# 6.003: Signals and Systems

## Signals and Systems

*September 8, 2011*

# 6.003: Signals and Systems

**Today's handouts:** Single package containing

- Slides for Lecture 1
- Subject Information & Calendar

**Lecturer:** Denny Freeman

**Instructors:** Elfar Adalsteinsson
Russ Tedrake

**TAs:** Phillip Nadeau
Wenbang Xu

**Website:** **mit.edu/6.003**

**Text:** *Signals and Systems* – Oppenheim and Willsky

# 6.003: Homework

Doing the homework is essential for understanding the content.

- where subject matter is/isn't learned
- equivalent to "practice" in sports or music

Weekly Homework Assignments

- Conventional Homework Problems plus
- **Engineering Design Problems** (Python/Matlab)

**Open Office Hours!**

- Stata Basement
- Mondays and Tuesdays, afternoons and early evenings

# 6.003: Signals and Systems

Collaboration Policy

- **Discussion** of concepts in homework is encouraged
- **Sharing** of homework or code is not permitted and will be reported to the COD

## Firm Deadlines

- Homework must be submitted by the published due date
- Each student can submit **one** late homework assignment without penalty.
- Grades on other late assignments will be multiplied by 0.5 (unless excused by an Instructor, Dean, or Medical Official).

# 6.003 At-A-Glance

| | Tuesday | | Wednesday | Thursday | Friday |
|---|---|---|---|---|---|
| Sep 6 | **Registration Day:** No Classes | | R1: Continuous & Discrete Systems | L1: Signals and Systems | R2: Difference Equations |
| Sep 13 | L2: Discrete-Time Systems | HW1 due | R3: Feedback, Cycles, and Modes | L3: Feedback, Cycles, and Modes | R4: CT Systems |
| Sep 20 | L4: CT Operator Representations | HW2 due | **Student Holiday:** No Recitation | L5: Laplace Transforms | R5: Laplace Transforms |
| Sep 27 | L6: Z Transforms | HW3 due | R6: Z Transforms | L7: Transform Properties | R7: Transform Properties |
| Oct 4 | L8: Convolution; Impulse Response | EX4 | **Exam 1** No Recitation | L9: Frequency Response | R8: Convolution and Freq. Resp. |
| Oct 11 | **Columbus Day:** No Lecture | HW5 due | R9: Bode Diagrams | L10: Bode Diagrams | R10: Feedback and Control |
| Oct 18 | L11: DT Feedback and Control | HW6 due | R11: CT Feedback and Control | L12: CT Feedback and Control | R12: CT Feedback and Control |
| Oct 25 | L13: CT Feedback and Control | HW7 | **Exam 2** No Recitation | L14: CT Fourier Series | R13: CT Fourier Series |
| Nov 1 | L15: CT Fourier Series | EX8 due | R14: CT Fourier Series | L16: CT Fourier Transform | R15: CT Fourier Transform |
| Nov 8 | L17: CT Fourier Transform | HW9 due | R16: DT Fourier Transform | L18: DT Fourier Transform | **Veterans Day:** No Recitation |
| Nov 15 | L19: DT Fourier Transform | HW10 | **Exam 3** No Recitation | L20: Fourier Relations | R17: Fourier Relations |
| Nov 22 | L21: Sampling | EX11 due | R18: Fourier Transforms | **Thanksgiving:** No Lecture | **Thanksgiving:** No-Recitation |
| Nov 29 | L22: Sampling | HW12 due | R19: Modulation | L23: Modulation | R20: Modulation |
| Dec 6 | L24: Modulation | EX13 | R21: Review | L25: Applications of 6.003 | Study Period |
| Dec 13 | Breakfast with Staff | EX13 | R22: Review | **Study Period:** No Lecture | **Final Exams:** No-Recitation |
| Dec 20 | **Final Examinations: No Classes** | | | | |

# 6.003: Signals and Systems

Weekly meetings with **class representatives**

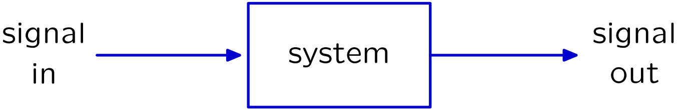
- help staff understand student perspective
- learn about teaching

Tentatively meet on Thursday afternoon

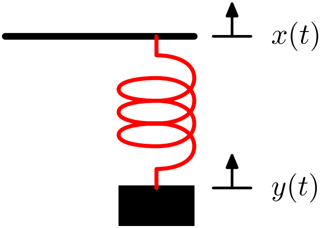
Interested? ...

