6.002 CIRCUITS AND ELECTRONICS

Superposition, Thévenin and Norton

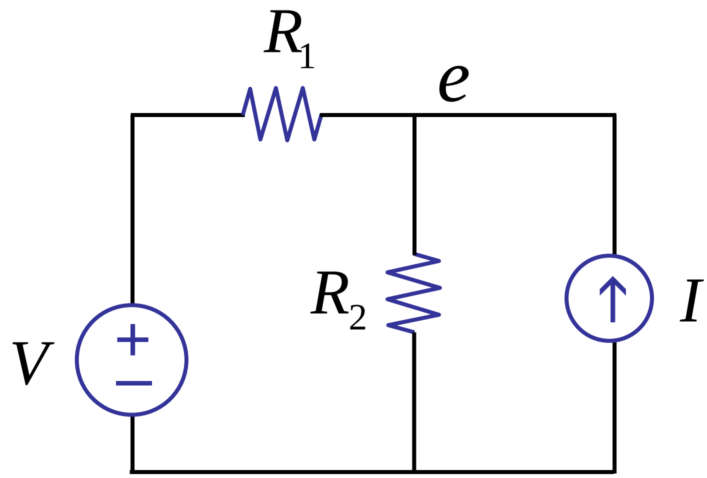
Review

Circuit Analysis Methods

- KVL: $\sum_{loop} V_i = 0$ KCL: $\sum_{node} I_i = 0$ VI
- Circuit composition rules
- Node method – the workhorse of 6.002
  KCL at nodes using V's referenced from ground
  (KVL implicit in "$(e_i - e_j)\ G$")

Linearity

Consider



Write node equations -

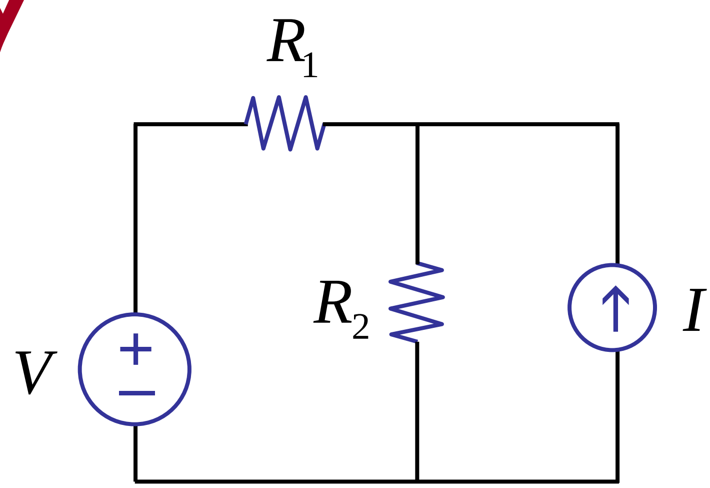
$$\frac{e-V}{R_1}+\frac{e}{R_2}-I=0$$

Notice:
linear in e, V, I

No eV, VI terms

Linearity

Consider



Write node equations --

$$\frac{e - V}{R_1} + \frac{e}{R_2} - I = 0$$

linear in e, V, I

Rearrange --

$$\left[\frac{1}{R_1} + \frac{1}{R_2}\right] e = \frac{V}{R_1} + I$$

conductance matrix — node voltages — linear sum of sources

$$G \quad e = S$$

Linearity

Write node equations --

$$\frac{e-V}{R_1}+\frac{e}{R_2}-I=0$$

linear in e, V, I

Rearrange --

$$\left[\frac{1}{R_1}+\frac{1}{R_2}\right]e=\frac{V}{R_1}+I$$

conductance matrix

node voltages

linear sum of sources

$$G \quad e = S$$

or

$$e=\frac{R_2}{R_1+R_2}V+\frac{R_1R_2}{R_1+R_2}I$$

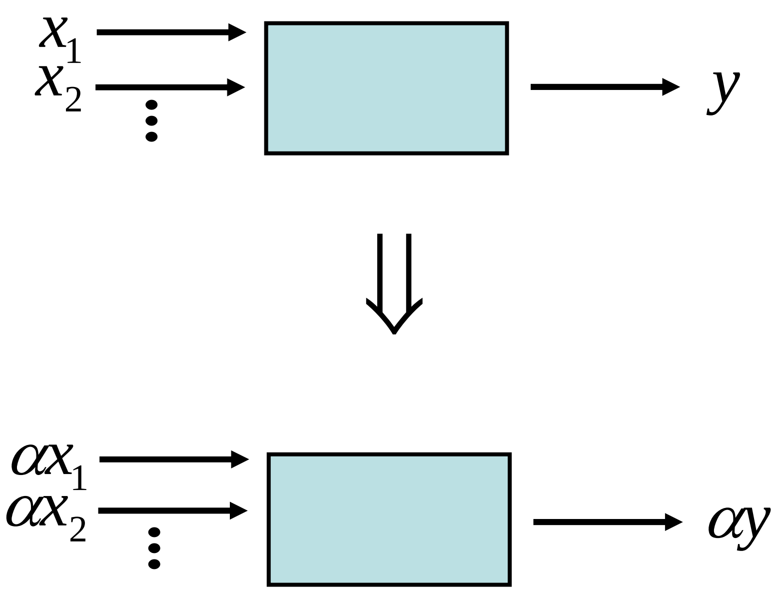
$$e=a_1V_1+a_2V_2+\ldots+b_1I_1+b_2I_2+\ldots$$

