q[n]$ are not independent. For example, for $M = 4$, if $a_i[n] = +a$ then $a_q[n]$ must be zero since $a_i[n] = +a$ implies that $\theta_n = 0$. A convenient way of looking at this is through what's referred to as an I-Q constellation as shown in Figure 12.8.

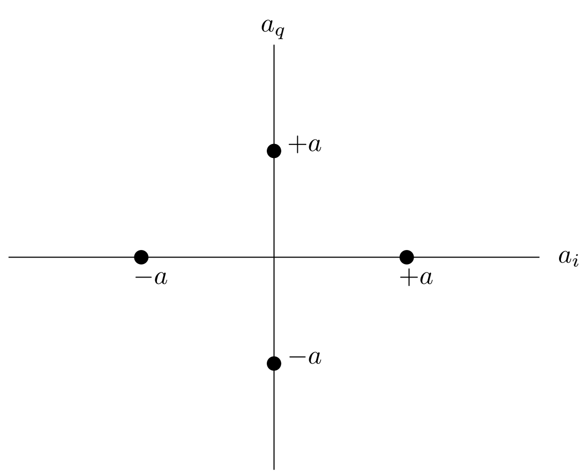

FIGURE 12.8 I-Q Constellation for QPSK.

Each point in the constellation represents a different symbol that can be encoded, and clearly with the constellation of Figure 12.8 one of four symbols can be encoded in each symbol interval (recall that for now, the amplitude $a[n]$ is constant. This will change when we expand the discussion shortly to QAM).

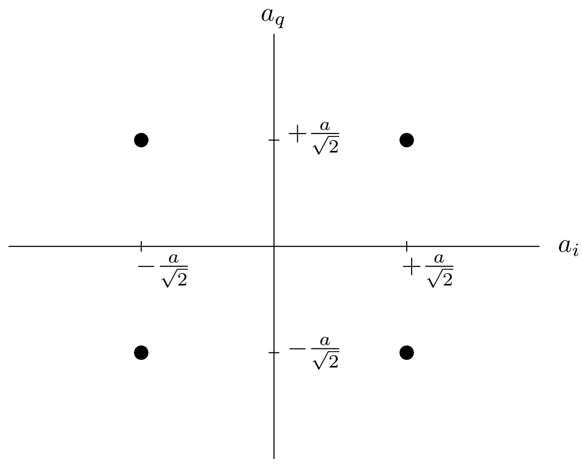

FIGURE 12.9 I-Q Constellation for quadrature phase-shift-keying (QPSK).

An alternative form with four-phase PSK is to choose

$$\theta_n = \frac{2\pi b_n}{4} + \frac{\pi}{4} \ ; \ b_n \text{ a non-negative integer} \tag{12.24}$$

in which case $a_i[n] = \pm\frac{a}{\sqrt{2}}$ and $a_q[n] = \pm\frac{a}{\sqrt{2}}$ resulting in the constellation in Figure 12.9.

In this case, the amplitude modulation of $I(t)$ and $Q(t)$ (equations 12.20 and 12.21) can be done independently. Modulation with this constellation is commonly referred to as QPSK (quadrature phase-shift keying).

In PSK as described above, $a[n]$ was assumed constant. By incorporating encoding in both the amplitude $a[n]$ and phase θ_n in equation 12.16 we are led to a richer form of modulation referred to as quadrature amplitude modulation (QAM). In the form of equations (12.19 - 12.21) we now allow $a_i[n]$ and $a_q[n]$ to be chosen from a richer constellation.

12.3.3 QAM

The QAM constellation diagram is shown in Figure 12.10 for the case where each set of amplitudes can take the values $\pm a$ and $\pm 3a$. The 16 different combinations that are available in this case can be used to code 4 bits, as shown in the figure. This particular constellation is what is used in the V.32ALT standard shown in the table of Figure 12.7. In this standard, the carrier frequency is 1,800 Hz, and the symbol frequency or baud rate ($1/T$) is 2,400 Hz. With 4 bits per symbol, this works out to the indicated 9,600 bits/second. One baseband pulse shape $p(t)$ that may be used is the square root of the cosine-transition pulse mentioned earlier, say with $\beta = 0.3$. This pulse contains frequencies as high as $1.3 \times 1,200 = 1,560$ Hz.