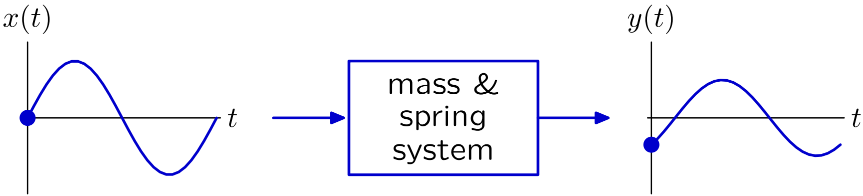
# The Signals and Systems Abstraction

Describe a **system** (physical, mathematical, or computational) by the way it transforms an **input signal** into an **output signal**.

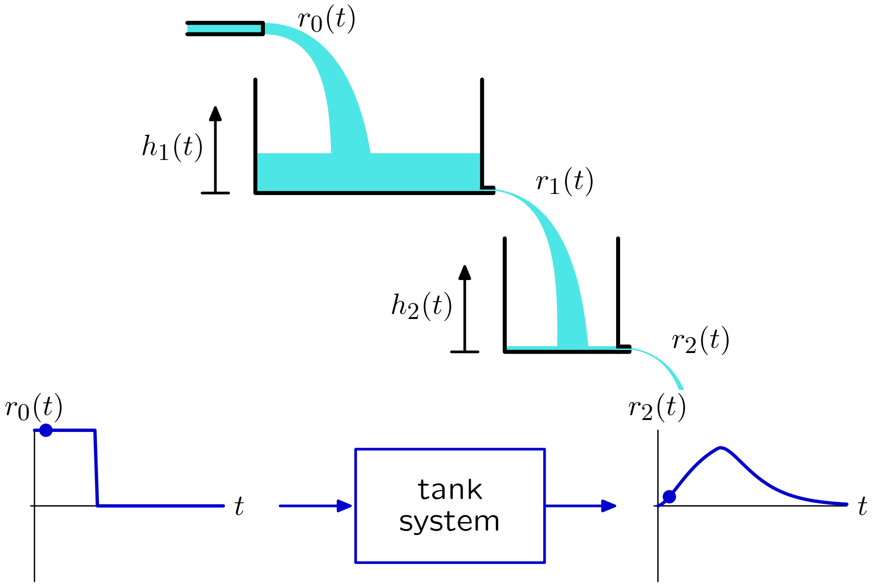


# Example: Mass and Spring

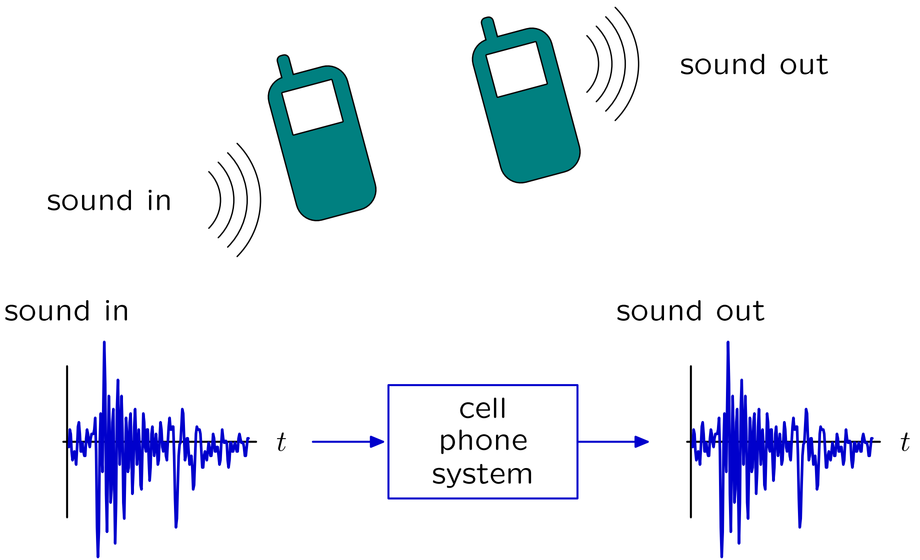


# Example: Tanks

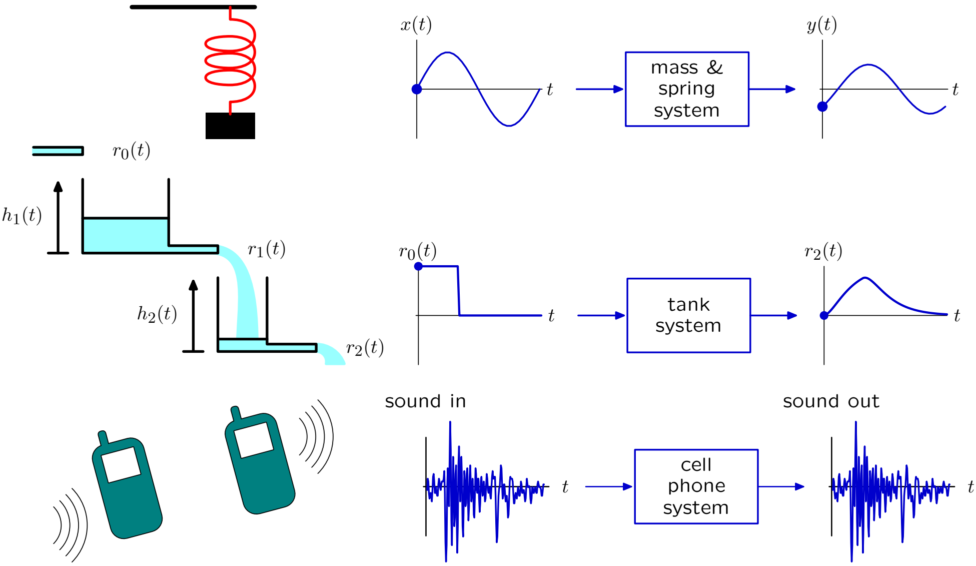


# Example: Cell Phone System



# Signals and Systems: Widely Applicable

The Signals and Systems approach has