Linear!

Linearity $\Rightarrow$ Homogeneity
Superposition

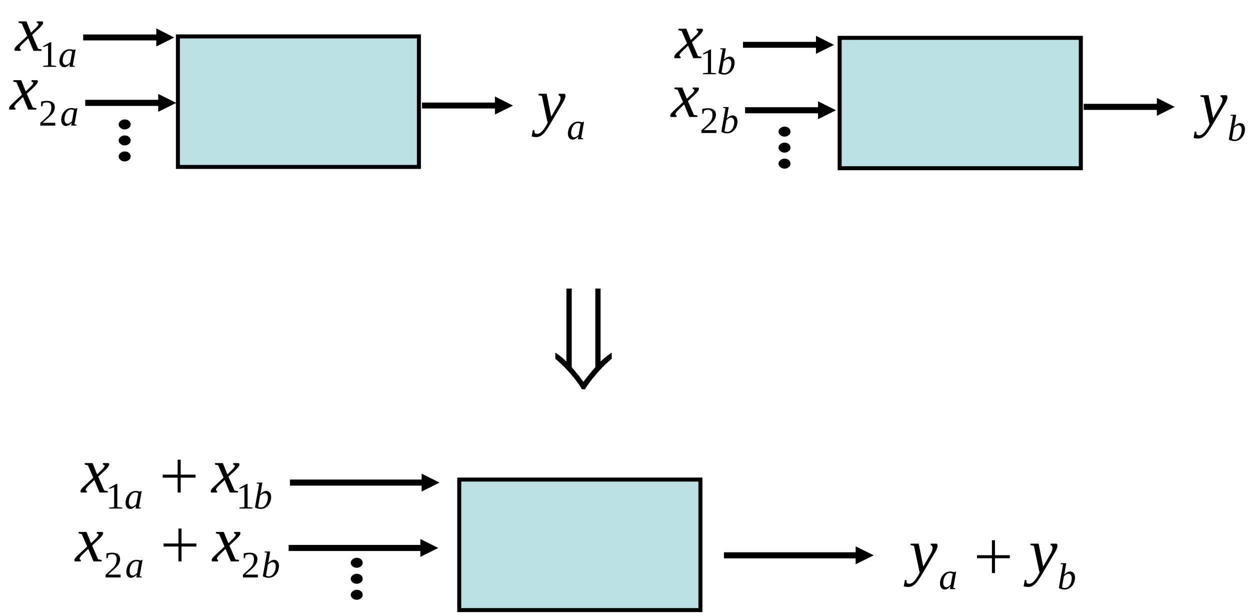
Linearity $\Rightarrow$ Homogeneity
Superposition