After modulation of the 1,800 Hz carrier, the signal occupies the band from 240 Hz to 3,360 Hz, which is right in the passband of the voice telephone channel.

The two faster modems shown in the table use more elaborate QAM-based schemes. The V.32bis standard involves 128QAM, which could in principle convey 7 bits per symbol, but at the price of greater sensitivity to noise (because the constellation points are more tightly clustered for a given signal power). However, the QAM in this case is actually combined with so-called *trellis-coded modulation* (TCM), which in effect codes in some redundancy (by introducing dependencies among the modulating amplitudes), leading to greater noise immunity and an effective rate of 6 bits per symbol (think of the TCM as in effect reserving a bit for error checking). The symbol rate here is still 2,400 Hz, so the transmission is at $6 \times 2,400 = 14,400$ bits/second. Similarly, the V.34 standard involves 1024QAM, which could convey 10 bits per symbol, although with more noise sensitivity. The combination with TCM introduces redundancy for error control, and the resulting bit rate is 28,800 bits/second (9 effective bits times a symbol frequency of 3,200 Hz).

Demodulation of Quadrature Modulated PAM signals:
The carrier modulated signals in the form of equations (12.19 - 12.23) can carry encoded data in both the I and Q components $I(t)$ and $Q(t)$. Therefore in demodulation we must be able to extract these seperately. This is done through quadrature demodulation as shown in Figure 12.11

In both the modulation and demodulation, it is assumed that the bandwidth of $p(t)$ is low compared with the carrier frequency w_c so that the bandwidth of $I(t)$ and $Q(t)$ are less than ω_c. The input signal $r_i(t)$ is

$$r_i(t) = I(t)cos^2(\omega_c t) - Q(t)sin(\omega_c t)cos(\omega_c t) \tag{12.25}$$

$$= \frac{1}{2}I(t) - \frac{1}{2}I(t)cos(2\omega_c t) - \frac{1}{2}Q(t)sin(2\omega_c t) \tag{12.26}$$

Similarly

$$r_q(t) = I(t)cos(\omega_c t)sin(\omega_c t) - Q(t)sin^2(\omega_c t) \tag{12.27}$$

$$= \frac{1}{2}I(t)sin(2\omega_c t) + \frac{1}{2}Q(t) - \frac{1}{2}Q(t)cos(2\omega_c t) \tag{12.28}$$

Choosing the cutoff frequency of the lowpass filters to be greater than the bandwidth of $p(t)$ (and therefore also greater than the bandwidth of $I(t)$ and $Q(t)$) but low enough to eliminate the components in $r_i(t)$ and $r_q(t)$ around $2\omega_c$, the outputs will be the quadrature signals $I(t)$ and $Q(t)$.