broad application: electrical, mechanical, optical, acoustic, biological, financial, ...

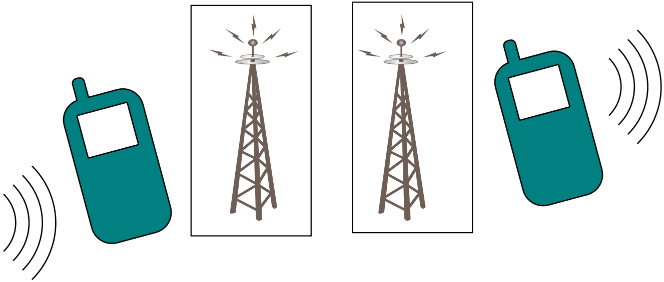


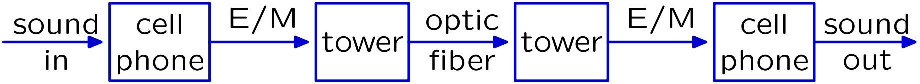
# Signals and Systems: Modular

The representation does not depend upon the physical substrate.

sound out

sound in



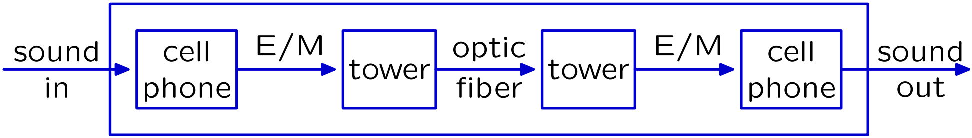



focuses on the flow of **information**, abstracts away everything else

# Signals and Systems: Hierarchical

Representations of component systems are easily combined.