Homogeneity



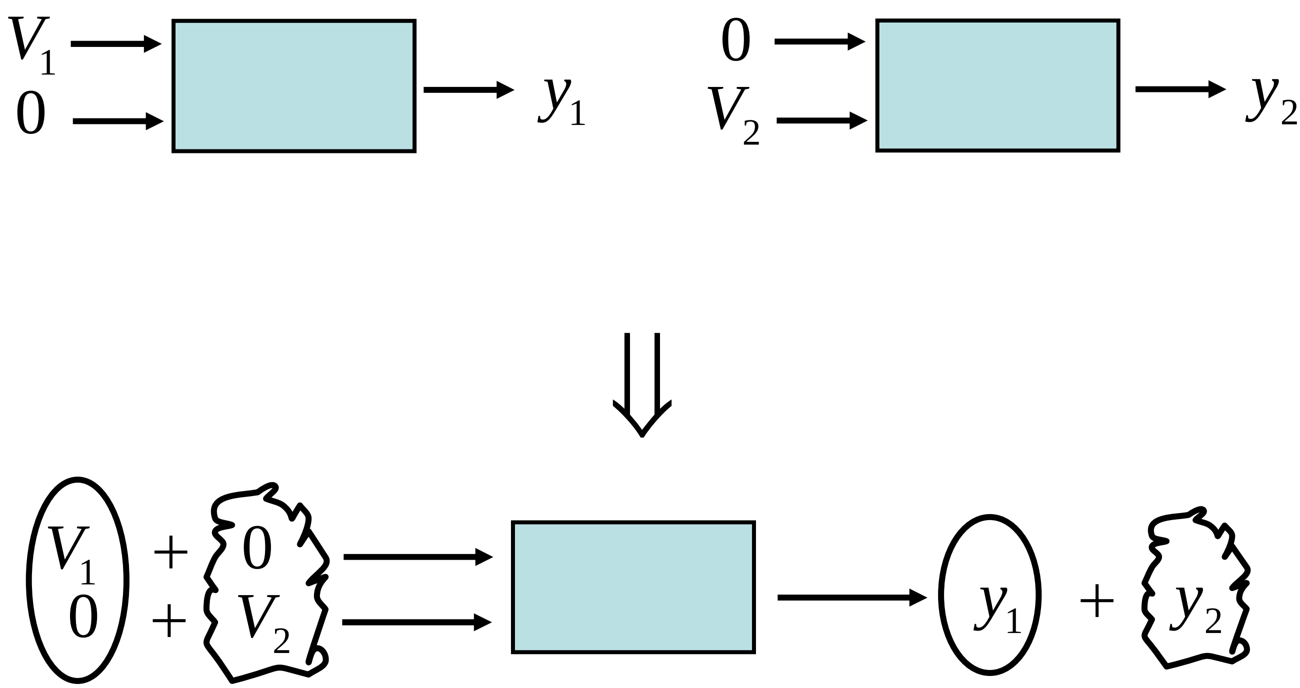
Linearity $\Rightarrow$ Homogeneity
Superposition

Superposition



Linearity $\Rightarrow$ Homogeneity
Superposition

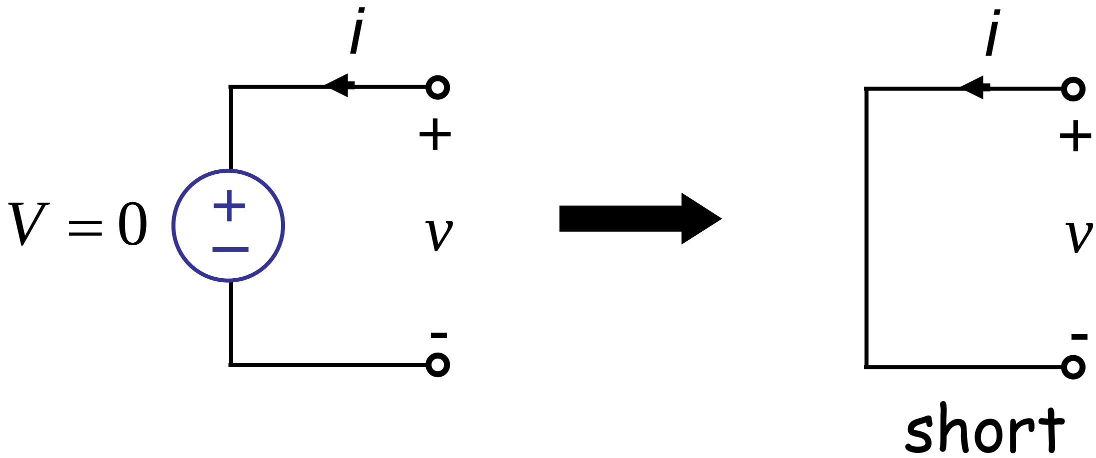
Specific superposition example:

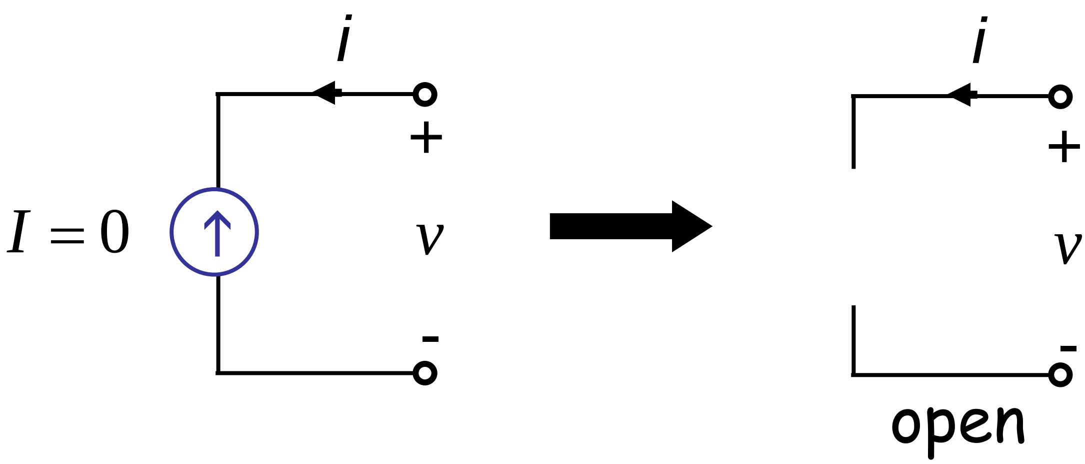


Method 4: Superposition method

The output of a circuit is determined by summing the responses to each source acting alone.

