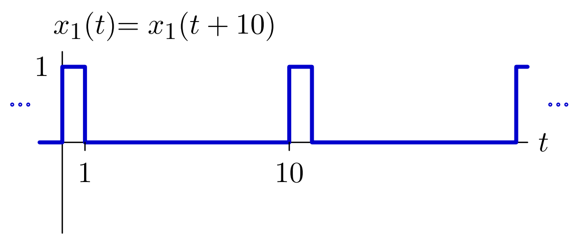
6.003 Homework #8

*Due at the beginning of recitation on **November 2, 2011**.*

Problems

1. Fourier Series

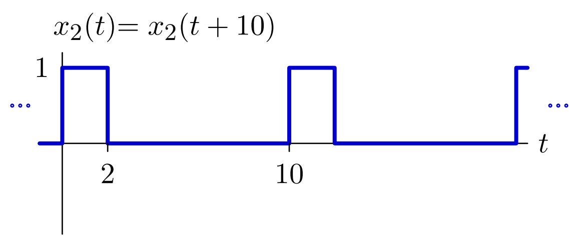
Determine the Fourier series coefficients a_k for $x_1(t)$ shown below.



$a_0 =$

$a_k =$ for $k \neq 0$

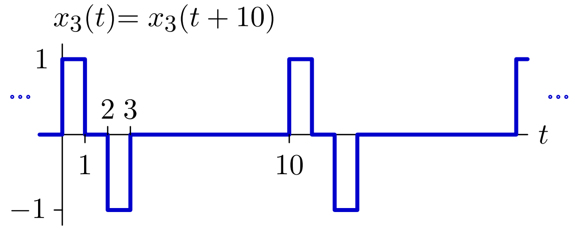
Determine the Fourier series coefficients b_k for $x_2(t)$ shown below.



$b_0 =$

$b_k =$ for $k \neq 0$

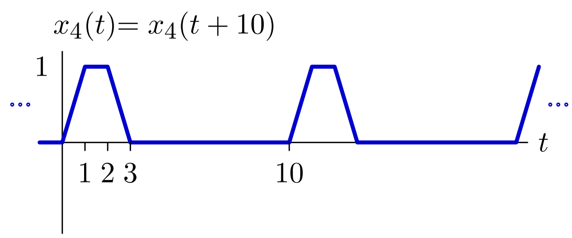
Determine the Fourier series coefficients c_k for $x_3(t)$ shown below.



$c_0 =$

$c_k =$ for $k \neq 0$

Determine the Fourier series coefficients d_k for $x_4(t)$ shown below.



$d_0 =$

$d_k =$ for $k \neq 0$

2. Inverse Fourier series

Determine the CT signals with the following Fourier series coefficients. Assume that the signals are periodic in $T = 4$. Enter an expression that is valid for $0 \le t < 4$ (other values can be found by periodic extension).