Example: cascade of component systems



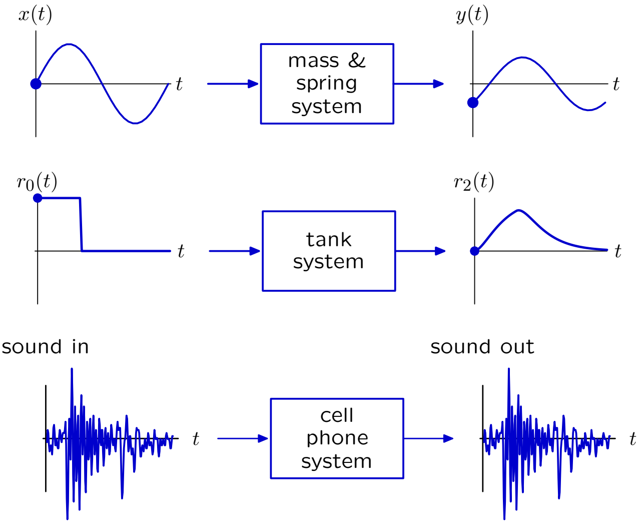

Composite system


Component and composite systems have the same form, and are analyzed with same methods.

# Signals and Systems

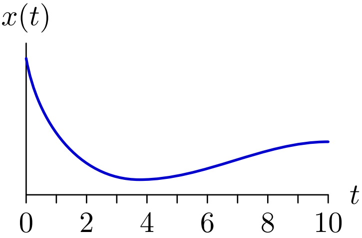
Signals are mathematical functions.

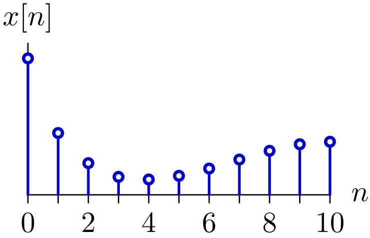
- independent variable = time
- dependent variable = voltage, flow rate, sound pressure



# Signals and Systems

continuous "time" (CT) and discrete "time" (DT)



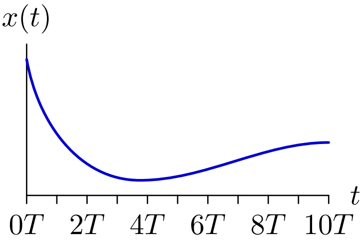

Signals from physical systems often functions of **continuous** time.

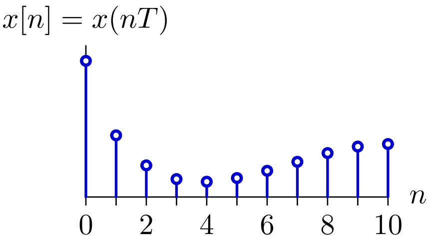
- mass and spring
- leaky tank

Signals from computation systems often functions of **discrete** time.

- state machines: given the current input and current state, what is the next output and next state.

## Sampling: converting CT signals to DT



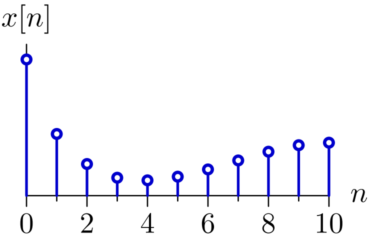

$T =$ sampling interval

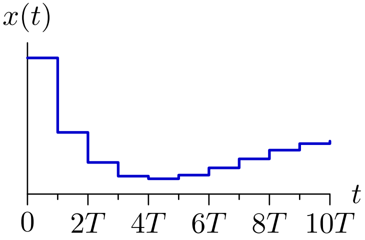
Important for computational manipulation of physical data.