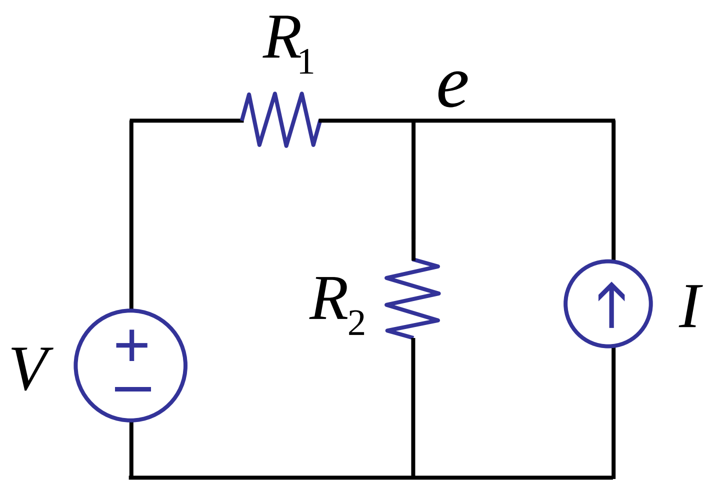
independent sources only


i
+
V = 0
v
-
i
+
v
-
short


i
+
I = 0
v
-
i
+
v
-
open

Back to the example

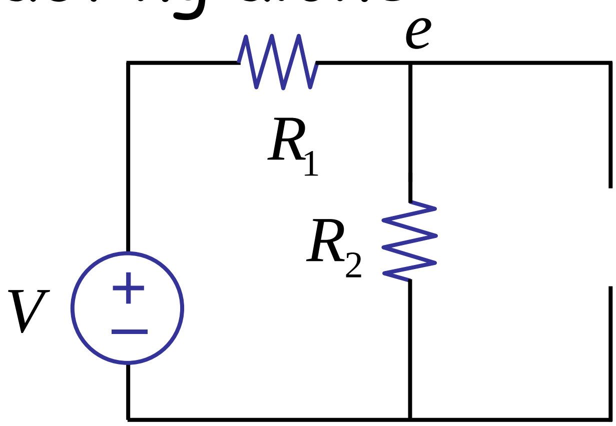
Use superposition method



Back to the example

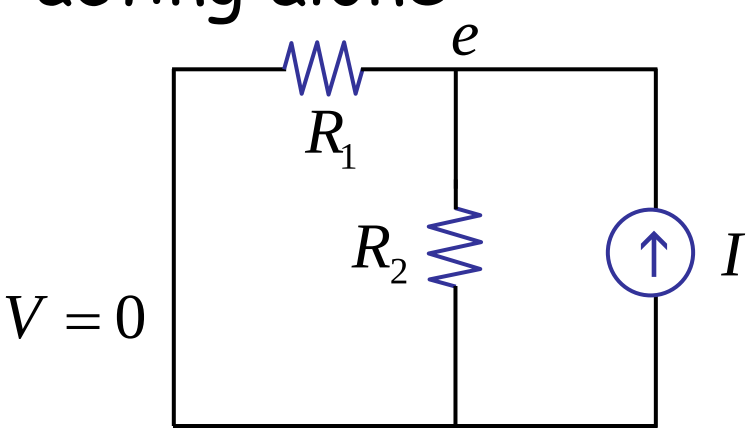
Use superposition method