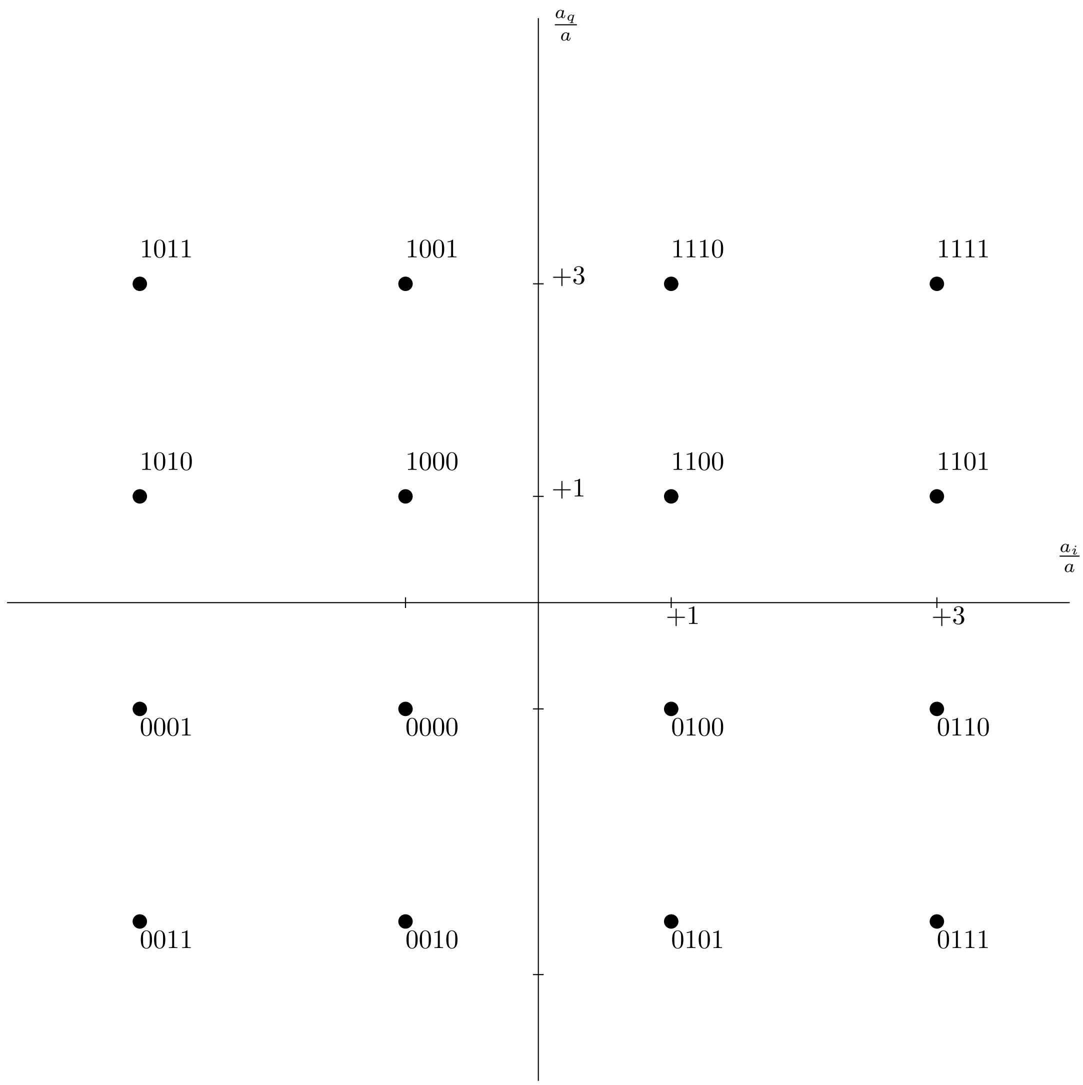

FIGURE 12.10 16 QAM constellation. (From *Digital Transmission Engineering* by J.B. Anderson, IEEE Press, 1999, p.96)

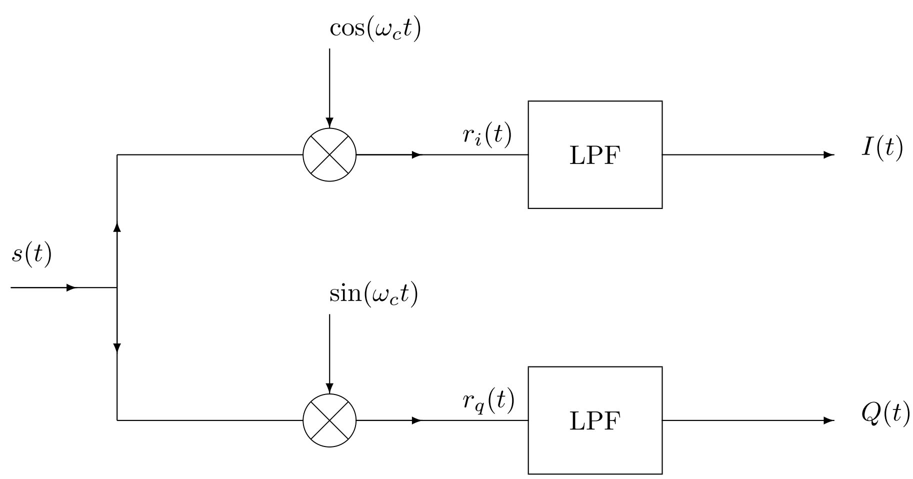

FIGURE 12.11 Demodulation scheme for a Quadrature Modulated PAM Signal.