- digital representations of audio signals (e.g., MP3)
- digital representations of images (e.g., JPEG)

Reconstruction: converting DT signals to CT

zero-order hold



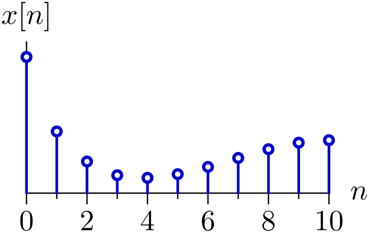



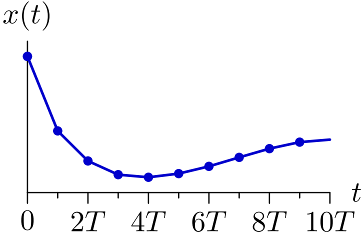

$T =$ sampling interval

commonly used in audio output devices such as CD players

Reconstruction: converting DT signals to CT

piecewise linear



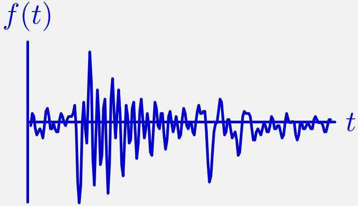




$T =$ sampling interval

commonly used in rendering images

# Check Yourself

Computer generated speech (by Robert Donovan)




Listen to the following four manipulated signals:

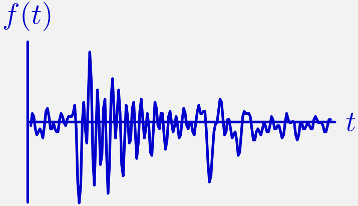
$$f_1(t),\ f_2(t),\ f_3(t),\ f_4(t).$$

How many of the following relations are true?

- $f_1(t) = f(2t)$
- $f_2(t) = -f(t)$
- $f_3(t) = f(2t)$
- $f_4(t) = \frac{1}{3}f(t)$

# Check Yourself

Computer generated speech (by Robert Donovan)



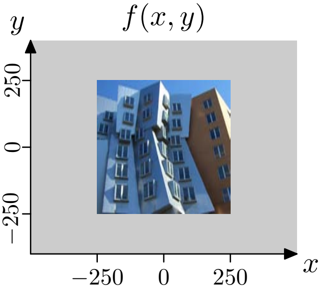

Listen to the following four manipulated signals:

$$f_1(t),\ f_2(t),\ f_3(t),\ f_4(t).$$

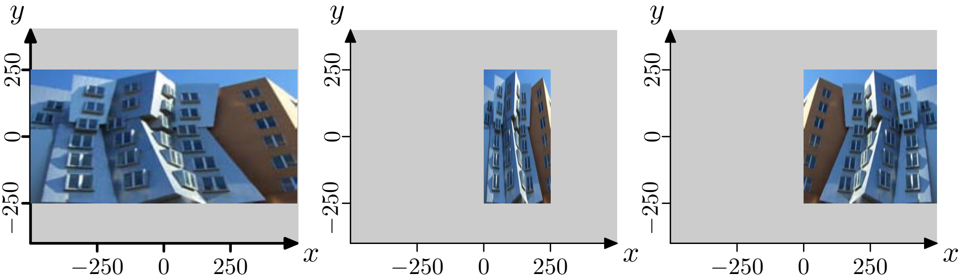
How many of the following relations are true? 2

- $f_1(t) = f(2t)$ $\surd$
- $f_2(t) = -f(t)$ $\times$
- $f_3(t) = f(2t)$ $\times$
- $f_4(t) = \frac{1}{3}f(t)$ $\surd$

# Check Yourself

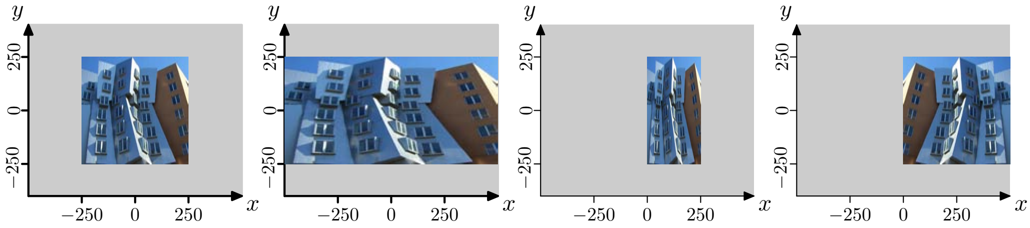



How many images match the expressions beneath them?




