6.003 (Fall 2011)

Quiz #2 *October 26, 2011*

Name:

Kerberos Username:

Please circle your section number:

Section	*Time*
2	11 am
3	1 pm
4	2 pm

Grades will be determined by the correctness of your answers (explanations are not required).

Partial credit will be given for ANSWERS that demonstrate some but not all of the important conceptual issues.

You have **two hours.**

Please put your initials on all subsequent sheets.

Enter your answers in the boxes.

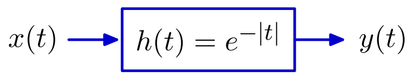
This quiz is closed book, but you may use two 8.5 × 11 sheets of paper (four sides total).

No calculators, computers, cell phones, music players, or other aids.

1	/15
2	/15
3	/30
4	/20
5	/20
Total	/100

1. Find the differential equation *[15 points]*

Determine a linear differential equation with constant coefficients to represent the relation between the input $x(t)$ and output $y(t)$ of the linear, time-invariant system whose impulse response is $h(t) = e^{-|t|}$.



differential equation: $y(t) - \ddot{y}(t) = 2x(t)$

The Laplace transform of $h(t) = e^{-|t|}$ is

$$H(s) = \int_{-\infty}^{0} e^t e^{-st} dt + \int_0^{\infty} e^{-t} e^{-st} dt = \left.\frac{e^{(1-s)t}}{1-s}\right|_{-\infty}^{0} + \left.\frac{e^{-(1+s)t}}{-(1+s)}\right|_{0}^{\infty} = \frac{2}{1-s^2} = \frac{Y(s)}{X(s)}$$

Therefore, $2X(s) = (1-s^2)Y(s) = Y(s) - s^2 Y(s)$, which corresponds to

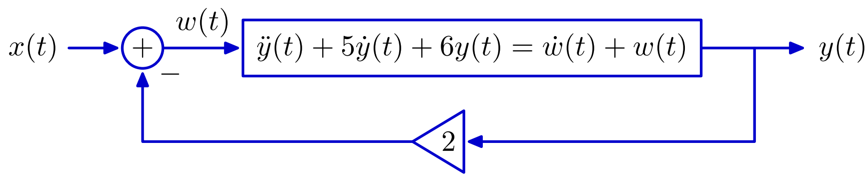
$$y(t) - \ddot{y}(t) = 2x(t)$$

2. Feedback *[15 points]*

A system with input $w(t)$ and output $y(t)$ is represented by the differential equation

$$\ddot{y}(t) + 5\dot{y}(t) + 6y(t) = \dot{w}(t) + w(t)\,.$$

This system is placed in a feedback loop as shown below.



Determine a differential equation that relates the input $x(t)$ and output $y(t)$ of the closed loop system. Your answer should **NOT** depend on $w(t)$ or any of its derivatives.

differential equation: $\ddot{y}(t) + 7\dot{y}(t) + 8y(t) = \dot{x}(t) + x(t)$

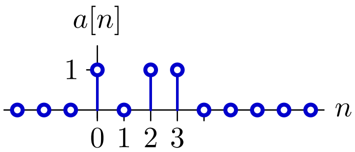
$$G(s) = \frac{Y(s)}{W(s)} = \frac{s+1}{s^2+5s+6}$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{G(s)}{1+2G(s)} = \frac{\frac{s+1}{s^2+5s+6}}{1+2\frac{s+1}{s^2+5s+6}} = \frac{s+1}{s^2+5s+6+2s+1} = \frac{s+1}{s^2+7s+8}$$

$$\ddot{y}(t) + 7\dot{y}(t) + 8y(t) = \dot{x}(t) + x(t)$$

3. Convolutions *[30 points]*

Consider the convolution of two of the following signals.

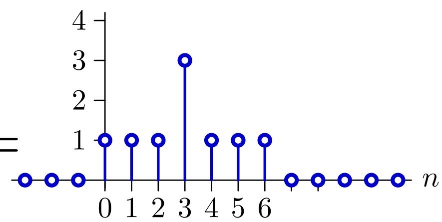


Determine if each of the following signals can be constructed by convolving (a or b or c) with (a or b or c). If it can, indicate which signals to convolve. If it cannot, put an X in both boxes.

Notice that there are ten possible answers: $(a * a)$, $(a * b)$, $(a * c)$, $(b * a)$, $(b * b)$, $(b * c)$, $(c * a)$, $(c * b)$, $(c * c)$, or (X,X). Notice also that the answer may not be unique.

[a / b] $*$ [b / a] =

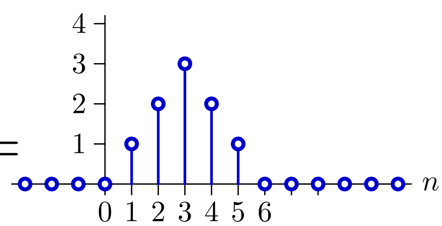
a or b or c or X — a or b or c or X



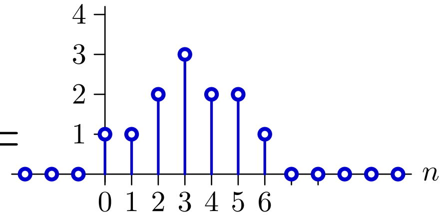
1101000 + 0011010 + 0001101 = 1113111

[X] $*$ [X] =

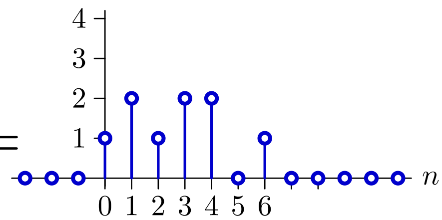
a or b or c or X — a or b or c or X



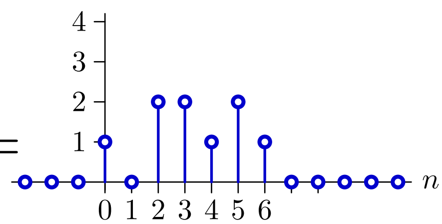
Since $a[0] = b[0] = c[0] = 1$, the result of convolving any two of these signals is 1 at $n = 0$. Similarly, since $a[3] = b[3] = c[3] = 1$, the result must be 1 at $n = 6$.