a. $a_k = \begin{cases} jk; & |k| < 3 \\ 0 & \text{otherwise} \end{cases}$

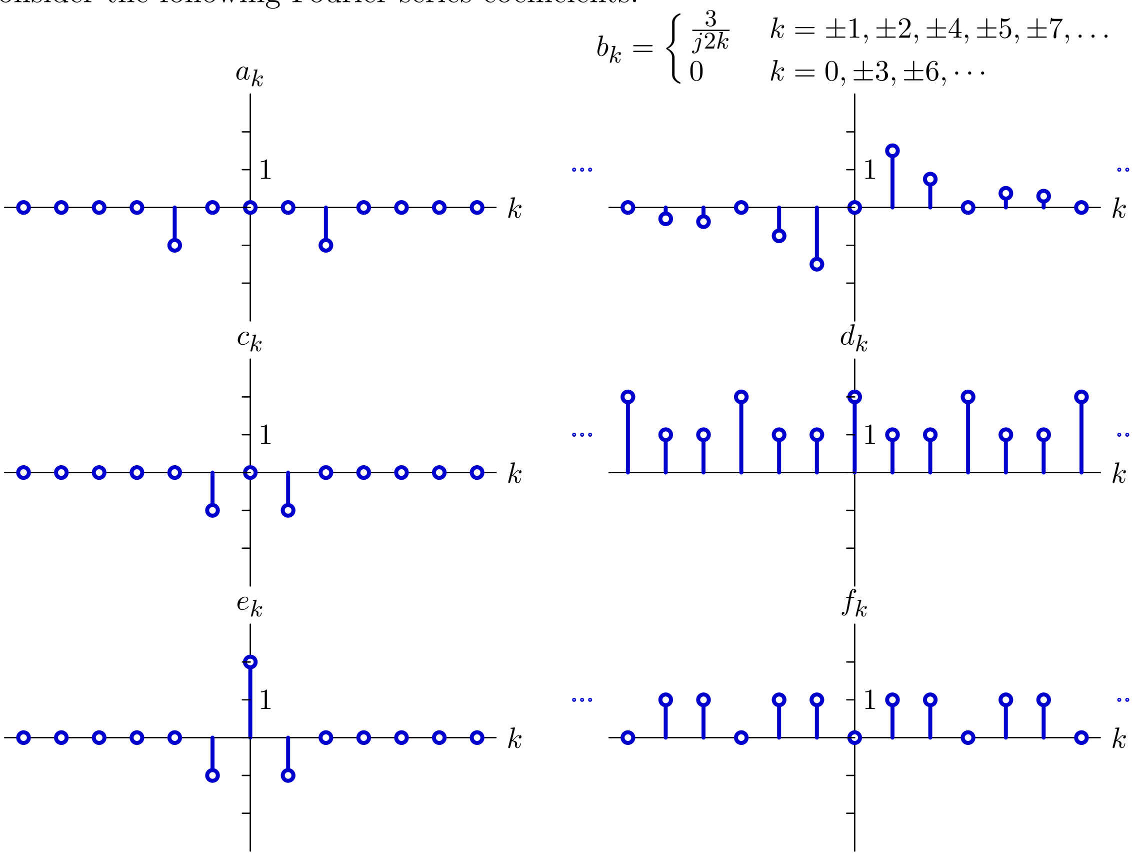
$x(t) =$ [ ] for $0 \le t < 4$.

b. $b_k = \begin{cases} 1; & k \text{ odd} \\ 0; & k \text{ even} \end{cases}$

$x(t) =$ [ ] for $0 \le t < 4$.

3. Matching

Consider the following Fourier series coefficients.

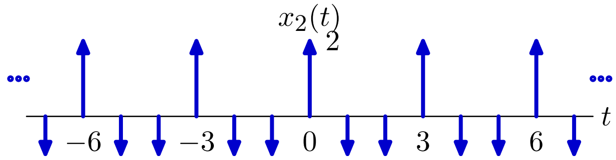


a. Which coefficients (if any) corresponds to the following periodic signal?

$$x_1(t) = 2 - 2\cos\left(\frac{2\pi}{3}\,t\right)$$

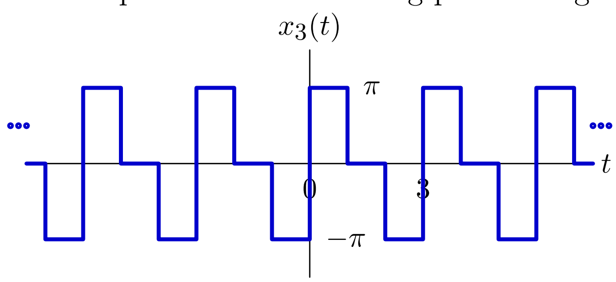
a_k, b_k, c_k, d_k, e_k, f_k, or **None**:

b. Which coefficients (if any) corresponds to the following periodic signal with period $T = 3$?



a_k, b_k, c_k, d_k, e_k, f_k, or **None**:

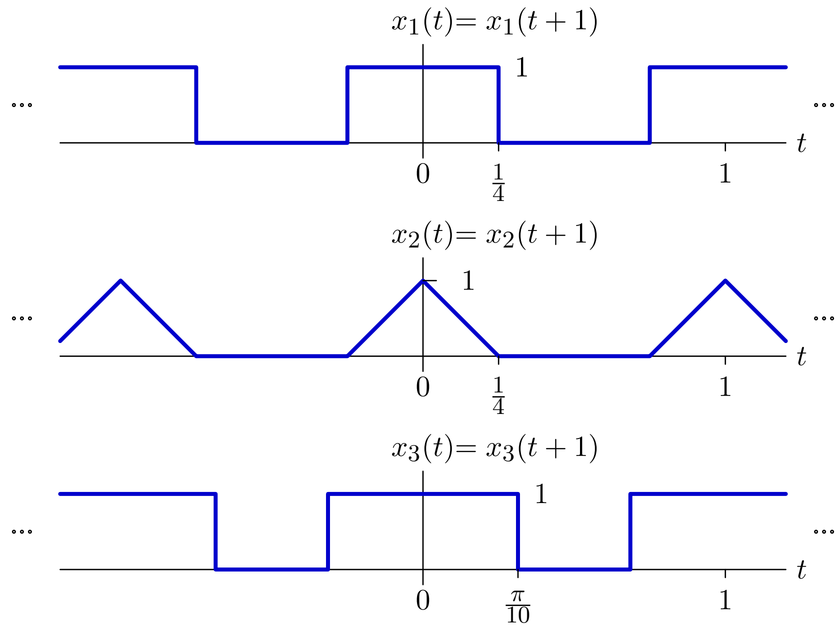
c. Which (if any) set corresponds to the following periodic signal with period $T = 3$?



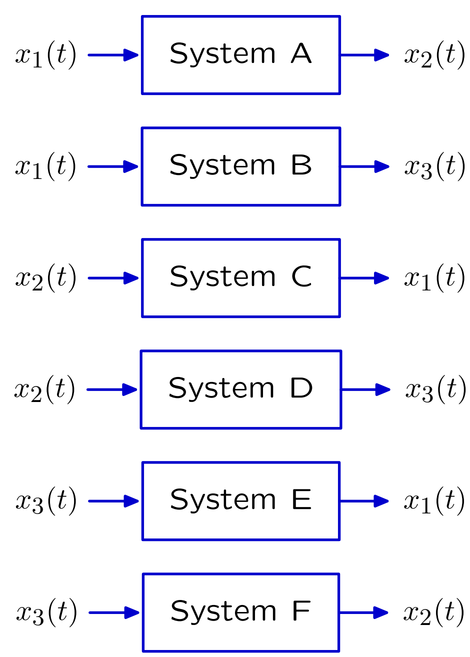
a_k, b_k, c_k, d_k, e_k, f_k, or **None**:

4. Input/Output Pairs

The following signals are periodic with period $T = 1$.



Determine if the following systems could or could not be linear and time-invariant (LTI).



Enter a list of the systems that could **NOT** be LTI. If your list is empty, enter **none**.

answer = [   ]

Engineering Design Problems

5. Overshoot

a. What function $f(t)$ has the Fourier series

$$\sum_{n=1}^{\infty} \frac{\sin nt}{n}?$$

You can evaluate the sum analytically or numerically. Either way, guess a closed form for $f(t)$ and then sketch it.

b. Confirm your conjecture for $f(t)$ by finding the Fourier series coefficients f_n for $f(t)$. Compare your result to the expression in the previous part. What happens to the cosine terms?

c. Define the partial sum

$$f_N(t) = \sum_{n=1}^{N} \frac{\sin nt}{n},$$

Plot some $f_N(t)$'s. By what fraction does $f_N(t)$ overshoot $f(t)$ at worst? Does that fraction tend to zero or to a finite value as $N \to \infty$? If it is a finite value, estimate it.

d. Now define the average of the partial sums:

$$F_N(t) = \frac{f_1(t) + f_2(t) + f_3(t) + \cdots + f_N(t)}{N}$$

Plot some $F_N(t)$'s. Compare your plots with those of $f_N(t)$ that you made in the previous part, and qualitatively explain any differences.