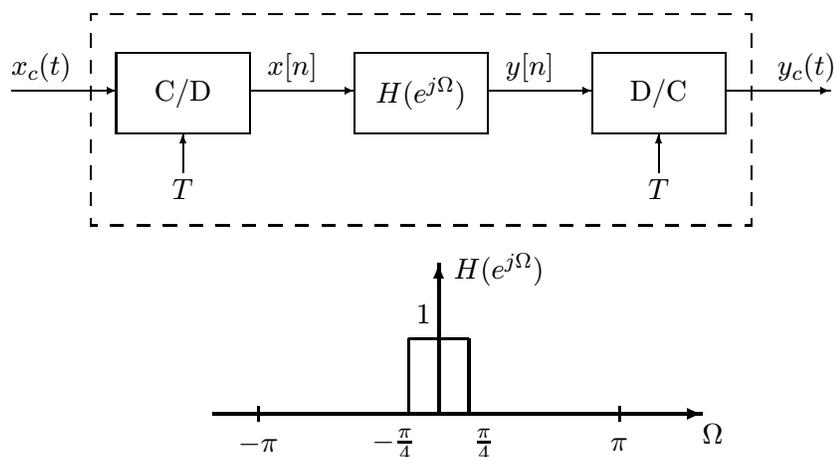


Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science
6.011: Introduction to Communication, Control and Signal Processing
QUIZ 2, November 24, 2004

Your Full Name:	
Recitation Instructor & Time :	a.m.

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- This quiz is **closed book**, but **two** sheet of notes are allowed. Calculators will not be necessary and are not allowed.
 - Put your name in the space indicated above, and your recitation time next to the name of your instructor.
 - Check that you have pages numbered up to **. The booklet contains spaces for **all** answers.
 - **Neat work and clear explanations count; show all relevant work and reasoning!** You may want to first work things through on scratch paper and then neatly transfer to this booklet the work you would like us to look at. Let us know if you need additional scratch paper. All scratch paper will be collected at the end of the quiz. However, **only** this booklet will be considered in the grading; **no additional answer or solution written elsewhere will be considered.** Absolutely no exceptions!
 - There are **two** problems, weighted as indicated on the question booklet.
 - **DO NOT DISCUSS THIS QUIZ WITH STUDENTS WHO HAVE NOT YET TAKEN IT TODAY!**

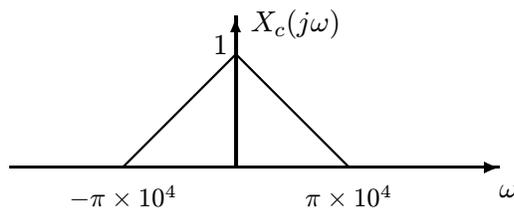
Problem	Your Score
1 (15 points)	
2 (35 points)	
Total (50 points)	

**Problem 1 (35 points)**

The three parts of this problem — (a), (b), and (c) — are somewhat independent, so if you get stuck on one part, move to another!

The figure above shows a continuous-time filter implemented using: (i) a C/D converter that samples the CT input at integer multiples of T ; (ii) a DT LTI ideal lowpass filter with frequency response $H(e^{j\Omega})$ whose cutoff frequency is $\frac{\pi}{4}$; and (iii) an ideal (i.e., bandlimited interpolating) D/C converter operating with reconstruction interval T .

- (a) Assume $x_c(t)$ is bandlimited to $\pi \times 10^4$, i.e., assume $X_c(j\omega) = 0$ for $|\omega| \geq \pi \times 10^4$. Also suppose $(1/T) = 8 \times 10^4$. First determine whether $x_c(t)$ is sampled without aliasing. Then express $y_c(t)$ in terms of $x_c(t)$, explaining your reasoning in detail. **Note:** We are **not** asking for $y_c(t)$ to be expressed as a function of time; just express its relation to $x_c(t)$.
- (b) Suppose that $X_c(j\omega)$ is as shown in the figure below, and that now $(1/T) = 2 \times 10^4$. Again determine whether $x_c(t)$ is sampled without aliasing. Then draw a **fully labeled** sketch of $Y_c(j\omega)$.



- (c) Suppose all we know about the set of possible input signals $x_c(t)$ is that they are constrained to be bandlimited to $\pi \times 10^4$. How small can the sampling rate ($1/T$) be while still ensuring that the system performs a CT LTI filtering operation on this class of inputs? Completely specify this LTI filtering operation, either in words or by drawing a **fully labeled** sketch of the effective CT frequency response $H_{eff}(j\omega)$.

Problem 2 (15 points)

The figure below shows a continuous-time filter implemented in the familiar way, i.e., using: (i) a C/D converter that samples the CT input at integer multiples of T ; (ii) a DT LTI filter with frequency response $H(e^{j\Omega})$; and (iii) an ideal (i.e., bandlimited interpolating) D/C converter operating with reconstruction interval T .

Suppose $H(e^{j\Omega}) = e^{-j0.4\Omega}$.

Assume

$$x_c(t) = 2 \frac{\sin(\pi(t - 0.3T)/T)}{(\pi(t - 0.3T)/T)}.$$

Find explicit expressions for $x[n]$, $y[n]$ and $y_c(t)$.