a / c	$*$	c / a	$=$	
a or b or c or X		a or b or c or X		

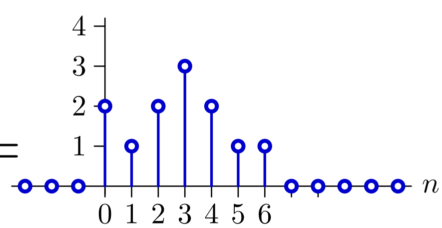
1111000 + 0011110 + 0001111 = 1123221

b	$*$	b	$=$	
a or b or c or X		a or b or c or X		

1101000 + 0110100 + 0001101 = 1212201

a	$*$	a	$=$	
a or b or c or X		a or b or c or X		

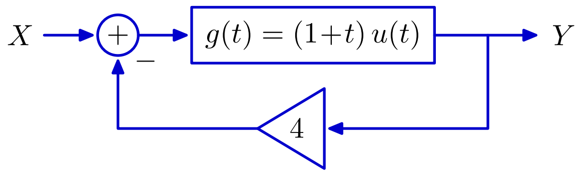
1011000 + 0010110 + 0001011 = 1022121

X	$*$	X	$=$	
a or b or c or X		a or b or c or X		

Since $a[0] = b[0] = c[0] = 1$, the result of convolving any two of these signals is 1 at $n = 0$.

4. Feedback *[20 points]*

Let $g(t) = (1+t)\,u(t)$ represent the impulse response of a linear, time-invariant system that is part of the following feedback system. Determine the poles and zeros of the closed-loop system $\frac{Y}{X}$.



Enter the number of poles and zeros and list their approximate values below. If a pole or zero is repeated k times, then enter that value k times. If there are more than 5 poles or zeros, enter just 5 of them. If there are fewer than 5 poles or zeros, write **none** in the remaining boxes.

# of poles:	2

poles:	-2	-2	none	none	none

# of zeros:	1

zeros:	-1	none	none	none	none

The Laplace transform of $u(t)$ is $\frac{1}{s}$. The Laplace transform of $tu(t)$ is $\frac{1}{s^2}$. Therefore

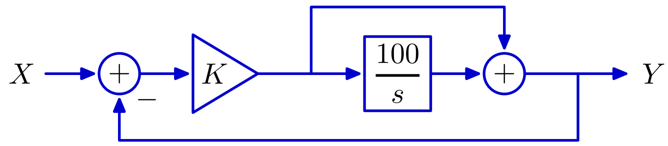
$$G(s) = \frac{1}{s} + \frac{1}{s^2} = \frac{s+1}{s^2}\,.$$

The closed loop system function is

$$H(s) = \frac{G(s)}{1+4G(s)} = \frac{\frac{s+1}{s^2}}{1+4\frac{s+1}{s^2}} = \frac{s+1}{s^2+4s+4} = \frac{s+1}{(s+2)(s+2)}$$

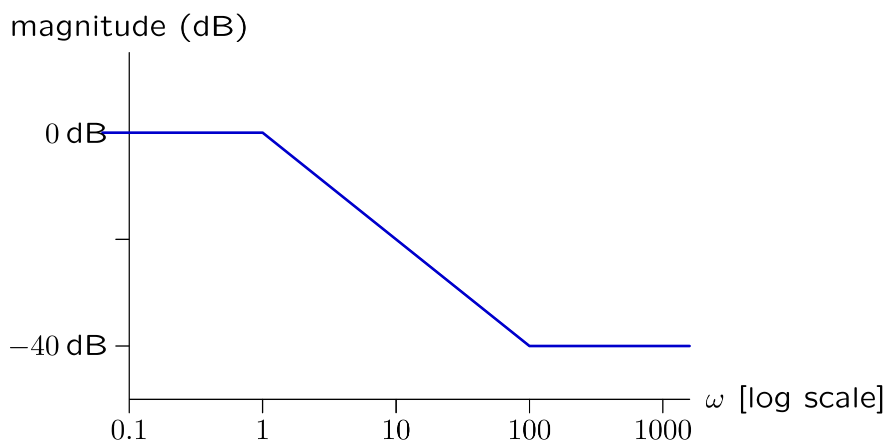
5. Frequency Response of Feedback System *[20 points]*

Consider the frequency response of the following system, where the input signal is X and the output signal is Y.



Part a. On the axes below, sketch the straight-line approximation (Bode plot) for the magnitude of the frequency response for the case $K = 0.01$.

Show **numerical values** for the magnitudes (dB) and frequencies ω for all of the points of intersection between adjacent straight-line segments.



$$1 + \frac{100}{s} = \frac{s+100}{s}$$

$$H(s) = \frac{\frac{K(s+100)}{s}}{1+\frac{K(s+100)}{s}} = \frac{K(s+100)}{s+Ks+100K} = \frac{0.01(s+100)}{s+0.01s+100\times 0.01} = \frac{0.01(s+100)}{1.01s+1}$$

Part b. On the axes below, sketch the straight-line approximation for the angle of the frequency response for the case $K = 0.01$.

> Show **numerical values** for the angles (radians) and frequencies ω for all of the points of intersection between adjacent straight-line segments.