$f_1(x,y) = f(2x,y)$ ?   $f_2(x,y) = f(2x-250,y)$ ?   $f_3(x,y) = f(-x-250,y)$ ?

# Check Yourself



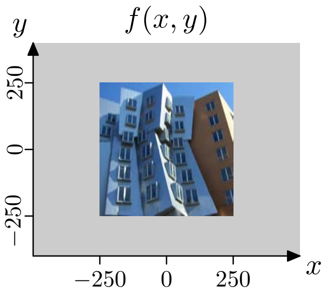

$f(x,y)$ $\quad f_1(x,y)=f(2x,y)$ ? $\quad f_2(x,y)=f(2x-250,y)$ ? $\quad f_3(x,y)=f(-x-250,y)$ ?

$$x=0 \quad \rightarrow f_1(0,y)=f(0,y) \quad \surd$$
$$x=250 \quad \rightarrow f_1(250,y)=f(500,y) \quad \times$$

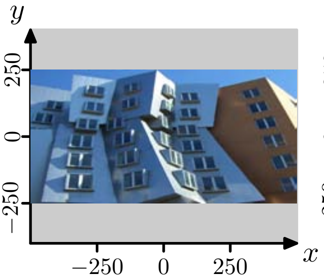
$$x=0 \quad \rightarrow f_2(0,y)=f(-250,y) \quad \surd$$
$$x=250 \quad \rightarrow f_2(250,y)=f(250,y) \quad \surd$$

$$x=0 \quad \rightarrow f_3(0,y)=f(-250,y) \quad \times$$
$$x=250 \quad \rightarrow f_3(250,y)=f(-500,y) \quad \times$$

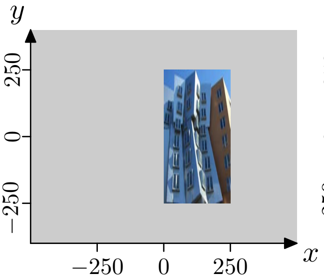
# Check Yourself



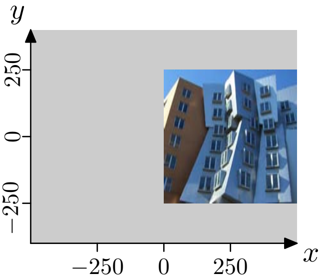

How many images match the expressions beneath them?




~~$f_1(x, y) = f(2x, y)$ ?~~



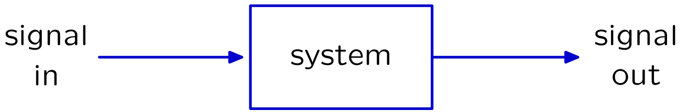

$f_2(x, y) = f(2x - 250, y)$ ?




~~$f_3(x, y) = f(-x - 250, y)$ ?~~

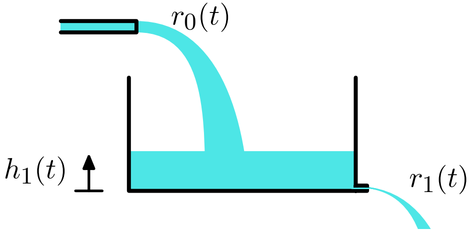
# The Signals and Systems Abstraction

Describe a **system** (physical, mathematical, or computational) by the way it transforms an **input signal** into an **output signal**.