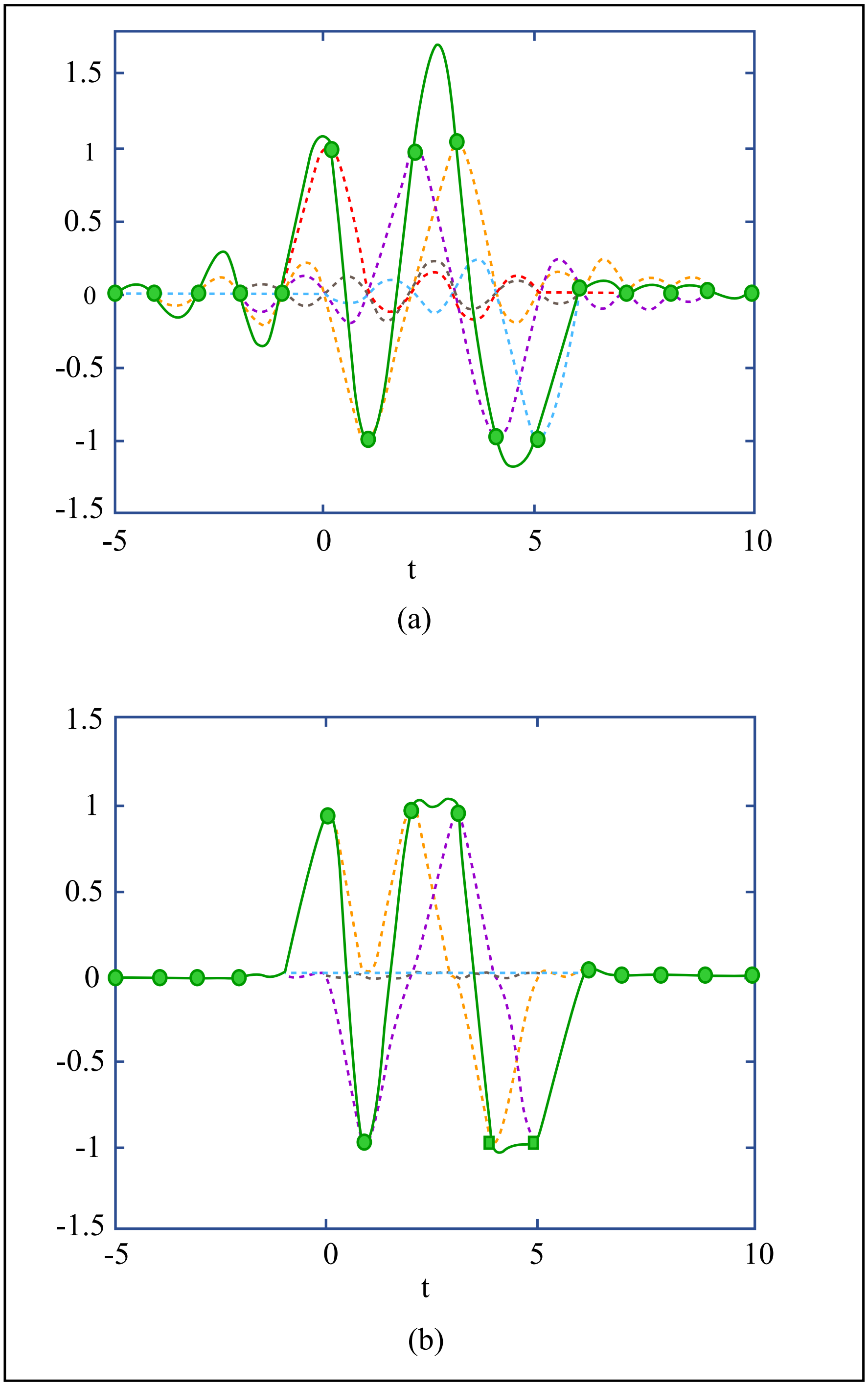

Image by MIT OpenCourseWare, adapted from *Digital Transmission Engineering*, John Anderson. IEEE Press, 1999.

FIGURE 12.12 (a) PAM signal with sinc pulse. (b) PAM signal with 'raised cosine' pulse. Note much larger tails and excursions in narrow band pulse of (a); tails may not be truncated without widening the bandwidth. (From J.B. Anderson, *Digital Transmission Engineering*, IEEE Press, 1999.)