V acting alone



$I = 0$ $\qquad e_V = \frac{R_2}{R_1 + R_2} V$

I acting alone

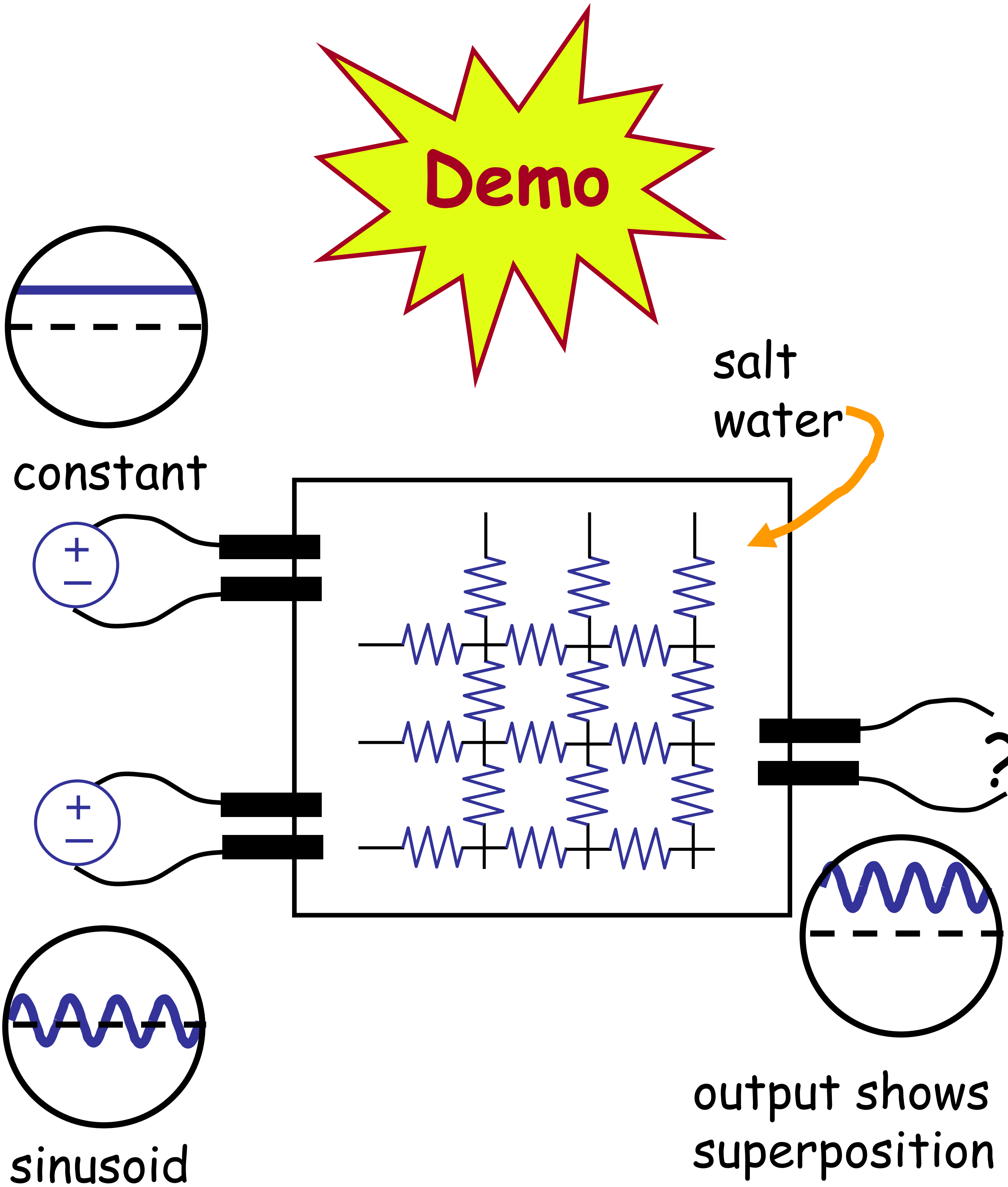


$$e_I = \frac{R_1 R_2}{R_1 + R_2} I$$

sum ⟶ superposition

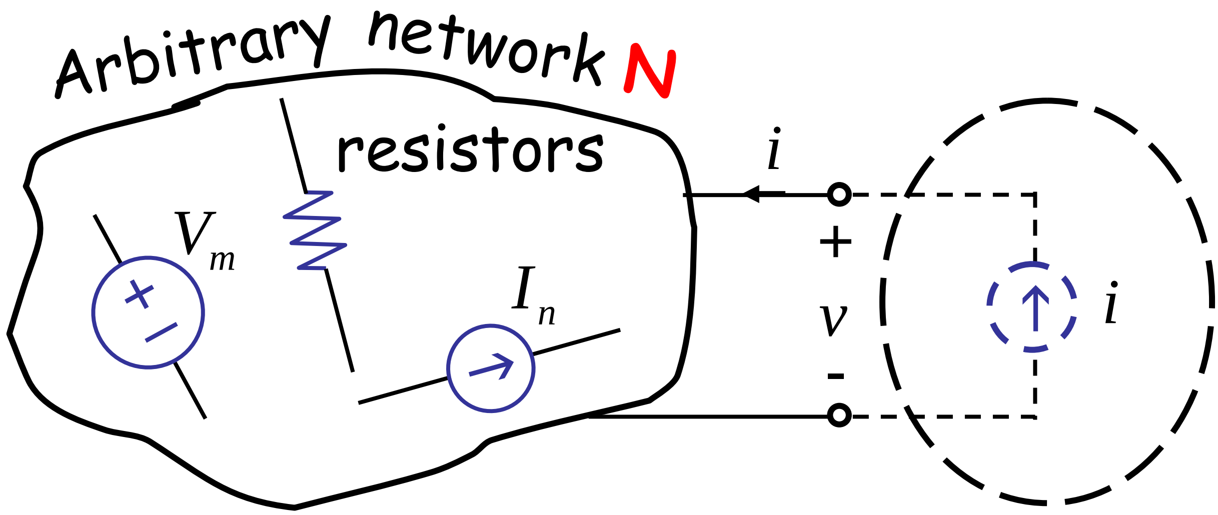
$$e = e_V + e_I = \frac{R_2}{R_1 + R_2} V + \frac{R_1 R_2}{R_1 + R_2} I$$