# Example System: Leaky Tank

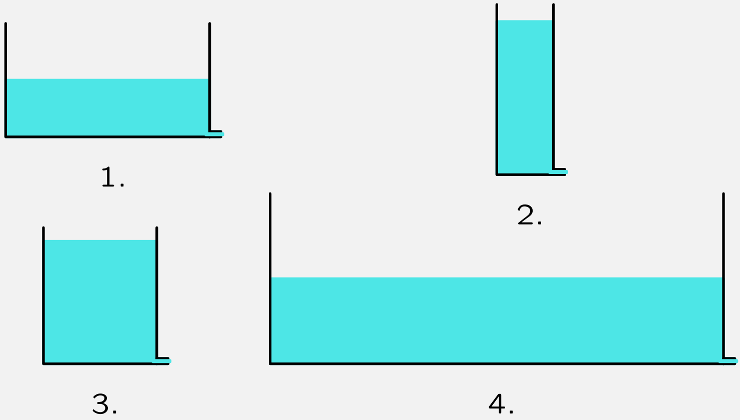
Formulate a mathematical description of this system.




What determines the leak rate?

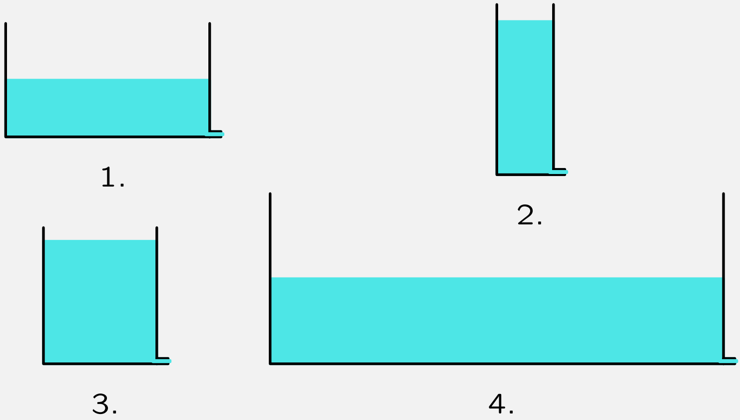
# Check Yourself

The holes in each of the following tanks have equal size. Which tank has the largest leak rate $r_1(t)$?



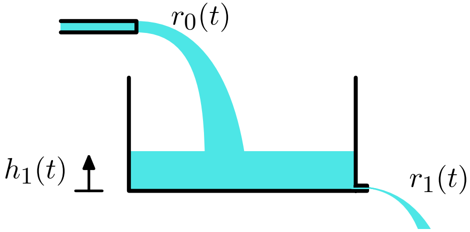
# Check Yourself

The holes in each of the following tanks have equal size. Which tank has the largest leak rate $r_1(t)$? 2



# Example System: Leaky Tank

Formulate a mathematical description of this system.

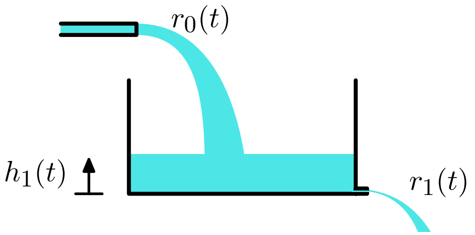



Assume linear leaking: $r_1(t) \propto h_1(t)$

What determines the height $h_1(t)$?

# Example System: Leaky Tank

Formulate a mathematical description of this system.




Assume linear leaking: $r_1(t) \propto h_1(t)$

Assume water is conserved: $\dfrac{dh_1(t)}{dt} \propto r_0(t) - r_1(t)$

Solve: $\dfrac{dr_1(t)}{dt} \propto r_0(t) - r_1(t)$

# Check Yourself

What are the dimensions of constant of proportionality $C$?

$$\frac{dr_1(t)}{dt} = C\Big(r_0(t) - r_1(t)\Big)$$

# Check Yourself

What are the dimensions of constant of proportionality $C$?
inverse time (to match dimensions of $dt$)

$$\frac{dr_1(t)}{dt} = C\Big(r_0(t) - r_1(t)\Big)$$

# Analysis of the Leaky Tank

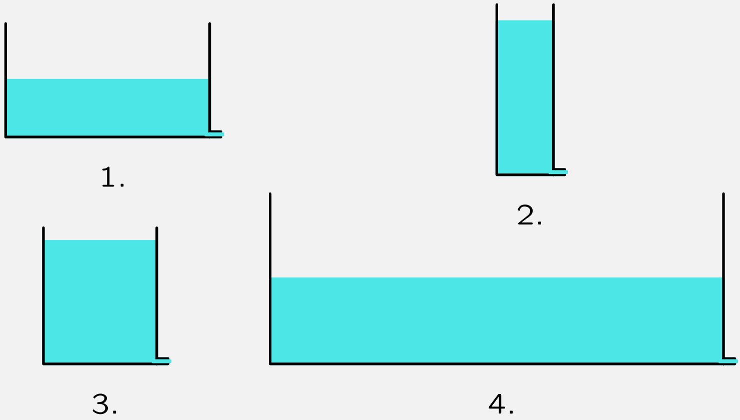
Call the constant of proportionality $1/\tau$.

Then $\tau$ is called the **time constant** of the system.

$$\frac{dr_1(t)}{dt} = \frac{r_0(t)}{\tau} - \frac{r_1(t)}{\tau}$$

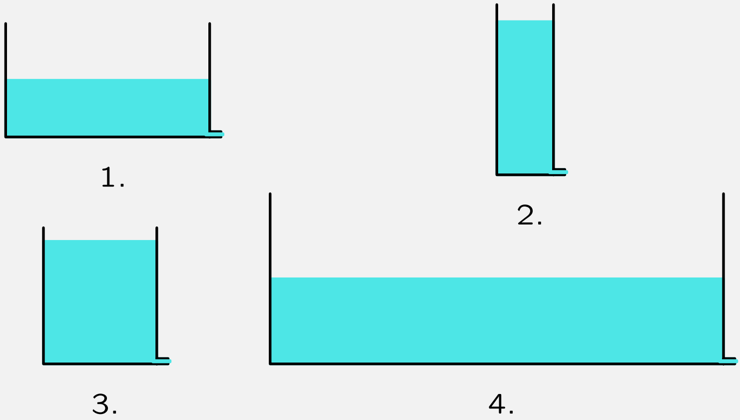
# Check Yourself

Which tank has the largest time constant $\tau$?



# Check Yourself

Which tank has the largest time constant $\tau$? 4



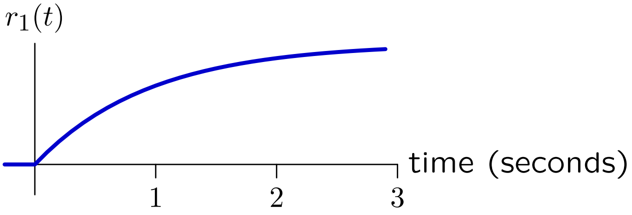
# Analysis of the Leaky Tank

Call the constant of proportionality $1/\tau$.

Then $\tau$ is called the **time constant** of the system.

$$\frac{dr_1(t)}{dt} = \frac{r_0(t)}{\tau} - \frac{r_1(t)}{\tau}$$

Assume that the tank is initially empty, and then water enters at a constant rate $r_0(t) = 1$. Determine the output rate $r_1(t)$.



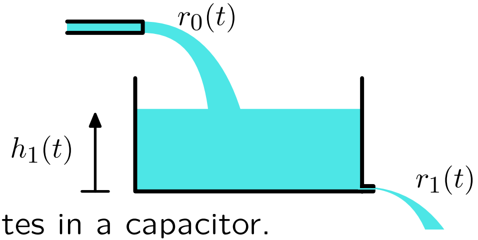

Explain the shape of this curve mathematically.

Explain the shape of this curve physically.

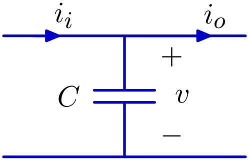
# Leaky Tanks and Capacitors

Although derived for a leaky tank, this sort of model can be used to represent a variety of physical systems.

Water accumulates in a leaky tank.




Charge accumulates in a capacitor.




$$\frac{dv}{dt} = \frac{i_i - i_o}{C} \propto i_i - i_o \qquad \text{analogous to} \qquad \frac{dh}{dt} \propto r_0 - r_1$$

6.003 Signals and Systems
Fall 2011