Voilà !


Demo
salt water
constant
sinusoid
?
output shows superposition

Yet another method...

Consider



By superposition

$$v = \sum_m \alpha_m V_m + \sum_n \beta_n I_n + Ri$$

α_m: no units

β_n, R: resistance units

Ri: also independent of external excitement & behaves like a resistor

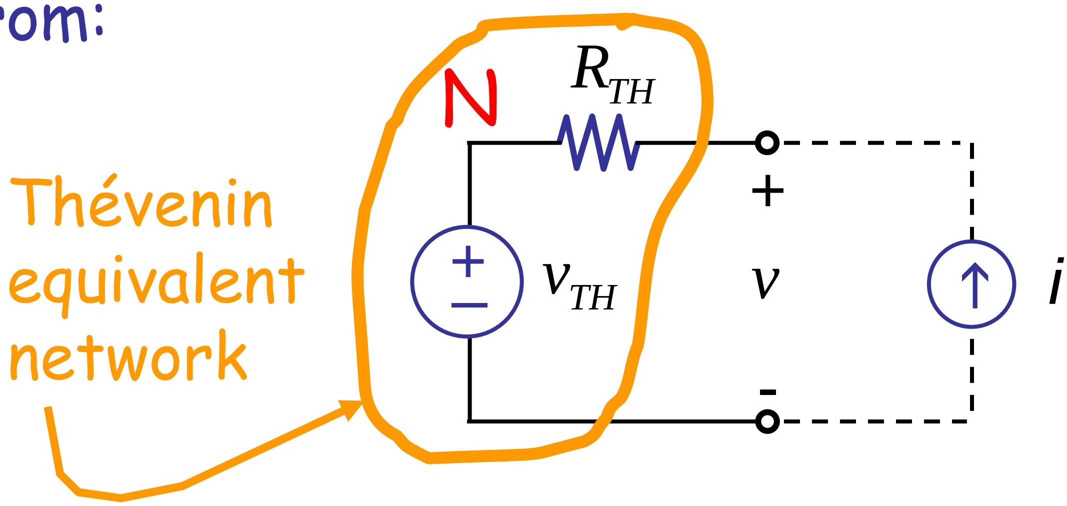
By setting $\forall_n I_n = 0$, $i = 0$ (no units); $\forall_m V_m = 0$, $i = 0$ (resistance units); All $\forall_n I_n = 0$, $\forall_m V_m = 0$

independent of external excitation and behaves like a voltage "v_{TH}"

Or

$$v = v_{TH} + R_{TH} i$$

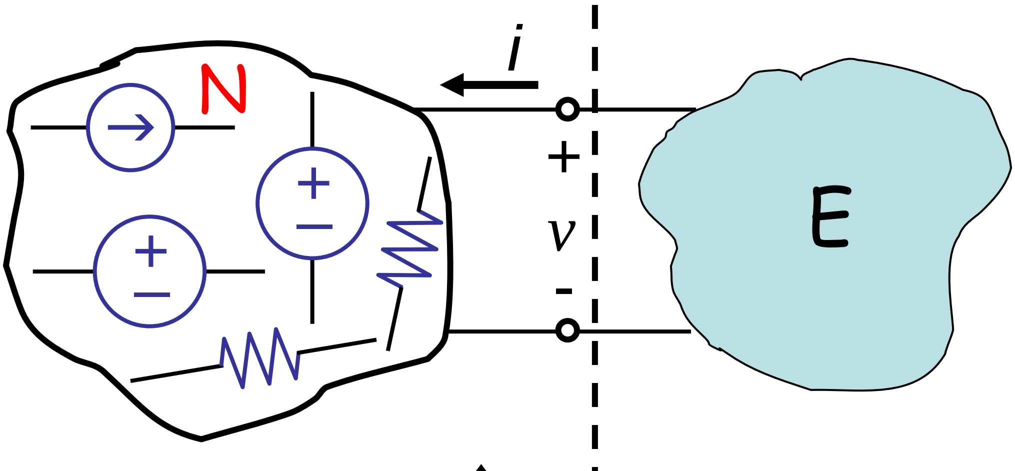
As far as the external world is concerned (for the purpose of I-V relation), "Arbitrary network N" is indistinguishable from:



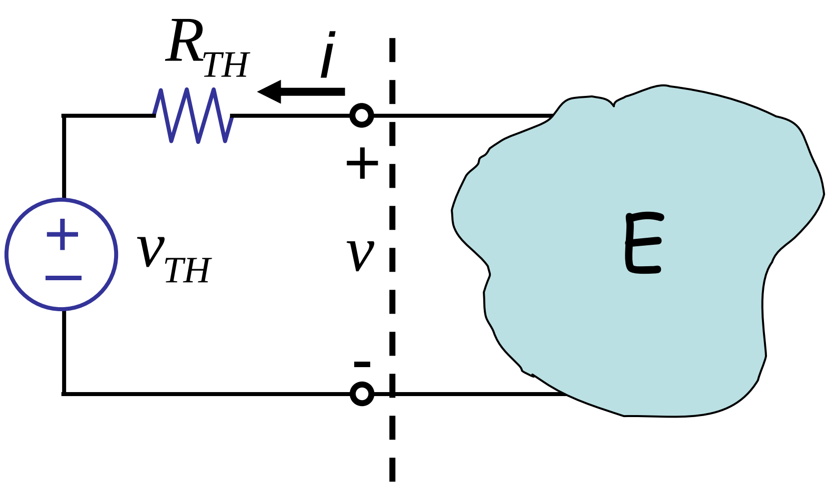
v_{TH} ⟶ open circuit voltage at terminal pair (a.k.a. port)

R_{TH} ⟶ resistance of network seen from port (V_m's, I_n's set to 0)

Method 4:
The Thévenin Method

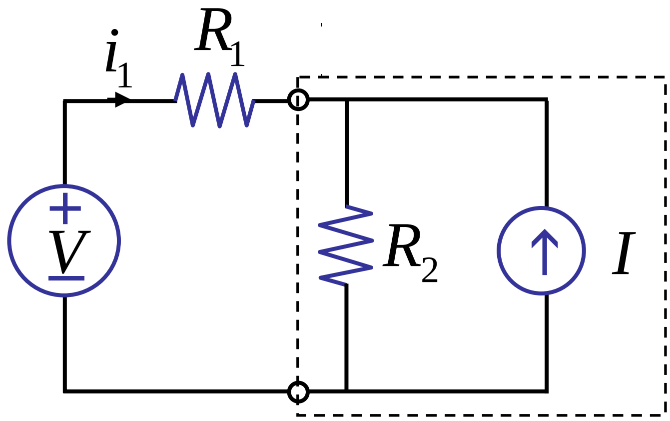


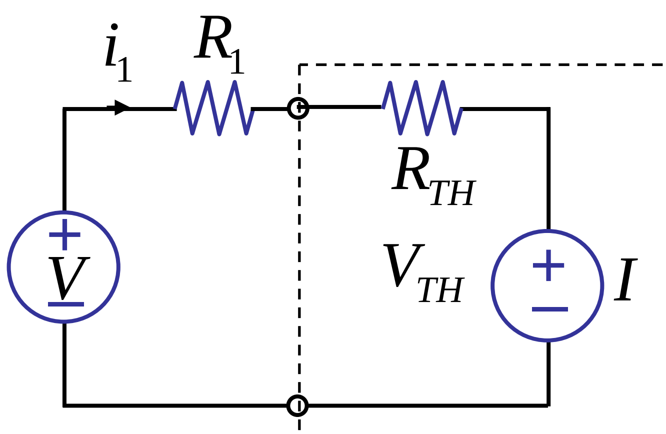
Thévenin equivalent



Replace network N with its Thévenin equivalent, then solve external network E.

Example:



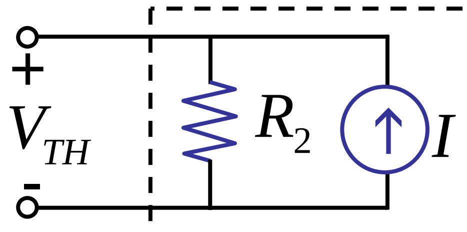


$$i_1 = \frac{V - V_{TH}}{R_1 + R_{TH}}$$

Example:

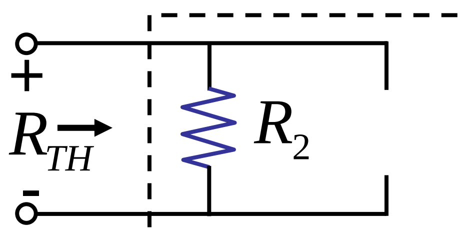
V_{TH} :

$$V_{TH} = IR_2$$

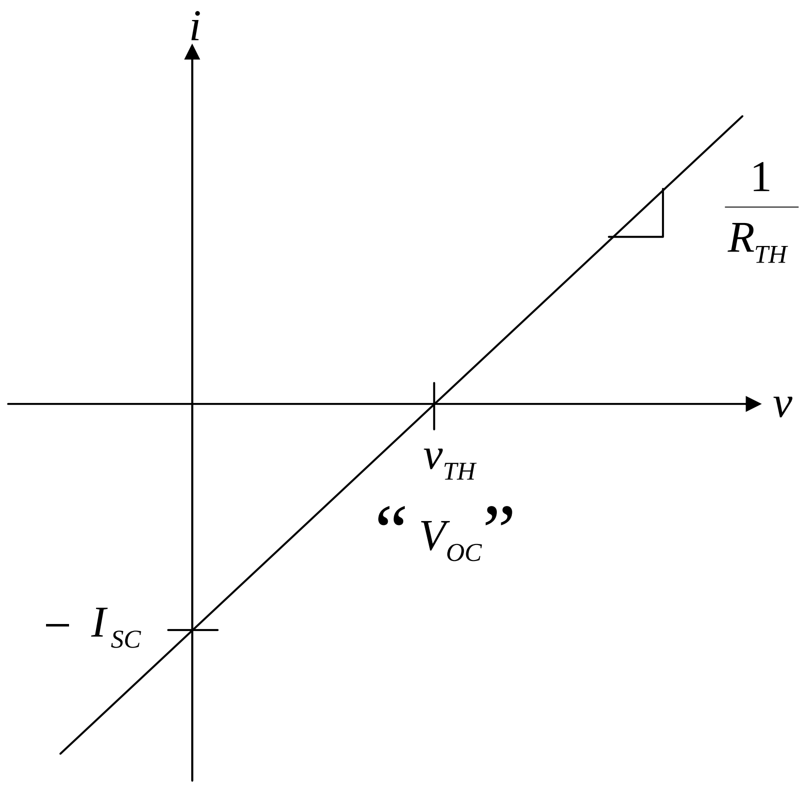


R_{TH} :

$$R_{TH} = R_2$$



Graphically, $v = v_{TH} + R_{TH}\, i$



Open circuit
$(i \equiv 0)$

$$v = v_{TH} \leftarrow V_{OC}$$

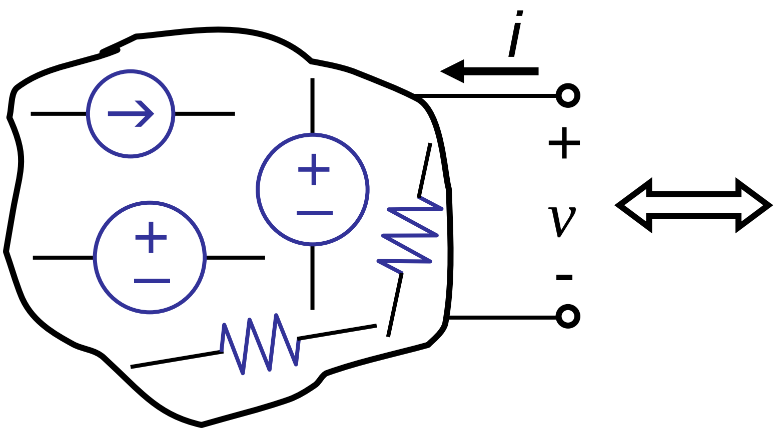
Short circuit
$(v \equiv 0)$

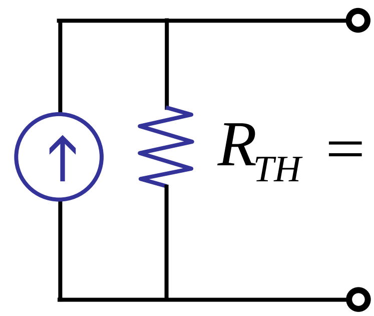
$$i = \frac{-v_{TH}}{R_{TH}} \leftarrow -I_{SC}$$

Method 5:

in recitation, see text

The Norton Method





Norton equivalent

$$I_N = \frac{V_{TH}}{R_{TH}}$$

Summary

- Discretize matter

  LMD → LCA

  Physics → EE

- R, I, V **Linear** networks

- Analysis methods (linear)
  KVL, KCL, I — V
  Combination rules
  Node method
  Superposition
  Thévenin
  Norton

- Next
  Nonlinear analysis
  Discretize voltage